Earned Income Tax Credit in a Disaggregated Labor Market with Minimum Wage Contracts*

Lars Haagen Pedersen and Peter Stephensen

November, 1999

Abstract

An earned income tax credit is introduced in a dynamic CGE model with overlapping generations and imperfectly competitive labor market segments. The model is calibrated to a 10 percent sample of a register of all persons and workplaces in Denmark and scaled to the level of National Accounts. Each labor market segment is covered by a collective bargaining agreement, except one that is competitive. In all segments a distribution of workers with different productivity exists.

A tax credit, which increases the net income of the poorest full-time employed workers in the economy by DKK 5000, reduces unemployment by 2.5 percentage points. Since the marginally employed have a productivity lower than the average worker, employment measured in productivity units increases only by approximately 1.5 percentage points. Women and younger generations tend to gain from the policy.

The effect of the policy depends crucially on the initial level of unemployment because the productivity distributions are highly non-linear: if the initial level of unemployment is halved to 5% so are the effects of unemployment and employment.

JEL classification: D58, J51, H24, D91.

---

* A short version of the present paper focusing on the results and basic ideas and omitting the documentation of the full modelling and calibration of the disaggregated labor market is forthcoming in Harrison, Jensen, Pedersen and Rutherford (eds.): *Using Dynamic General Equilibrium Models for Policy Analysis* from North Holland Publishers.

We wish to thank an anonymous referee (to the short version of the paper) for comments and suggestion that also have benefited this paper. Thanks are also due to Morten Lobedanz Sørensen for research assistance. E-mail: lps@dst.dk and psp@dst.dk. Address: DREAM Model; Statistics Denmark; Sejrøgade 11; DK-2100 Copenhagen; Denmark.

Table of Contents:

1. Introduction

Persistent unemployment has been a major problem in most European countries for almost three decades. Therefore, measures that potentially reduce this structural unemployment problem are on the political agenda in most European countries. During the 1990s earned income tax credit (EITC) schemes have become an issue in the European unemployment policy discussion (see e.g. European Commission (1994) and OECD (1996)).[1]

The expected employment effects of an EITC scheme depend crucially on the institutions in the economy into which the scheme is introduced. In this paper we will argue that the institutions of the Danish labor market are likely to generate relatively large employment effects of EITC schemes. The basic reasons for this are that the Danish labor market is highly organized and that bargaining contracts are specified as minimum wage contracts. Furthermore, Danish pre-tax unemployment benefits are indexed to the pre-tax wage rate, and therefore, introduction of an EITC is a way to reduce the replacement ratio (the ratio of unemployment benefits after tax to wages after tax) for a given pre-tax wage.

The literature on the employment effects of EITC schemes focuses on the following issues: The effect of the EITC scheme on the replacement ratio, the financing of the EITC scheme, and the effect of changes in the progressivity of the tax system on wage formation. To these issues the present paper adds the type of wage contract considered.

The interaction between the tax system and the unemployment benefit system is the topic in, e.g., Bovenberg and Van der Ploeg (1994) and Pissarides (1998). These papers consider the effect on unemployment of reductions in the employment tax rate and find that it depends crucially on the indexation rules of the unemployment benefit scheme. If unemployment benefits (after-tax) are indexed to the after-tax wages, then reducing the average tax on wages is not likely to reduce unemployment significantly. The reason is that the increase in wages after tax is passed on the benefits after tax, and therefore, the incentives to become employed are not increased from a lower replacement ratio. However, if utility from leisure is non-zero, then unemployment may still be reduced by the reduction in the average tax on wage income as the total reward from being employed has increased relatively to the alternative. Nevertheless the first policy conclusion is that employment tax cuts are (most) effective in reducing unemployment if they lead to a reduction in the replacement ratio.

[1] Also in Denmark various EITC proposals have been discussed (see e.g. the Economic Council (1997) and Danish Ministry of Business *et al.* (1996)).

The second issue with respect to the employment effects of the EITC scheme is the financing of the scheme. The analysis of Pissarides (1998) explicitly abstracts from the fact that revenue requirements of the public sector might be fixed. Introducing the constraint that the revenue is fixed leads to the question whether shifting the tax burden from labor (or specifically from employed labor) to other parts of the economy may decrease unemployment. This is one of the themes in, e.g., the discussion of the so-called green tax reforms; see, e.g., Nielsen, Pedersen and Sørensen (1995), Bovenberg and Van der Ploeg (1998), Holmlund and Kolm (1999) for a discussion of the possible positive employment effects of green tax reforms.

In the present analysis, we abstract from the possibility of reducing the employment tax by shifting the tax burden to other types of taxes and focus solely on a restructuring of the income tax system. This leads to the third issue with respect to the effect of the EITC scheme, which is the potential employment effects of changing marginal versus average tax rates. Recent literature on progressive taxation (see Lockwood and Manning (1993) and Bovenberg and Van der Ploeg (1994)) yield the general result that increased marginal tax rates reduce the wage rate, and increased average tax rates increase the wage rate. These results hold for imperfect labor markets with exogenous labor supply, where the imperfection is caused by either the presence of unions, the existence of efficiency wage formation, or the presence of frictions in the labor market (the so-called search models of unemployment). Sørensen (1997) provides an overview of the theoretical effects in various standard models.

In the theoretical literature, the change in the marginal tax rate is considered a marginal effect, i.e., the average tax rate is assumed constant. To implement these results in real world tax systems is no simple task. In standard theoretical models, where the wage rate is unique, a constant average tax rate for a variable marginal tax rate is obtainable by having two tax instruments, e.g., a simple linear (progressive) tax system with a marginal tax rate and a fixed income treshold. In real world data, where a distribution over wage income exist, the number of tax instrument is, however, not sufficient to keep the average tax rate constant for all income groups when the marginal tax rate of a given income group is changed. Pedersen, Smith and Stephensen (1998) consider a simple linear tax system and a continuous distribution of wages and defines a policy experiment that increases in the marginal tax rate for a fixed average tax rate of the employed with the lowest wage income. Given a set of labor market institutions that are similar to the one used in the present paper, including bargaining over minimum wages, it is shown that if employers' bargaining power is sufficiently high, then the increase in the marginal tax rate may increase the minimum wage, and therefore the total wage of all employed persons. This leads to the conclusion that if a distribution over wages exist and there is a limited number of tax instruments, then restructuring the income tax system by introducing an employment tax credit and increasing the marginal tax rate need not generate a general reduction in the wages in the economy and therefore also not a general increase in employment.[2]

The qualification implies that the effects of marginal and average tax rates on wage formation become an empirical question. Wage equations containing measures of average and marginal tax rates have been estimated for several countries, see Sørensen (1997) for an overview of results. The estimated relations are generally based on macroeconomic time series with relatively few observations. Therefore one should probably not emphasize the results too much. However, in general the standard result of the imperfectly competitive model is verified. An exception is found in Hansen, Pedersen and Sløk (1999), where a higher marginal tax rate tends to increase the wage rate for white collar workers in Denmark. Pedersen *et al.* (1998) estimate wage equations for different union groups based on panel data of 1 percent of the Danish population from 1981-1990. These estimations based on micro data tend to give a significantly lower effect of taxation on wage formation than wage equations based on macroeconomic time series. Furthermore, the results concerning the effect of the marginal tax rate are mixed. For female workers the increased marginal tax rate tend to have a positive effect on the wage, whereas the opposite is true for most male groups. Overall the effect of the marginal tax rate is small.

If the progressivity effects on wage formation are small, one may infer that an EITC scheme that is financed through restructuring of the income tax system does not necessarily lead to a significant increase in employment if the replacement ratio is not reduced and if no part of tax burden is shifted away from wage income. Positive employment effects will mainly appear through composition effects between different groups of employed. Sørensen (1997) simulates a static CGE model with three types of labor (unskilled, skilled, and white collar workers), three official sectors and an underground economy. Sørensen considers an EITC scheme that amounts to one percent of the wage income of low wage earners and two different financing schemes. The first financing schemes includes a higher marginal tax rate on labor and transfer income. Given this financing rule

[2]An additional qualification to the general result of increased progressivity in imperfect labor markets is given in Hansen *et al.* (1995, 1999) where the union and the employers' confederation bargain over both the wage and the length of the working day. In this case increasing the marginal tax rate for given average tax rate may increase the wage rate if the labor supply elasticity sufficiently low and the bargaining power of the employers confederation sufficiently strong.

the EITC increases the replacement ration and shifts a part of the tax burden away from the wage earners. Depending on the elasticities of the wage equation with respect to tax changes, the reduction in the official unemployment is between 0.1 and 0.6 percentage points. Given the second financing rule, the EITC does not shift the tax burden to away from wage earners and maintains the real value of the unemployment benefits after tax. In this case the simulation reveals that if progressivity effects on the wage formation are sufficiently strong then unemployment is still reduced by 0.4 percents. If progressivity effects are weak, the reduction is turned into an increase in unemployment by 0.1 percent.

