Full Integration of a CGE model in a Microsimulation Model: A Recipe

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ABSTRACT: A method for full integration of a static Computable General Equilibrium model (a CGE model) in a Microsimulation model is proposed. The microsimulation model is a static model with a tax-benefit-module. Each household is described as a utility maximizing agent with a non-linear tax system and endogenous heterogeneous labor supplies. Conditions are specified such that sufficient aggregation is possible. The CGE model is a static model for a small open economy. It can have many good markets (up to several hundreds) and multiple labor markets. It is demonstrated that equilibrium in the total system can be calculated using 3 algorithms: 1) a Gauss-Seidel-like algorithm to solve for equilibrium good prices, 2) a linear algebra algorithm to solve for the quantities in the CGE-model and 3) an outer algorithm that calculates the wage rates and a public tax instrument that secures a balanced budget.

1 INTRODUCTION

When analyzing major economic policy reforms emphasis is put on both the macro effects and the distributional effects. Typically, the macro effects are measured through some kind of macro model, whereas the distributional effects are measured through a microsimulation model. The problem with this approach is that most aspects at the micro level is not considered at the macro level and vice versa. As an example, changes in relative wages at the macro level can have important impacts on the income distribution. Likewise, the way the public sector chooses to finance a given policy can have important distributional impacts. You could say, that if you for any macroeconomic policy change want to point out the losers and winners, you need a tool for distributional analysis.

An attractive way to solve this is problem is to integrate macroeconomic models and microsimulation models (Agenor et al., 2006; Cockburn, 2006; Cogneau and Robilliard, 2006; Bourguignon and Savard, 2008; Davies, 2009; Colombo, 2010; Cockburn et al., 2010). This integration can be more or less total. In this paper we are considering full integration. In the existing examples of a full integration, the microsimulation model is typically integrated into a macroeconomic version of a Computable General Equilibrium model (a CGE model; for an overview, see Cockburn et al., 2014). The strategy of the present paper is the opposite: we will propose a procedure to do a full integration of a static CGE model in a microsimulation model.

This implies that the researcher or programmer that builds the microsimulation model also has to build a CGE model. This may sound like a lot of work, but as will be argued it is actually a manageable project.

In the following pages we sketch a method for full integration. The model is not implemented except for a test implementation of one of the used algorithms. It is therefore not demonstrated how to calibrate the CGE model to national accounting data. This should be done using standard data calibration methods (Shoven and Whalley, 1992). The rest of the paper is organized as follows. In section 2 the model is outlined. The microsimulation model is a static model with a tax-benefit-module. Each household is described as a utility maximizing agent with a non-linear tax system and an endogenous heterogeneous labor supply. Conditions are specified such that sufficient aggregation is possible. The CGE-model is a static model for a small open economy. It can have many good markets (up to several hundreds) and multiple labor markets. In section 3 a method for solving the model is described. It is demonstrated that equilibrium in the total system can be calculated using 3 algorithms: 1) a Gauss-Seidel-like algorithm to solve for good prices, 2) a linear algebra algorithm to solve for the quantities in the CGE-model and 3) an outer algorithm that calculates the wage rates and a public tax instrument that secures a balanced budget. Section 4 is the conclusion.

2 THE MODEL SYSTEM

The system that is outlined here is kept as simple as possible. The model is therefore assumed to be *static*. It would be possible to make the system dynamic as long as the agents are not forward looking, i.e., base these actions on current available or past data. As the model is static, there is no capital or investments. The households do not save and do therefore consume all income. In this version there is no consumption- or production taxes, although these could easily be introduced into the model. We assume a simple public sector that has no employers and no good demand. The only function of the public sector is to issue taxes and pay transfers to the households. The model is closed by an assumption of a balanced public budget. Some tax parameter is endogenous to satisfy this (can be a lumpsum tax).

2.1 Microsimulation

We are considering an economy with multiple goods, labor types and households. Especially when it comes to the modelling of the households, the assumptions on functional forms are very important for the properties of the model. For the model to be tractable we need to be able to do some aggregation.

We assume that the utility of household i is given by:

$$U_i = u(c_i) - \sum_{j=1}^{n_i} \eta_{ij} l_{ij}^{\gamma_{ij}} \tag{2.1}$$

where $c_i = (c_{i1}, ..., c_{in})$ is a vector of consumed goods, n_i is the number of adults in the household i and l_{ij} is the labor supply of adult j in household i. The additivity of this utility function is a core assumption that (as we shall see) protects us against the worst aggregation problems. Observe that we assume the good-utility-function $u(c_i)$ to be the same for everybody, but that the disutility of labor can have heterogeneous elasticities γ_{ij}, e.g. differences among gender or income groups.