This latter result is rather discouraging from the point of view of economic policy as EITC is introduced to increase employment while maintaining the standard of living for individuals receiving unemployment benefits and individuals receiving other transfers. The analysis presented here strikes a more positive note.

From the presentation of the previously reported results, it is obvious that the assumptions concerning the tax effects on the wage curve are crucial to the results. As the empirical estimates of these effects differ both in the direction and the size the present analysis takes a different route by explicitly calculating tax effects for the whole distribution of wages. Therefore, the main new aspects of the present analysis is an introduction of heterogeneity in the form of distributions of individual productivity within a given labor market segment, and a bargaining institution that is in line with Danish labor market institutions by having bargaining over minimum wages. A related major difference is the representation of the income tax system. In previous studies, like Sørensen's (1997), the income tax system is presented by a marginal and an average tax rate in the wage curves representing the bargaining behavior in each labor market segment. In our modelling a distribution of wage income exists in each labor market segment. For a given distribution of gross wage incomes a distribution of wage incomes after tax is calculated using a representation of the entire income tax system in Denmark. This is difference particularly relevant in policy evaluation like the present one, where the structure of taxation is changed using a limited number of tax instruments. This type of policy shifts the tax burden from one part of the wage distribution to another and therefore, union behavior is depends upon the shape of the distribution of wage income for the particular union in question. Union behavior may be affected even if both the marginal and the average tax rate of average union member is unaffected.

The introduction of bargaining over minimum wages imply that (long term) unemployment tend to be concentrated among individuals with low levels of productivity within each labor market segment. This in line with data from Danish labor market and the main reason why EITC schemes is considered a potential policy instrument. An EITC scheme becomes a more effective instrument if unemployment is concentrated at the low income groups as the scheme is designed to increase the employment incentives of this particular group of individuals. Therefore, ceteris paribus, our analysis tends to increase the positive employment effects of EITC schemes relative to previous analyses.

The type of EITC scheme considered here is defined as an extra income threshold to wage-earners. In addition, we reduce the proportional labor market contribution tax rate by 1 percentage point. The scheme is financed by progressively increasing in the marginal tax rates of the Danish personal income tax system of 1995. High income wage earners are net contributors to the financing of the tax credit. The EITC introduces an extra income treshold for wage earners of 11,250 DKK. This policy shock is 5 to 6 times larger than the shock in Sørensen (1997).

The EITC scheme and the associated financing rule reduces the replacement rate and generates a minor increase in the real unemployment benefits after tax. Therefore employment incentives are increased without reducing the standard of living of unemployed persons. There is no direct shifting of the tax burden from wage earners to other groups. However, the EITC scheme and the associated financing rule generates an increase in the public deficit in short run, whereas, it is fully financed in the long run. Therefore, dynamic effects are part of the financing of the scheme. These effects include increased revenues from indirect taxes, due to a long run increase in consumption. Observe that the revenue requirement is not fixed, since the automatic reduction in expenditures from the increased employment is part of the financing of the EITC scheme. In the long run, this financing rule is comparable to the most restrictive case in Sørensen (1997).

The result of our analysis is that introduction of the EITC leads to a reduction in the minimum wages in all segments of the labor market. This generates increased employment of individuals with low productivity in the economy. Therefore, average productivity is reduced, so that employment measured in number of individuals increases significantly more than employment in productivity corrected units. Therefore, production is less affected by this type of policy than unemployment measured in persons. In this way the EITC scheme is a (partial) cure for unemployment for the low income groups.

The size of the EITC scheme in the present simulation is large. Given the initial level of activity and income distribution, the revenue effect of the unfinanced scheme is 2.3 per cent of GDP in 1995. Our base simulation results are that unemployment is reduced by 2.5 percentage points both in the long and in the short run. Employment measured in productivity corrected units is increased by 1.5 percentage points. The distribution of income after tax becomes more compressed after the introduction of the tax credit, as the high-income earners experience a deterioration of the net income, whereas, the low-income group experience a gain with respect to net income. Concerning the distribution of income across gender and age the result is that the average woman, in general, becomes better off, irrespectively of her age. The same is not true for men. The average man in the forties and fifties experiences a loss in net income, and the gain for the average man in the younger generations is lower than the gain for the average women belonging to the same generation. These distributional effects appear because the average man in every generation, a priori, has a higher net income than the average woman. Especially the negative effects experienced among men in the forties and fifties are due to the fact that the high-income group in the society is concentrated in these age groups of men.

Due to the non-linear distribution of productivity the effect of the EITC scheme becomes highly dependent on the initial level of unemployment in the present analysis. Therefore, an alternative scenario, which has a level of unemployment which is approximately half of the size in the base scenario, has been analyzed. In this case the increase in employment is reduced to 0.6 percent while in rate of unemployment is reduced by 1.4 percentage points. Thus in this case the effect approximately half the size as the effect in the base scenario.

Comparing to the macroeconomic results in Sørensen (1997) by scaling the results 5 times to correct for the size of the policy change, one finds that measured in the productivity corrected units of employment our base scenario effect is smaller than the optimistic case of the similar financing scheme in Sørensen (1997). Measured in the reduction in unemployment, our base scenario effect is larger than the effect in Sørensen.[3]

The rest of the paper is organized as follows: Section 2 gives an overall description of the Danish labor market, including a general characterization of the 8 segments that the labor market is divided into in the present analysis. The size of each segment measured in number of individuals and the wage distributions extracted from the empirical sample are presented. The theoretical model of the labor market is presented formally in the second half of the section. The macro economic features of the DREAM model is described in section 3, whereas the

[3] The Danish Economic Council (1997) finds that an EITC that is limited to low income groups and has a financing rule which is comparable to the rule in the present experiment will generate a 1.3 percentage point increase in employment. The revenue effect of the scheme is approximately half of the effect of the scheme presented here. The analysis of the Danish Economic Council is based on an adjusted version of the model in Sørensen (1997).

calibration of the labor market is extensively described in Appendix 1. The specific earned income tax credit scheme and the effects from introducing the scheme is presented in section 4 and 5 respectively. Section 5 also includes a discussion of the sensibility with respect to central assumptions. Concluding remarks are given in section 6.

2. The modelling of the Danish labor market

The Danish labor market is characterized by high rates of organization on both sides of the market. Union coverage measured as union members relative to the total number of wage earners is 88.0 percent in 1997. Total coverage of firms organized in employers' associations is 52.7 percent in 1997. (Danish Employers' Confederation; 1998). The coverage of collective bargaining agreements defined as the number of employed in firms where more than 50 percent of the work force is covered by collective bargaining agreements relative to the total employment, was 83 percent (Statistics Denmark; 1997).

The employers' associations are organized according to line of business. Traditionally the trade unions are organized as crafts unions. However, a shift towards organization according to industry has been made in some unions. Therefore, the union structure in Denmark is a mix of organization according to crafts and organization according to industry.

Collective bargaining agreements are decentralized in the sense that the employers' associations tend to have an agreement with each union, and these agreements are negotiated individually between the two parties. Bargaining agreements covering non-piecework contracts include 3 different types of wage formation: 1) A standard wage contract where the wage of a given category of workers is set in the contract. 2) A minimum wage contract, where collective bargaining determines the minimum wage. The total wage of each employee is given as the sum of the minimum wage and a personal raise. 3) A contract without collective bargaining over the wage. In this case, the wage is determined by local negotiations within a specific firm. The negotiation may be a purely personal matter between the employee and the firm, or it may be a bargaining between the shop steward and the managers of the firm.

Table 1 shows the coverage of each of the three types of contracts in the three last rounds of collective bargaining between member associations of DA and unions organized in LO from 1993, 1995, and 1997/98.[4] The minimum wage contract

[4] DA is the Danish Federation of Employers, and LO is Danish Federation of Trade Unions,

is, by far, the most widespread type of contract. The coverage is measured as number of employees with a given contract relative to the number of employees with a non-piecework contract in firms organized in DA.

	1993	1995	1997/98
Standard wage	16	16	16
Minimum wage	80	73	67
Without wage bargaining	4	11	17

Source: Danish Employers' Confederation (1998)

Table 1: The coverage of different types of contracts.

The Danish labor market is modelled as an imperfectly competitive and segmented market with bargaining over minimum wages. Wage formation is specific for each segment. Therefore, segments reflect the coverage of the main bargaining agreements. The private labor market of unskilled workers is divided into five bargaining agreements, each covering a specific type of craft. The types are: Manufactural workers, clerical workers, shop assistants, tradesmen, and unskilled workers. In addition to these 5 labor market segments, the model contains a segment of white collar workers that consists mainly of employees in the financial sector; a segment for pure public workers, who negotiate their wage contracts with the employers in the public sector; and a "high income segment" defined as employees with an academic master's degree and company managers. The latter segment is assumed to be perfectly competitive.

The data used to generate the empirical distribution of employment in each of the 8 labor market segments are based on a 10 percent sample for 1995 taken from the Danish longitudinal data base, IDA. IDA is register-based database of all persons and all workplaces in Denmark. (See Statistics Denmark; 1991).

Individuals are distributed across the 8 segments of the labor market according to the DISCO-classification code (Danish version of International Standard Classification of Occupations), which is attached to each observation. The DISCO-code is a 4-digit code specifying the occupation based on level of education, the line of business and the type of job. (See Statistics Denmark; 1996) Table 2 summarizes the results of the depiction from DISCO-code to labor market segments and shows the relative size of the of each labor market segment. The table also shows the average observed wage rate for each segment. Appendix 2 gives a documentation of the DISCO codes attached to each labor market segment.

that is, a federation of unions organizing blue-collar workers.

	Number of observations		Average wage (DKK)
Manufactural workers	37.501	19%	139
Clerical workers	32.539	17%	128
Shop assistants	18.989	10%	124
Tradesmen	11.343	6%	139
Unskilled workers	17.591	9%	122
Pure public workers	44.230	23%	133
High-income workers	23.104	12%	203
White collar workers	7.778	4%	163
Total/average	193.075	100%	141

Table 2: Summary of empirical input

Empirical wage distributions are constructed on the basis of information of hourly wages for each individual in the sample. These distributions are shown in figure 1. The distributions are shown as discrete columns, where the first column represents the average wage of the 4 percent of workers who has the highest hourly wage, the second column represents the average wage of the next 4 per cent and so on.

In each segment with imperfect competition an employers' associations and a union bargains over minimum wages, taking as given minimum wages in other labor market segments. The bargaining is modelled using the Nash-bargaining solution; see Binmoore, Rubinstein and Wolinsky (1986). The element that distinguishes the present modelling from the standard Nash-bargaining model is the fact that individual members of a given unions receive individual wages. The wage of an individual worker is the sum of the minimum wage and a personal raise. The personal raise depends upon the productivity of the worker in question. The productivity of a specific worker is observable both to the union and the employers.

Bargaining solutions in dynamic models may involve a discussion of commitment and the question of time-consistency, see e.g. Van der Ploeg (1987). In the present analysis, we employ the dynamic CGE model of Statistics Denmark, DREAM (see below for a presentation of the model). The fact that the firm faces convex cost of installation of capital in DREAM implies that the question arises in the present analysis. This is because the investment decision of the firm *inter alia* depends on the vector of future wages when the firm faces convex cost of installation of capital.

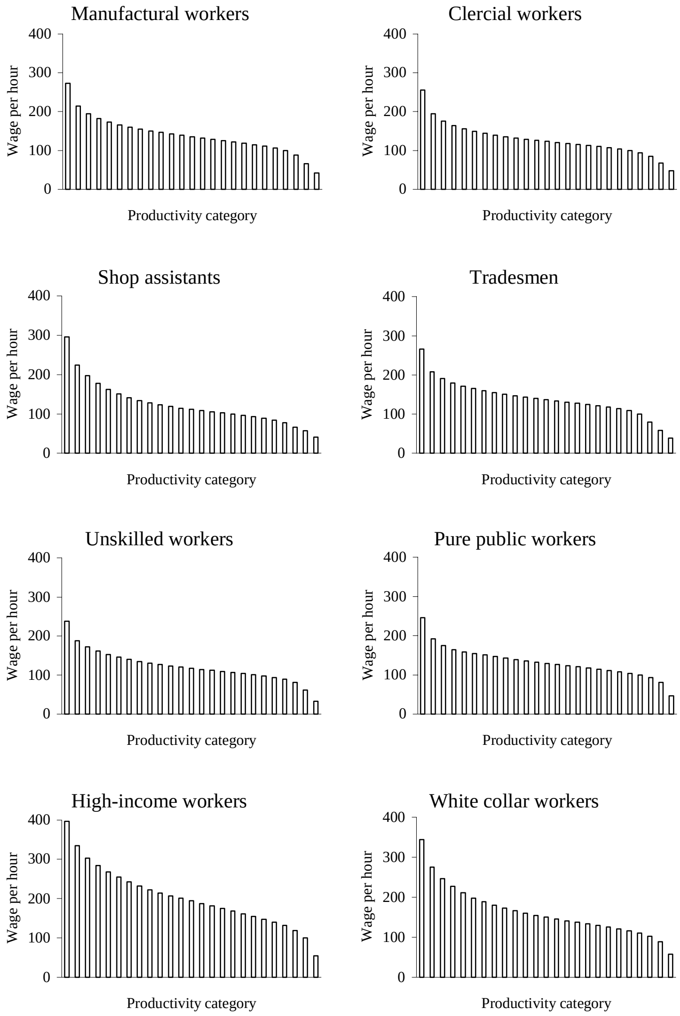


Figure 1: Wage distributions

Therefore, a wage profile with low future wage rates increases the incentive to invest (ceteris paribus). However, once the capital has been installed, the low wage rates are not time-consistent with respect to the object of the union, since the union can exploit the fact that it is costly for the firm to reduce the capital stock. If the union is not committed to the future wage rates, then it may, under these circumstances, be optimal to break the contract and raise the wage claims. In this analysis we assume that all wage contracts are time consistent, i.e., there is no commitment to sign wage contracts which last more than one period (i.e. 5

years). This implies that the equilibrium is sub-game perfect.

Finally, we assume that the firms have a "right to manage" so that for a given wage rate, the employment decision is made unilaterally by the firm.

In the next subsections, the theoretical model of the labor market is presented. For a less formal description and a simplified version of the labor market structure, see Pedersen, Smith and Stephensen (1998).

2.1. Demand for labor in the private sector

Figure 2 illustrates the production technology of the private producers. Private production is not divided into sectors. The upper part of the figure outlines the assumed production function, which is specified as two-factor nested CES function with a nest structure as indicated in the figure. Gross output is produced by combining the index of materials, M_t^P and the value added index, H_t^P. Materials are obtained by combining governmentally produced materials and privately produced materials. Finally, privately produced materials can either be of domestic or foreign origin. The index of value added consists of input of labor and capital. The capital stock, K_{t-1}^P is changed by investments that may be of either domestic or foreign origin. The index of labor input, L_t^P is a CES function with 7 different inputs. Each of these inputs, L_t^{Pj} represents labor from a given wage bargaining segment. Within each bargaining segment there exists a continuum of workers with different productivity levels. We assume for the sake of simplicity that workers from a given bargaining segment are perfect substitutes. Workers from different bargaining segments are imperfect substitutes, and the elasticity of substitution, σ_{PL}, is identical between any two pairs of labor categories.