The budget is given by:

$$\sum_{s=1}^{n} P_s c_{is} = \sum_{j=1}^{n_i} \left(w_{ij} l_{ij} + TR_{ij} \right) - T\left(w_{i1} l_{i1}, ..., w_{i,n_i} l_{i,n_i}, TR_1, ..., TR_{n_i} \right) \equiv Y_i^D \tag{2.2}$$

where w_{ij} is the wage for adult j in household i, TR_{ij} is public transfers to adult j in household i and P_s is the price of good s. $T(...)$ is a general, potentially non-linear, tax function of all family members incomes. We assume that this wage depends on individual productivity and education (more on this later).

We assume that the utility function $u()$ is CES and the same for all households:

$$u(c_i) = \left[\sum_{s=1}^{n} \left(\gamma_s^c \right)^{\frac{1}{E_c}} c_{is}^{\frac{E_c - 1}{E_c}} \right]^{\frac{E_c}{E_c - 1}} \tag{2.3}$$

For given labor supply the right side of the budget (2.2) is given. It is well-known that maximizing a CES-function like (2.3) given the budget (2.2) yields:

$$c_{is} = \gamma_s^c \left(\frac{P_s}{P^C} \right)^{-Ec} \frac{Y_i^D}{P^C} \tag{2.4}$$

where the CES price index P^C is given by

$$P^C = \left[\sum_{s=1}^{n} \gamma_s^c P_s^{1-E_c} \right]^{\frac{1}{1-E_c}}$$

This is the consumer price index of the model. As we have assumed that all households has the same CES-utility of good-consumption, this price index is identical for all households such that we have a well defined macro consumer price index.

If the solution (2.4) is substitutet into (2.3) we have:

$$u\left(c_i\right)=\frac{Y_i^D}{P^C}$$

such that the households indirect utility function is given by

$$U_i=\frac{Y_i^D}{P^C}-\sum_{j=1}^{n_i}\eta_{ij}l_{ij}^{\gamma_{ij}}$$

or

$$U_i=\frac{\sum_{j=1}^{n_i}\left(w_{ij}l_{ij}+TR_{ij}\right)-T\left(w_{i1}l_{i1},...,w_{i,n_i}l_{i,n_i},TR_1,...,TR_{n_i}\right)}{P^C}-\sum_{j=1}^{n_i}\eta_{ij}l_{ij}^{\gamma_{ij}}$$

The labor supply is calculated by maximizing this utility function. Let us say the solution is given by:

$$l_{ij}=l_{ij}\left(w_{i1},...,w_{i,n_i},TR_{i1},...,TR_{i,n_i},P^C\right) \tag{2.5}$$

Substituting this into the definition of Y_i^D in (2.2) yields:

$$Y_i^D=Y_i^D\left(w_{i1},...,w_{i,n_i},TR_{i1},...,TR_{i,n_i},P^C\right)$$

If we aggregate the good demands (2.4) we get:

$$c_s=\gamma_s^c\left(\frac{p_s}{P^C}\right)^{-Ec}\frac{Y^D}{P^C} \tag{2.6}$$

where

$$Y^D=\sum_i Y_i^D\left(w_{i1},...,w_{i,n_i},TR_{i1},...,TR_{i,n_i},P^C\right) \tag{2.7}$$

The variable Y^D is total disposable income.

The micro simulation part of the model is a standard static model with a tax-benefit-module. The basic outputs from the microsimulation model is after-tax-income (determinant for consumption in the CGE part) and labor supply. The input to the microsimulation model is wage rates and some tax instruments.

We have E types of educations, and after aggregation of (2.5) we assume that the supply of labor with education e is given by:

$$L_e^S=L_e^S\left(W_1,...,W_E,P^C,\tau\right),\, e=1,...,E$$

where $W_1,...,W$ are the wage rates and τ is the tax instrument. We allow the supply of labor to depends on the wage rates and the tax instrument although it is not nescessary. In the calculation in the microsimulation model of $L_e(...)$, individual j (with education e) has the wage

$$w_j = \rho_j W_e$$

where ρ_j is calibrated from micro data. In this way the wage rates from the CGE-part $(W_1, ..., W_E)$ enters the microsimulation part.

After aggregation over all individuals we can calculate the total disposable income

$$Y^D = Y^D\left(W_1, ..., W_E; P^C, \tau\right)$$

Here we assume that the microsimulation part (for given values of the CGE wage rates $W_1, , , .W_E$, the consumer price index P^C and the tax rate τ) has been used to calculate the total income after tax. In this calculation the tax-benefit system as well as the individual wage incomes has been used. Assumptions on wage regulation of public transfers should also be added here.