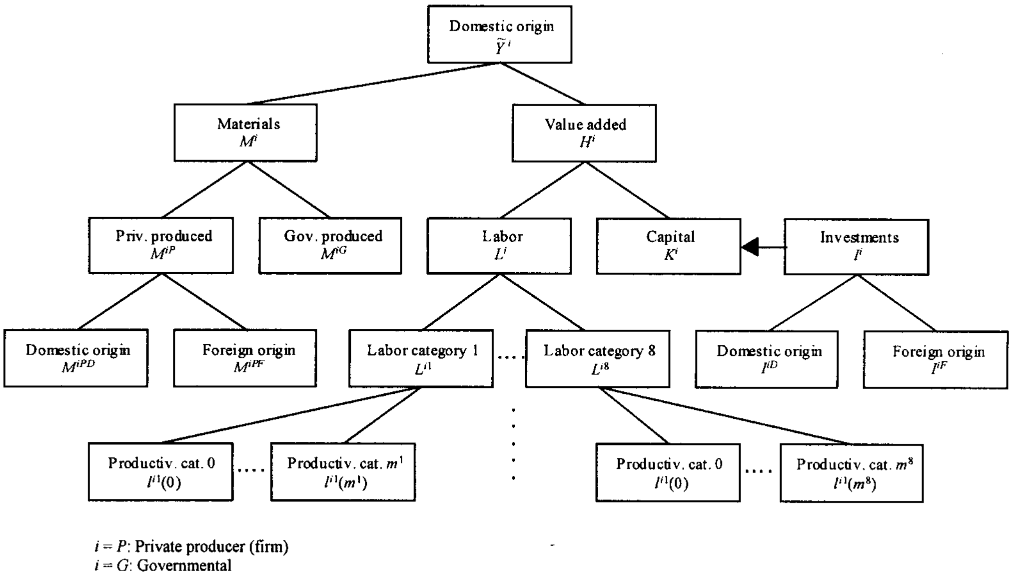


Figure 2: Production structure

In the bargaining game, the two parties consider the price of the produced good and the price of materials as given. The capital stock, K_{t-1}^P is pre-determined, i.e. we assume that wage negotiations take place after the capital stock has been up-dated but before employment decisions are made. Given these assumptions we may derive the following demand for the aggregate labor index, which is considered common knowledge.[5]

$$L_t^P = \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P \tag{2.1}$$

where $\eta^P \left(W_t^P\right)$ is function of the wage index of the private sector, W_t^P with $\eta^{P\prime} \left(W_t^P\right) < 0$. The explicit expression is given in appendix 3. The wage index of the private sector is defined as

$$W_t^P = \left(\sum_{j=1}^{7} \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(W_t^{Pj}\right)^{1-\sigma_L^P}\right)^{\frac{1}{1-\sigma_L^P}} \tag{2.2}$$

where W_t^{Pj} is the wage index of workers from labor market segment j. σ_L^P is the constant elasticity of substitution between workers from different labor market

[5] The derivation is given in appendix 3.

segments in the production function of the representative firm of the private sector, $\mu_{Lj}^{P} > 0$ is a weight parameter of input of workers from labor market segment j in the CES production function.

The demand for the aggregate labor index, L_t^P has the standard properties that demand is decreasing in the aggregate wage index and increasing in the predetermined stock of capital.

To decompose the demand for the aggregate labor index into demand for the different categories of labor observe that the aggregate labor index is defined as a CES function of the labor indexes of different categories of labor.

$$L_t^P = \left(\sum_{j=1}^{7} \mu_{Lj}^{P} \left(L_t^{Pj} \right)^{\frac{\sigma_L^P - 1}{\sigma_L^P}} \right)^{\frac{\sigma_L^P}{\sigma_L^P - 1}} \tag{2.3}$$

where L_t^{Pj} is input from union j. The standard CES demand relationship implies that the demand for labor of category j yields the standard isoelastic demand relationship for labor of category j. The demand relation is given by

$$L_t^{Pj} = \left(\mu_{Lj}^{P} \right)^{\sigma_L^P} \left(\frac{W_t^{Pj}}{W_t^P} \right)^{-\sigma_L^P} L_t^P \tag{2.4}$$

Finally, the labor index of category L_t^{Pj}, is given as the sum of labor inputs adjusted for the productivity of the different workers. To simplify the mathematical exposition, we assume that there is infinitely many different productivity levels, so that the total index may be represented by the following integral

$$L_t^{Pj} = \int_0^{m_t^j} \rho^j(i)\, l_t^{Pj}(i)\, di \tag{2.5}$$

where $\rho^j(i)$ is a productivity parameter that measures the productivity of a specific type i of workers of category j. As a convention $\rho^{j\prime}(i) < 0$, so that $i = 0$ is the category with the highest productivity. Figure 1 is a representation of the empirical properties of the $\rho^j(i)$ functions. $l_t^{Pj}(i)$ is the demand for labor of productivity-type i in labor category j in the private sector. m_t^j is a measure of the number of workers of category j.

When choosing the demand for labor of each productivity-type of labor category j the firm is assumed to minimize the cost of a given level of the labor index of category j for given wages of each productivity-type. This minimization problem may be stated as

$$\min_{l^{Pj}(i)} \int_0^{m_t^j} w_t^{Pj}(i)\, l_t^{Pj}(i)\, di$$

subject to

$$\int_0^{m_t^j} \rho^j(i)\, l_t^{Pj}(i)\, di = \bar{L}_t^{Pj}$$

where $w_t^{Pj}(i)$ is the wage rate paid to workers of productivity-type i in labor category j, and $\bar{L}_t^{Pj}$ is a fixed level of the aggregate employment of category j. This yields the following condition for optimality for each productivity-type

$$\lambda\rho^j(i) = w_t^{Pj}(i), \quad \text{iff } l_t^{Pj}(i) > 0 \tag{2.6}$$

where λ is the Lagrange-multiplier associated with the minimization problem. The condition holds for all productivity types that are employed. Dividing one first order condition with another yields

$$\frac{w_t^{Pj}(i)}{\rho^j(i)} = \frac{w_t^{Pj}(h)}{\rho^{Pj}(h)} \equiv q_t^{Pj}, \quad \text{for all } i, j \text{ where } l_t^{Pj}(i), l_t^{Pj}(h) > 0 \tag{2.7}$$

(2.7) states that since any two different categories of productivity are perfect substitutes then if both types are to be used, the wages corrected for productivity have to be identical across the groups. The productivity-corrected wage is defined as q_t^{Pj}. The wage of the individual employed category is given by:

$$w_t^{Pj}(i) = \rho^j(i)\, q_t^{Pj} \tag{2.8}$$

Productivity-types not employed by the firm are identified by:

$$w_t^{Pj}(h) > \rho^j(h)\, q_t^{Pj}$$

which simply states that productivity-type h of category j is not productive enough to become employed.

The productivity-corrected wage for labor category j, q_t^{Pj}, is equal to the wage index, W_t^{Pj} of category j. To see this we define the wage cost of labor of category j as

$$W_t^{Pj} L_t^{Pj} = \int_0^{m_t^j} w_t^{Pj}(i)\, l_t^{Pj}(i)\, di$$

inserting the cost minimizing rule (2.8) yields

$$W_t^{Pj} L_t^{Pj} = \int_0^{m_t^j} \rho^j(i)\, q_t^{Pj} l_t^{Pj}(i)\, di = q_t^{Pj} \int_0^{m_t^j} \rho^j(i)\, l_t^{Pj}(i)\, di = q_t^{Pj} L_t^{Pj}$$

where the last equality follows from relation (2.5). Therefore

$$W_t^{Pj} = q_t^{Pj} \tag{2.9}$$

Using the demand relationship for the different nests of labor input we may use the relations (2.1), (2.4), and (2.5) to derive the following equilibrium condition for the private sub-labor market of labor category j

$$\int_0^{m_t^j} \rho^j(i)\, l_t^{Pj}(i)\, di = \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{W_t^{Pj}}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P \tag{2.10}$$

2.2. Demand for labor in the public sector

The public sector has a production technology that is similar (but not identical) to the private sector. The nest-structures of the two technologies are the same but the elasticities of substitution between the various types of input differ between the two sectors. In addition there exists a category of labor which is only present in the production function of the public sector.

To define a demand relationship for each of the inputs of the public production sector we assume that the public sector maximizes production of the public good given an exogenous constraint on the annual public spending, which is assumed to be politically decided. Furthermore, we assume that also investments in the public sector are given exogenously by the political process. Therefore public managers choose the optimal input of materials and the different categories of labor to solve the following maximization problem

$$\begin{aligned} &\max \quad F\left(M_t^G, H^G\left(\bar{K}_{t-1}^G, L_t^G\right)\right) \\ &\text{subject to} \\ &\quad P_t^{GM} M_t^G + W_t^G L_t^G = \bar{R}_t \end{aligned}$$

where $F\left(M_t^G, H^G\left(\bar{K}_{t-1}^G, L_t^G\right)\right)$ is the production function of the representative production unit in the public sector. superscript G denotes public sector, so that e.g. P_t^{GM} is the price index of materials used in the public sector. Finally, $\bar{R}_t$, is the exogenously given budget constraint of the public production.

Using the first order conditions to this problem and using the nested-CES structure of the production technology and the budget constraint we derive the following implicit demand for the aggregate labor supply in the public sector[6]

$$\left(\bar{R}_t - W_t^G L_t^G\right)^{-\frac{1}{\sigma_Y^G}} W_t^G \left(L_t^G\right)^{\frac{1}{\sigma_H^G}} H^G \left(\bar{K}_{t-1}^G, L_t^G\right)^{\beta} = \frac{\mu_{HL}^G \mu_{YH}^G}{\mu_{YM}^G} \left(P_t^{GM}\right)^{\frac{\sigma_Y^G - 1}{\sigma_Y^G}}$$

where σ_Y^G is the constant elasticity of substitution between materials and the value added index, H^G. σ_H^G is the constant elasticity of substitution between capital and the index of aggregate labor in the public sector. μ_{HL}^G, μ_{YH}^G, μ_{YM}^G are positive parameters of the CES specification of the production function. Finally, $\beta > 0$ is a parameter defined as

$$\beta \equiv \frac{\sigma_H^G - 1}{\sigma_H^G} - \frac{\sigma_Y^G - 1}{\sigma_Y^G}$$

so that we may define the demand for aggregate labor as the following function

$$L_t^G = \eta^G \left(W_t^G, \bar{K}_{t-1}^G, \bar{R}_t, P_t^{GM}\right) \tag{2.11}$$

Since the nest-structure of the public sector is identical to the nest-structure of the private sector, it follows that the aggregate labor index may be divided into labor of different categories, each of which again may be divided into a continuum of productivity-types. The demand relations for each of the sub-divisions of the labor force follows the procedure outlined in the previous sub-section. The equilibrium condition for the sub-labor market that is similar to relation (2.10) of the private sector is given as

$$\int_0^{m_t^j} \rho^j(i)\, l_t^{Gj}(i)\, di = \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{W_t^{Gj}}{W_t^G}\right)^{-\sigma_L^G} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \tag{2.12}$$

2.3. Characterization of a partial equilibrium for a given minimum wage

As explained, we assume that the wage contracts in the economy are of the minimum wage type, where a labor union and an employers' association negotiate a minimum wage. The wages of the employed persons are higher than or equal to this minimum wage. As an initial step we consider the partial equilibrium of a segment of the labor market for a given exogenous minimum wage. For persons who have a productivity sufficiently high to become employed, given the minimum wage, we assume that a sub-labor market with perfect competition exists so that all persons with sufficiently high productivity are employed and receive the wage

[6] The explicit derivation is shown in appendix 4.

that clears the sub-labor market given the level of the minimum wage. Using the description of the firm, the wages under a minimum wage system as defined above may be defined by

$$w_t^{sj}(i) = \max\left\{w_t^{\min,sj}, \rho^j(i) W_t^{sj}\right\}, \quad s = P, G \tag{2.13}$$

where $w_t^{\min,sj}$ is the minimum wage for the relevant segment of the labor market identified both by the category of labor and the sector in the economy (private, P, or public, G). If $w_t^{\min,sj} < \rho^{sj}(i) W_t^{sj}$ all workers of productivity i receive the (contingent competitive) wage $\rho^{sj}(i) W_t^{sj}$ and are all employed. On the other hand, if $w_t^{\min,sj} > \rho^{sj}(i) W_t^{sj}$ then workers of category i "receive" the minimum wage and are not employed $\left(l_t^{sj}(i) = 0\right)$. The border between these two regimes is given by a category of workers r^{sj}, called the marginally employed productivity category defined by:

$$w_t^{\min,sj} \equiv \rho^j\left(r^{sj}\right) W_t^{sj} \tag{2.14}$$

Due to our convention that $\rho^{j\prime}(i) < 0$, employment in sector s will be given by

$$l_t^{sj}(i) = \begin{cases} l_t^{sj} \text{ for } i \leq r_t^{sj} \\ 0 \text{ for } i > r_t^{sj} \end{cases}, \quad s = P, G \tag{2.15}$$

2.4. Perfect mobility between sectors

We assume that there is perfect mobility between sectors in the economy of all types of productivity in all categories of labor.[7] Therefore the wage of an individual productivity type must be identical between sectors if the productivity type in question is employed in all sectors.

$$w_t^{Pj} = w_t^{Gj} \quad \text{if} \quad l_t^{Pj} > 0 \wedge l_t^{Gj} > 0$$

Since the individual productivity $\rho^j(i)$ is not sector-specific we have that

$$w^{Pj}(i) = \rho^j(i) W_t^{Pj} = w^{Gj}(i) = \rho^j(i) W_t^{Gj}$$

so that

$$W_t^{Pj} = W_t^{Gj} = W_t^j \quad \forall s, v \text{ where } l_t^{Pj} > 0 \wedge l_t^{Gj} > 0 \tag{2.16}$$

[7] Observe that there exists a group of workers that have skills, which are only demanded by the public sector. For this group the present subsection is of course not relevant.

Relation (2.16) states that the productivity corrected wage for a given category of workers is identical in all sectors of the economy.

By assuming that the minimum wage is the same in all sectors,

$$w_t^{\min,Pj} = w_t^{\min,Gj} \tag{2.17}$$

it follows from (2.14), (2.16) and (2.17) that the marginal productivity-type, r_t^{sj} is the same in all sectors.

$$r_t^{Pj} = r_t^{Gj} \equiv r_t^j \tag{2.18}$$

We take this to be the case in the following.

Since we assume that there exists a perfect market for productivity types who fulfill the minimum productivity requirement of the minimum wage, the total employment of these productivity types is equal to the total labor supply.

Finally, we assume that there is one representative agent for each category of productivity and that the individual labor supply of this agent is given as $\bar{l}$. This could be the case if the length of the working day is fixed by a long-term contract between the union and the confederation of employers, as is the case in Denmark, and if we ignore the possibility of part-time jobs.

In this case we may write total employment as

$$l_t^{Pj} + l_t^{Gj} = \begin{cases} \bar{l} \text{ for } i \leq r_t^j \\ 0 \text{ for } i > r_t^j \end{cases} \tag{2.19}$$

2.5. Demand for labor of category j in partial equilibrium

Given the description of the behavior of the firms in the economy, the assumption of perfect mobility between sectors and the fact that there is a unique minimum wage for each category of workers, we may derive a relation between the minimum wage of the specific category of workers and the demand for this category. The relationship is found by adding (2.10) and (2.12) and using (2.14) and (2.19).[8]

[8] The derivation is shown is Appendix 5.

$$\bar{l}\int_0^{r_t^j} \rho^j(i)\, di = \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{\frac{w_t^{\min,j}}{\rho^j\left(r_t^j\right)}}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P\left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P +$$

$$\left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{\frac{w_t^{\min,j}}{\rho^j\left(r_t^j\right)}}{W_t^G}\right)^{-\sigma_L^G} \eta^G\left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \qquad (2.20)$$

Equation (2.20) determines the demand relationship between the minimum wage and the border-category r for a given price level. An increase in $w^{\min}$ implies a decrease in r_t^j, so that the number of full-employed categories decreases. Relation (2.20) is the labor demand relationship in traditional wage curve - labor demand representation of the partial equilibrium of the labor market for category j.

2.6. Union behavior

To determine the wage curve we have to set up the Nash-bargaining problem. First consider the behavior of the union of labor of category j. The union is assumed to be utilitaristic, i.e. to bargain for a minimum wage, $w^{\min,j}$, so that the sum of the workers' utilities is maximized. The incentive for the union to raise the minimum wage is that this raises the total wage of those becoming employed and this may outweigh the loss due to the unemployment that is the consequence of the minimum wage. The increase in the total wage of the employed appears because the productivity of a given worker depends both on her individual productivity and on the overall level of employment in the economy. The mechanism from total employment to individual productivity is that increased employment reduces the capital labor ratio, which reduces productivity of the individual worker.