2.2 The CGE Module

The CGE modul decribes a small open economy. As mentioned earlier, for simplicity there is no capital, investments or savings. The demand for each good is given by (2.6) repeated here:

$$c_i = \gamma_i^c \left(\frac{P_i}{P^C}\right)^{-E^c} \frac{Y^D}{P^C} \tag{2.8}$$

where the consumer price P^C is given as the CES price index and Y^D is total after tax income defined above.

$$P^C = \left[\sum_{i=1}^{n} \gamma_i^c P_i^{1-E^c}\right]^{\frac{1}{1-E^c}}$$

The demanded good c_i is split into demand after domestic and foreign goods:

$$c_i^D = \gamma_i^{cD} \left(\frac{p_i}{P_i}\right)^{-E^{cDF}} c_i \tag{2.9}$$

$$c_i^F = \gamma_i^{cF} \left(\frac{\overline{p}_i^F}{P_i}\right)^{-E^{cDF}} c_i$$

$$P_i = \left[\gamma_i^{cD} p_i^{1-E^{cDF}} + \gamma_i^{cF} \left(\overline{p}_i^F\right)^{1-E^{cDF}}\right]^{\frac{1}{1-E^{cDF}}}$$

where p_i is the domestic output price on good i and $\overline{p}_i^F$ is the foreign price.

The firms produce with materials (domestic and foreign) and the E types of labor as input. We assume the top-nest-demand is given by

$$M_i = \mu_i^{YM} \left(\frac{P_i^M}{p_i} \right)^{-E^Y} y_i \tag{2.10}$$

$$H_i = \mu_i^{YH} \left(\frac{P_i^H}{p_i} \right)^{-E^Y} y_i \tag{2.11}$$

$$p_i = \left[\mu_i^{YM} \left(P_i^M \right)^{1-E^Y} + \mu_i^{YH} \left(P_i^H \right)^{1-E^Y} \right]^{\frac{1}{1-E^Y}} \tag{2.12}$$

where M_i is an aggregate of inputs in sector i from all sectors, H_i is an aggregate of labor inputs in sector i, and (2.12) is the cost determined output price, given by a CES price index.

The input of good j in sector i is given by

$$x_{ji} = \mu_{ji}^x \left(\frac{P_{ji}^x}{P_i^M} \right)^{-E_i^M} M_i,\ j = 1, ..., n \tag{2.13}$$

where the CES price index P_i^M is given by

$$P_i^M = \left[\sum_{j=1}^{n} \mu_{ji}^x \left(P_{ji}^x \right)^{1-E_i^M} \right]^{\frac{1}{1-E_i^M}} \tag{2.14}$$

The demand for domestic and foreign goods are given by

$$x_{ji}^D = \mu_{ji}^{xD} \left(\frac{p_j}{P_{ji}^x} \right)^{-E^x} x_{ji} \tag{2.15}$$

$$x_{ji}^F = \mu_{ji}^{xF} \left(\frac{\overline{p}_j^F}{P_{ji}^x} \right)^{-E^x} x_{ji} \tag{2.16}$$

$$P_{ji}^x = \left[\mu_{ji}^{xD} p_j^{1-E^x} + \mu_{ji}^{xF} \left(\overline{p}_j^F \right)^{1-E^x} \right]^{\frac{1}{1-E^x}} \tag{2.17}$$

The demand for labor with education e in sector i is given by

$$L_{ei} = \mu_{ei}^H \left(\frac{W_e}{P_i^H} \right)^{-E^H} H_i,\ e = 1, ..., E \tag{2.18}$$

$$P_i^H = \left[\sum_{e=1}^{E} \mu_{ei}^H W_e^{1-E^H} \right]^{\frac{1}{1-E^H}} \tag{2.19}$$

We assume that export of good i is given by the Armington-specification (Armington, 1969):

$$X_i = \varphi_i \left(\frac{p_i}{\overline{p}_i^F} \right)^{-E^{Ex}} \tag{2.20}$$

Equilibrium in the good markets implies:

$$y_i = \sum_{j=1}^{n} x_{ij}^D + c_i^D + X_i, i = 1, ..., n \tag{2.21}$$

Equilibrium in the labor markets implies

$$L_e^S = \sum_{j=1}^{n} L_{ej} \tag{2.22}$$

The public sector only has costs on transfers to households. Its income is the revenue from τ. We assume that τ makes sure that the public budget is in balance.

3 SOLVING THE MODEL SYSTEM: THE ALGORITMS

There are 3 parts in the solution of the model system. We start by assuming we know the wage rates and the endogeneous tax rate (the tax rate that makes sure that the public budget is balanced). In algorithm 1 the good prices are calculated given the wage rates. This is done by an iterative process using the CES price indices. When we know the wage rates and the good prices we determine the good and labor quantities in algorithm 2. This is basically a question of linear algebra. The final algorithm redo the first two in a iterative process to determine the wage rates and the endogeneous tax rate. The only time consuming element in this process is the microsimulation model. It is therefore expected that this approach takes 10-20 times longer then running the static microsimulation model.

3.1 Algorithm 1: Determination of good prices

If you substitute the CES price indexes (2.14), (2.17) and (2.19) into (2.12) you get something like:

$$p_i = \phi_i \left(p_1, ..., p_n, \overline{p}_1^F, ..., \overline{p}_n^F, W_1, ..., W_E \right), i = 1, ..., n \tag{3.1}$$

For given values of the wage rates $(W_1, ..., W_E)$ and the foreign prices $(\overline{p}_1^F, ..., \overline{p}_n^F)$ you have a fix-point problem. This problem can be solved by an iterative Gauss-Seidel-like process:

$$p_i^t = \phi_i \left(p_1^{t-1}, ..., p_n^{t-1}, \overline{p}_1^F, ..., \overline{p}_n^F, W_1, ..., W_E \right), i = 1, ..., n \tag{3.2}$$

The method is very simple to implement. You start with some initial values of prices $p^0 = \left(p_1^0,...,p_n^0\right)$. For given values of forign prices and wages you can calculate p^t from (3.2) again and again. After a while p^t converges to a vector that satifies (3.1).

We will demonstrate on artificial data that this method is very efficient (R-code is in appendix). Even with many hundred sectors and many educations, the method converge in less than 50 iterations.

The actual implementation of (3.2) is given by:

$$P_i^{H,t} = \left[\sum_{e=1}^{E} \mu_{ei}^H W_e^{1-E^H}\right]^{\frac{1}{1-E^H}} \tag{3.3}$$

$$P_{ji}^{x,t} = \left[\mu_{ji}^{xD}\left(p_t^t\right)^{1-E^x} + \mu_{ji}^{xF}\left(\overline{p}_j^F\right)^{1-E^x}\right]^{\frac{1}{1-E^x}} \tag{3.4}$$

$$P_i^{M,t} = \left[\sum_{j=1}^{n} \mu_{ji}^x \left(P_{ji}^{x,t}\right)^{1-E_i^M}\right]^{\frac{1}{1-E_i^M}} \tag{3.5}$$

$$p_i^t = \left[\mu_i^{YM}\left(P_i^{M,t-1}\right)^{1-E^Y} + \mu_i^{YH}\left(P_i^{H,t-1}\right)^{1-E^Y}\right]^{\frac{1}{1-E^Y}} \tag{3.6}$$

To get some data, we choose randomly $\mu_{ei}^H, \mu_{ji}^{xD}, \mu_{ji}^{xF}, \mu_{ji}^x, \mu_i^{YM}$ and μ_i^{YH} such that for all i and j in $(1,...,n)$:

$$\sum_{e=1}^{E} \mu_{ei}^H = 1$$

$$\mu_{ji}^{xD} + \mu_{ji}^{xF} = 1$$

$$\sum_{j=1}^{n} \mu_{ji}^x = 1$$

$$\mu_i^{YM} + \mu_i^{YH} = 1$$

This would be that case if the μ's was calibrated from a system where we assumed that all prices and wages equals 1. This is a standard assumption i CGE modelling.

If

$$W_e = 1$$

and

$$\overline{p}_i^F = 1$$

we therefore knows that we have an equilibrium is and only if:

$$p_i = 1$$

Solution time (sec.)	0.0294
Number of iterations	8.80

Table 3.1: Average of solving the system 1.000 times

We can therefore test our system by starting with a p^0 with elements far from 1, and then calculate succicive values of p^t from (3.3)-(3.6). We assume that we have a large CGE model with 100 sectors and 10 educations:

$$n = 100$$

$$E = 10$$

The error of the agoritm is calculated by Mean Absolute Error (MAE):

$$MAE = \frac{1}{n}\sum_{j=1}^{n}|p_j - 1|$$

The algorithm runs until $MAE \leq 0.0001$.