We assume that the unions set the minimum wages in each period to maximize the sum of their members life-time utility. However, we assume that unions cannot commit themselves to specific wage profiles in the future and therefore only time-consistent wage claims are credible. This fact and the assumption that the utility is additively separable in leisure imply that the unions set the minimum wage to maximize the sum of their members income after tax net of disutility of work in each period.

2.6.1. Income after tax

Before proceeding we need to define the tax system. The Danish personal income tax system is a piecemeal linear, progressive tax system. Taxes are collected both by the State and by local governments (i.e. municipality and county). Table 3 shows the tax rates imposed at different income levels as well as the income threshold and the basic allowances in 1995.

Personal income (1000 DKK)	Marginal income tax
0 - 36.3 (*)	0
36.3 - 130.9	42.9
130.9 - 174.3	47.9
174.3 - 236.6	50.9
236.6 -	64.1 (**)

Note (*) *the income threshold is a sum of the general allowance (29.4) and the average conveyance allowance (6.7)*

Note (**) *the marginal tax rate is reduced due to the marginal tax rate ceiling, which is reached in the majority of municipalities*

Table 3: The personal income tax system (1995).

In the tax system, personal income, X_t, is defined as the sum of taxable social transfers, unemployment benefits, and labor income net of the labor market contribution tax. The modelling of the labor market implies that a given person is either fully employed or fully unemployed. Therefore a given person cannot receive both unemployment benefits and wage income. Furthermore, we make the simplifying assumption that persons receiving wage income do not receive taxable social transfers.[9] Thus for workers of category j personal income is defined as

$$X_t^j = \left(1 - t_t^l\right) w_t^j(i)\, l_t^j(i) + b_t\left(\bar{l} - l_t^j(i)\right)$$

where the first term is the income taxable wage income whereas the last term is the income taxable income from unemployment benefits. As noted a specific person either receives the first or the latter. t_t^l is the labor market contribution tax rate, which is 6 percent in 1995. Only employees pay the labor market contribution tax. However, gross social transfers are indexed to the wage rate net of the labor market contribution tax, thus an increase in t_t^l leads to reduction in

[9] Since the representative housholds in DREAM do receive taxable social transfers, we effectively assume that taxation of wage income and taxation of taxable social transfers are additively seperable.

unemployment benefits (per hour), b_t. $\left(\bar{l} - l_t^j(i)\right)$ is the part of the individuals of category j that is unemployed during year t.

For computational reasons we approximate the Danish tax system the following differentiable function

$$T_t\left(X_t^j\right) = a_t\left(X_t^j\right)^{b_t} - d_t \quad (2.21)$$

The parameters (a_t, b_t, d_t) of the function are estimated using 1995 data of the income tax system and personal income levels fro 80,000 to 700,000 DKK. Figure 3 shows the actual and the estimated tax function.

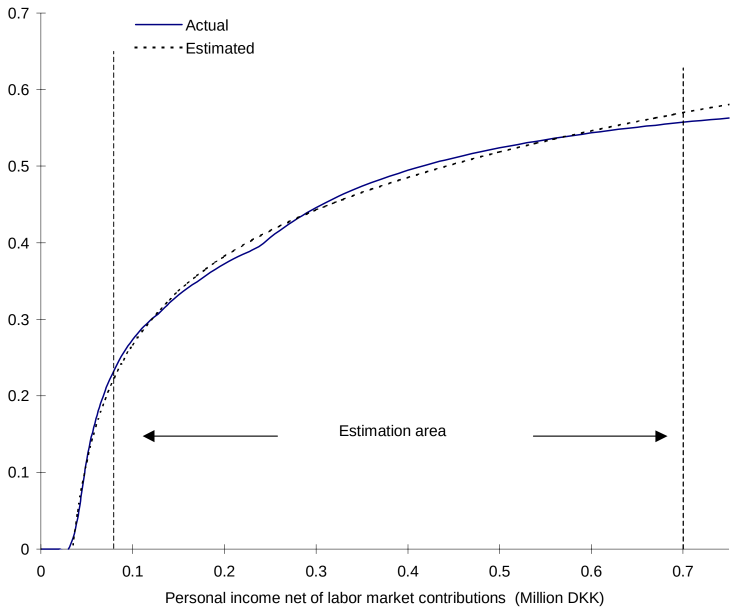


Figure 3: Average tax rate. Actual and estimated

The estimated parameters are

$$T_t\left(X_t^j\right) = 0.6363\left(X_t^j\right)^{1.2428} - 0.00968$$

Income after tax for a worker of category j is defined as

$$NI_t\left(X_t^j\right) = X_t^j - a_t\left(X_t^j\right)^{b_t} + d_t$$

The union's objective function As mentioned the union maximizes the sum of its members' income after tax net of disutility of work in each period. Therefore we may write the utility function of the union of workers of category j, V_t^j, as follows

$$V_t^j = \int_0^{m_t^j} \left(\frac{NI_t\left(X_t^j\right)}{P_t^C} - \gamma_1 \frac{\gamma}{1+\gamma} l_t^j\left(i\right)^{\frac{1+\gamma}{\gamma}} \right) di \tag{2.22}$$

where $\gamma_1 \frac{\gamma}{1+\gamma} l_t^j\left(i\right)^{\frac{1+\gamma}{\gamma}}$ is the disutility of work for a member who works l_t^j hours in year t. γ is the labor supply elasticity of the individual and $\gamma_1 > 0$ is a constant.

The threatpoint of the union, $\bar{V}$, which measures the outcome of the union if no agreement is reached in the negotiations, is defined as the real after tax unemployment benefits for each worker.[10]

Using the definition of the threatpoint and the definition of the tax function in relation (2.21) it may be shown that the union objective is given as[11]

$$\begin{aligned} V_t^j - \bar{V} &= \frac{\left(1-t_t^l\right) W_t^j \bar{l}}{P_t^C} \int_0^{r_t^j} \rho_j\left(i\right) di - a_t \frac{\left[\left(1-t_t^l\right) W_t^j \bar{l}\right]^{b_t}}{P_t^C} \int_0^{r_t^j} \rho_j\left(i\right)^{b_t} di \\ &\quad - r_t^j \left(\gamma_1 \frac{\gamma}{1+\gamma} \bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t \bar{l}\right)}{P_t^C} - \frac{d_t}{P_t^C} \right) \end{aligned} \tag{2.23}$$

Relation (2.23) is simply the sum of the net-income after tax for the employed members (the first line) minus the sum of their disutility of work plus the sum of the net-income of the unemployed members minus the threatpoint.

2.7. Behavior of the employers' association

We assume that there are no binding contracts and that negotiators have no commitment, so the relevant equilibrium is a subgame perfect equilibrium. Therefore the employers' association seeks to maximize the gross profits and takes the investments and the capital stock as given. Furthermore, we assume that the union of workers of category j and the relevant employers' association take the output price of the private sector and the wage rates of other categories of workers as given when bargaining over the minimum wage of a specific category of workers.

[10] This is a standard assumption in the literature on wage bargaining. According to Danish law there is a 5-week period of quarantine before workers can receive unemployment benefits after being laid off due to a conflict. However, the quarantine is ignored here.

[11] See Appendix 6 for a derivation

Using the CES-nesting structure of the private sector we may write gross profits as a function of the aggregate wage level in the private sector, W_t^P. [12]

$$\frac{\pi_t^P}{P_t^C} = \frac{(1-t_t^c)}{P_t^C}\left(\left(\bar{p}_t\kappa_2 - P_t^{PM}\kappa_1\right)\left(\mu_{HL}^P\right)^{-\sigma_H^P}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H^P}\right)\eta^P\left(W_t^P\right)^{\sigma_H^P}\bar{K}_{t-1}^P$$
$$-\frac{(1-t_t^c)}{P_t^c}\left((1+t_t^a)\,W_t^P\eta^P\left(W_t^P\right)^{\sigma_H^P}\bar{K}_{t-1}^P + p_t\Phi(\bar{I}_t^P,\bar{K}_{t-1}^P)\right) \qquad (2.24)$$

where π_t^P is the gross profits of a representative firm in the private sector. κ_1, κ_2 are constants (defined in appendix 5), t_t^a is the pay roll tax, $\Phi(\bar{I}_t^P,\bar{K}_{t-1}^P)$ is the cost of installation of capital.

For simplicity we assume that the threatpoint of the employers' association is zero.

2.8. Nash-bargaining and labor supply

The wage for a given level of productivity has to be identical in the two sectors if a category of workers is to be employed in both sectors of the economy. This is due to the assumption of perfect mobility between the sectors. Therefore we assume that the minimum wage is set in the negotiation between the unions and the employers' associations for categories of labor that may potentially be employed in both sectors. This simplification is justified by the fact that Danish wage contracts of public employed often are formulated in such a way that wage increases in the private sector to a certain extent automatically is passed on to the public employee (with a time-lag).

When negotiating with the employers of the private sector the labor unions take into account the effect on demand labor (of their type) from the public sector. Therefore the indirect utility of the labor union is found using the total demand for labor in the economy, even if the employers who are the direct counterpart in the negotiation only employ a fraction of the total employment of the category of workers in question. Thus the bargaining power of the employers' associations in the private sector determines the wages of the categories of public employed who may potentially be employed in the private sector.

[12] A derivation is found in appendix 7.

The Nash-bargaining problem is then given as

$$\arg\max_{r_t^j} \left\{ \left(V_t^j - \bar{V}_t\right)^{\lambda^j} \left(\frac{\pi_t^P}{P_t^C}\right)^{\left(1-\lambda^j\right)} \right\} \tag{2.25}$$

where $0 \leq \lambda^j \leq 1$, is the bargaining power of the union of workers of category j.

Substituting the relevant demand for labor functions into V_t^j and π_t^P respectively imply that both object functions become a function of the level of employment measured by r_t^j. As is clear from the demand for labor of category j, relation (2.20), choosing the minimum wage for category j implicitly sets the level of employment of category j measured by r_t^j. Therefore we may formulate the problem as in (2.25) where the Nash product is maximized by choosing the marginally employed productivity group, r_t^j.

The first order condition to the problem may be given as

$$\lambda^j \frac{\partial \left(V_t^j - \bar{V}_t\right)}{\partial r_t^j} \frac{1}{\left(V_t^j - \bar{V}_t\right)} + \left(1 - \lambda^j\right) \frac{\partial \left(\frac{\pi_t^P}{P_t^C}\right)}{\partial r_t^j} \frac{1}{\left(\frac{\pi_t^P}{P_t^C}\right)} = 0 \tag{2.26}$$

The explicit solution to the minimum wage cannot be derived in the present formulation. This is due to the general formulation of the tax-system in the economy, which implies that the derivative of the object function of the union is not a closed form solution. It is however possible to derive a numerical solution which is used in the solution to the CGE-model.[13]

In Pedersen *et al.* (1998) a similar problem is solved for the case of a single union and given a simplified tax system with a constant marginal tax that makes it possible to derive an explicit solution to (2.22). It is shown that in this case the minimum wage depends positively upon the opportunity costs of employment as perceived by the union (as in standard union models). In addition the minimum wage depends positively upon the productivity of the marginally employed relative to the average productivity. This implies that the larger the differences between the intramarginal workers the lower is the minimum wage. This relationship is the wage-curve in a partial equilibrium description of the labor market. The partial equilibrium is given by the intersection of this curve and the demand for labor of the category in question.

The employers' of the public sector bargain with the (constructed) cartel of (pure) public employees. The public employers take wages for all other categories of

[13] See Appendix 8 for a derivation of the expression used in the model

labor given, since the wage of these groups are determined in the private sector. The objective of the public sector is to maximize public production subject to the budget constraint and given the pre-determined level of capital. The union objective function is given by (2.23). Therefore the solution to this maximization problem is also only possible to find by numerical simulations. The result is similar to the bargain solutions for the segments where the wages are determined in the private sector. The wage-curve of public employees is given as the combinations of wages and employment (measured by the marginal productivity type r_t^j) of this category that fulfills the first order condition to the Nash bargaining problem with the public employers. The demand for labor is simpler in this case as only the public sector has demand for this type of labor. The demand curve is given as

$$\bar{l}\int_0^{r_t^j} \rho^j(i)\, di = \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{\frac{w_t^{\min,j}}{\rho^j\left(r_t^j\right)}}{W_t^G}\right)^{-\sigma_L^G} \eta^G\left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \tag{2.27}$$

where category j is (pure) public employees.

Finally we need to determine the partial equilibrium the labor market of the high-income employees. The demand for this category is given in relation (2.20) and is similar to the demand for other categories of labor employed in both the private and the public sector. The labor supply curve is particularly simple since we assume that the labor supply is fixed. Therefore there exists a reservation wage determined by the fact that the net income determined by the reservation wage is equal to the disutility of work given that the person in question has to work $\bar{l}$ hours. The intersection of the labor demand curve and this horizontal labor supply curve determines the wage of the marginally employed productivity group. Since all productivity groups of the high-income category are perfect substitutes (like productivity groups of any other category) this intersection determines the wage per produced unit. Each productivity type therefore receives a wage equal to their productivity times the wage per productivity. Any productivity type with a productivity higher than the marginally employed type is therefore fully employed and any productivity type with a productivity lower than the marginally employed is voluntarily unemployed.

3. Macroeconomic features of the DREAM model

This section gives an overview of the non-labor market parts of the DREAM model that is used to simulate the effects of the employment tax cut. The presentation is kept to a minimum and serves as a remedy for analyzing how the effects from the labor market behavior are fed into the economy through changes in the individual behavior. A formal (but preliminary) documentation of the model in a version with competitive labor markets is available in Knudsen *et al.* (1998).

DREAM is a model of a small open economy with perfect capital mobility. The model features overlapping generations of households and firms that face convex cost of installation of capital. The government produces a public good, distribute age-dependent transfers, social pensions and unemployment benefits. The activities of the government are financed through a range of different tax instruments. The government obeys its intertemporal budget constraint. This is accomplished by using a specific policy rule for either public expenditures or taxes. In the initial equilibrium the government is assumed to run a balanced budget in every period. In this presentation a lump sum tax is used to balance the budget initially.

3.1. Firms

The fundamental behavioral assumption of the corporate sector is that firms strive to maximize the value of the outstanding stock of shares. It is assumed that the firm finances investments by an exogenous combination of debt and retained profits. The value of shares is given by the discounted stream of future dividends. By assumption shares of domestic firms are owned by domestic citizens. Thus economic policy that affects future dividends and therefore the value of firms will generate a wealth effect of domestic households on impact, which again will affect the pattern of consumption.

The convex cost of installation of capital (and also of deploying existing capital) imply that an individual firm wants to adjust its capital stock gradually towards a stationary capital stock, if the firm is in an environment with constant prices and demand. Investments are driven by the so-called marginal q-theory of investment. No turnover costs are present in the case of labor (and materials). The demand for labor functions are derived in the section on the labor market.

3.2. Households

The household sector consists of overlapping generations of households with a finite and deterministic time horizon. Households are defined as representative couples with children. There is one representative household for each cohort of women, and the age of the woman defines the age of the household. Each cohort of women are associated with a group of men and a group of children. The age distribution of these men varies according to the observed pattern of age-differences in couples. Children below 18 years are distributed to the household of their mother. Thus, the total number of individuals in a household equals the sum of the number of women, men and children. To account for the fact that children consume less than adults, the size of the household is converted into *adult equivalents*, by multiplying the number of children with $\frac{1}{2}$.

The size of the representative household is affected by the exogenous age and gender specific death probabilities of the members, and by the age specific fertility rates of women. The adult members of the households are divided into workers and pensioners, according to an exogenously given retirement age. The explicit incorporation of both children and a retirement period gives rise to a life-cycle motive for saving (and dissaving) in the model. At the end of the planning horizon - when the woman turns 78 years - each generation of households leave a bequest to their children. Agents may be alive *after* the end of their planning horizon. These persons receive public pensions and age specific transfers, pay taxes and consume the residual income.

Younger generations of household are assumed to be utilitaristic i.e. to maximize the sum of utilities of household members. The households derive utility from consumption of the domestic and the foreign good, and incur disutility from time spent working. Bequest also has a positive effect on the donor's utility.

In this application of the model we assume that labor supply is fixed and therefore the problem of the families is to determine the optimal aggregate consumption path and thereby also the optimal bequest. Given the optimal aggregate consumption path the consumption of the different goods is found using a standard CES split.