The entire setup (random draws of μ's and initial p^0 and solving the system) is run 1.000 times on a standard labtop. The result is shown in table 1. Solving for the 100 equilibrium prices takes less than 0.03 seconds. The number of iterations is typically 9 and 8 on rare occations.

3.2 Algorithm 2: Determination of quantities

In this section we assumes that we know the wage rates and the good prices (derived in last section). Equilibrium in the good markets implies that:

$$y_i = \sum_{j=1}^{n} x_{ij}^D + c_i^D + X_i$$

Substituting (2.15), (2.13) and (2.10) implies:

$$y_i = \sum_{j=1}^{n} a_{ij} y_j + f_i \tag{3.7}$$

where

$$a_{ij} \equiv \mu_{ij}^{xD}\left(\frac{p_i}{P_{ij}^x}\right)^{-E^x}\mu_{ij}^x\left(\frac{P_{ij}^x}{P_j^M}\right)^{-E_i^M}\mu_j^{YM}\left(\frac{P_j^M}{p_j}\right)^{-E^Y} \tag{3.8}$$

and

$$f_i \equiv c_i^D + X_i$$

The vector f_i defines the final demand. If we know the total disposal income Y^D (form the microsimulation model) and the good prices, the final demand is given according to (2.8), (2.9) and (2.20).

In matrix notation we can write (3.7) as:

$$y = Ay + f$$

The elements in the matrix A is given by (3.8). For known good prices these elements are constants. We can therefore calculate the quantities y by

$$y = (I - A)^{-1} f$$

The demand for labor can be derived from (2.18) and (2.11):

$$L_{ei} = \mu_{ei}^{H} \left(\frac{W_e}{P_i^H} \right)^{-E^H} \mu_i^{YH} \left(\frac{P_i^H}{p_i} \right)^{-E^Y} y_i$$

3.3 Algorithm 3: Clearing the labor markets and budget constraint of the public sector

Equilibrium in the labor market is defined as

$$L_e^S (W_1, ..., W_E) = \sum_{i=1}^{n} L_{ei}$$

Both sides of the equation is dependent on the wage rates. After running algorithm 1 and 2 we well typically have that

$$L_e^S (W_1, ..., W_E) \neq \sum_{i=1}^{n} L_{ei}$$

Similarly the public budget will typically not be in balance. We therefore need to find $(W_1, ..., W_E)$ and τ such that the labor markets are in equlibrium and such that the public budget is balancen. This is $E+1$ equations with $E+1$ unknown. If we only have 1 labor marked (which is often the case) we have reduced the huge model to 2 equations with 2 unknown! This could actually be solved by trial and error.

In the general case we need to embed algorithm 1 and 2 in a loop that solves this problem. Some opens source equation solver can be used (java, R, Python or what ever). Preferable a solver method that do not need the calculation of the Jocobi matrix. Typically there will be less than 10 educations in the model. We therefore have less than 11 variables to determine in algorithm 3.

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Appendix A: R-code to test Gauss-Seidel-like method for calculation of equilibrum prices in the CGE model

```
n = 100  # Number of sectors
E = 10   # Number of educations
M = 1000 # Number of measurements
itt = rep(NA, M)
dt = rep(NA, M)

for(j in 1:M)
{
    print(j)

    # Artificial data and calibration
    myYM = runif(n)
    myYH = 1 - myYM
    myH = matrix(runif(n*E), ncol=E)
    myH = myH / rowSums(myH)
    myx = matrix(runif(n*n), ncol=n)
    myx = myx / rowSums(myx)
    myxD = matrix(runif(n*n), ncol=n)
    myxF = 1 - myxD

    # Initial wages and forign prices = 1
    w = rep(1, E)
    pF = rep(1,n)

    # Initial prices NOT equal to 1
    p = 0.5 + runif(n)

    # Elasticities of substitution
    Ex = 0.8
    EM = 1.5
    EH = 1.5
    EY = 0.8

    errOK = 0.0001
    err=1000
    i=0
    t0=Sys.time()
    while(err>errOK)
    {
        Px = (myxD*p^(1-Ex) + myxF*pF^(1-Ex))^(1/(1-Ex))
        PM = (rowSums(myx*Px^(1-EM)))^(1/(1-EM))
        PH = rowSums(myH*w^(1-EH))^(1/(1-EH))
        p = (myYM *PM^(1-EY) + myYH * PH^(1-EY))^(1/(1-EY))

        err = sum(abs(p-1))/n
        i=i+1
    }
    itt[j] = i
    dt[j] = Sys.time()-t0
}

mean(itt) # Mean number of iterations
mean(dt)  # Mean time use
```