Given the assumption of perfect foresight and perfect capital markets the aggregate consumption is determined as function of the sum of human and non human wealth and the price of current consumption relative to the index of discounted future consumption prices. The non-human wealth of households is defined as the sum of the stock of shares in domestic firms held by the household and the stock of bonds. This is a forward-looking variable due to the present of the stock of shares. The human capital is defined as the discounted stream of non-interest income of the household and is by definition also a forward looking variable. Since the stock of human capital is the main channel through which labor market policy affects the consumption behavior of the household the following section describes the definition of household non-interest income in some detail.

3.3. The composition of family income

Income for the consumers arises from 8 main sources. The first is wages for those who are employed, the second is unemployment benefits for those who are unemployed. The third source of income is age dependent income transfers from the public sector, such as child care transfers, education benefits, and sickness benefits. The fourth source of income is pension, which is given to all persons who are between 61 and 78 years old. The pension is identical for all pensioners. The fifth source of income is a (negative) lump sum transfer that is introduced to balance the budget of the government in the initial equilibrium. The sixth type of income arises from bonds and shares held by the consumers. The seventh source of net income is net transfers from abroad, such as aid transfers to developing countries and net transfers from the European Union. The final type of income arises from inheritance left by the parent household.

The wage income differ according to both the category of worker and the productivity type. For simplicity we assume that there exists a representative household for each generation, such that only intra-generational differences between households exist. To find the total net income of persons with gender, J, where $J \in \{F, M\}$, and of age, a, we need the following definitions:

Let $A^J_{a,j}(s)\, ds$ be the number of persons with gender J, age a, labor category j and productivity type s and let $A^J_a = \sum_{j=1}^{8} \int_{i=0}^{m^j_t} A^J_{a,j}(i)\, di$ be the total number of persons with gender J and of age a. Define

$$\phi^J_{a,,j}(s) = \frac{A^J_{a,j}(s)}{A^J_a}$$

so that $\phi^J_{a,,j}(s)$ is the fraction of persons with gender J, age a, who is of productivity type s in labor category j. By definition we have that

$$\sum_{j=1}^{8} \int_{i=0}^{m^j_t} \phi^J_{a,,j}(i)\, di \equiv 1$$

Therefore average net income of persons with gender J and age a is given as

$$NI^J_{a,t} = \sum_{j=1}^{8} \int_{i=0}^{m^j_t} NI_{j,t}(i)\, \phi^J_{a,,j}(i)\, di$$

where $NI_{j,t}(i)$ is the net income for workers with productivity i of labor category j. Observe that for persons with sufficiently low productivity the net income consists of unemployment benefits after tax.

Total income per adult for a household of age b at time t, $y_{b,t}$, is given by

$$
\begin{aligned}
y_{b,t} \quad = \quad & \frac{1}{N_{b,t}^{AF}} \left\{ N_{b,t}^{FW} \left(NI_{b,t}^{F} + B_{b,t}^{inF} \right) \right. \\
& + \left(N_{b,t}^{F} - N_{b,t}^{FW} \right) \left[f_t^P - T_t^P \left(f_t^P \right) \right] + N_{b,t}^{F} \cdot TR_{b,t}^{F} \\
& + \sum_{a=18}^{60} N_{a,t}^{M} \cdot \omega_{a,b} \cdot \left(NI_{a,t}^{M} + B_{a,t}^{inM} \right) \\
& + \sum_{a=18}^{\tilde{A}} N_{a,t}^{M} \cdot \omega_{a,b} \cdot TR_{a,t}^{M} \\
& \left. + \left[f_t^P - T_t^P \left(f_t^P \right) \right] \sum_{a=61}^{\tilde{A}} N_{a,t}^{M} \cdot \omega_{a,b} \right\} + \tau_t^W + \tau_t
\end{aligned}
\tag{3.1}
$$

where $N_{b,t}^{AF}$ is the number of adults in the representative household of age b at time t. $N_{b,t}^{FW}$ is the number of female workers in the household, and $N_{b,t}^{F}$ is the number of females in the households. By definition $N_{b,t}^{FW} = N_{b,t}^{F}$ for $b < 61$, $N_{b,t}^{FW} = 0$ for $b \geq 61$. $B_{b,t}^{inF}$ is the inheritage of females of age b at time t. The fist line in the expression above is therefore the sum of net income and the inheritance per female(i.e. the first line) multiplied by the number of female workers in the representative household The second line is the sum of pension after tax $\left[f_t^P - T_t^P \left(f_t^P \right) \right]$ multiplied by the number of female pensioners $\left(N_{b,t}^{F} - N_{b,t}^{FW} \right)$ and the age and gender specific public transfers per female multiplied by the total number of women in the household.

For households where the women are pensioners ($b \geq 61$), the first line of expression (3.1) cancels out because the number of female workers equals zero. Correspondingly the term $N_{b,t}^{F} - N_{b,t}^{FW}$ in the second line reduces to $N_{b,t}^{F}$. On the other hand in the case of $18 \leq b < 61$, this last term cancels out (i.e. no pension) while the first line remains.

The first two lines (inside the braces) represent the total non-interest income after tax of women in the household whereas the rest of the expression inside the braces relates to the income of men in the household. This part of the expression becomes more complicated, as the men in the representative household are not of the same age. Some men are retired, whereas others are not.

Therefore the men in the household - regardless of the age of the woman - contribute both with the income terms related to workers and to pensioners. The total income arising from men in the household is found by multiplying the number of men in each age group with their respective income terms and then summarizing these expressions over the relevant interval of ages. In the third line of the expression, $N^M_{a,t}$, is the number of men with age a at time t. $\omega_{a,b}$ is the share of men of age a who are married to women of age b. $\sum\limits_{a=18}^{60} N^M_{a,t} \cdot \omega_{a,b}$ therefore is the income total number of men in the household. $NI^M_{a,t}$ is the average net income from labor market participation for men of age a at time t, $B^{inM}_{b,t}$ is the average inheritage of men of age a at time t. The third line therefor measures the sum of labor market net income and inheritage for men in the household. Similarly the fourth and the fifth line measure the age and gender specific public transfers and the public pensions received by men in the household.

To measure per adult we divide the sum of income from men and women in the household, by the number of adults, $N^{AF}_{b,t}$. To this we add the two lump sum transfers, τ^W_t and τ_t, which are defined per adult, and we finally obtain the stream of non-interest income per adult belonging to a household of a specific age, b, at a given moment in time, t.

4. The specific earned income tax credit scheme

An EITC scheme may be either general or targeted at low income groups. The advantage of a targeted scheme of course is that the dead weight loss is minimized. The possibility of targeting an EITC scheme in the Danish economy by different ways of phasing out the income treshold with increasing income is discussed in Danish Ministry of Business *et al.* (1996). The conclusion is that limiting the tax credit to low-income groups leads to prohibitively high effective marginal tax rates (close 100 percent) for middle-income groups. Another way of limiting the earned income tax credit is to select receivers from special groups of persons e.g. lone-mother like the EITC system in the US. This possibility is not considered here. The reason is that we look for general effects on the level of unemployment of the employment tax credit.

We consider a general EITC defined as a DKK 11,250 income threshold on wage income for wage-earners with an annual gross income of DKK 80,000 and above.[14] In addition the labor market contribution tax is reduced 1 percentage point.

The tax credit is financed through general increases in marginal tax rates at a progressive scale according to the scheme presented in table 4. This financing rule implies that the scheme is fully financed in the long run. On impact the total tax

[14]The income threshold is deductible at the marginal tax rate of the lowest income bracket. This implies that the net-income effect of the unfinanced tax credit is equal to DKK 6,425.

reductions are not fully financed through the tax increases. By assumption the government uses a lump sum tax to balance the budget in each period. Therefore the lump sum tax is increased on impact. Gradually the lump sum tax is reduced to zero.

	Marginal tax rate	
Personal income (1000 DKK)	*before*	*after*
0 - 36.3 (*)	0	0
36.3 - 130.9	42.9	43.9
130.9 - 174.3	47.9	49.9
174.3 - 236.6	50.9	52.9
236.6 -	64.1	67.1

Table 4: The financing scheme.

Since the personal income tax system applies to all kinds of personal income including e.g. social pensions and unemployment benefits these groups contribute to the financing of the tax credit. In the simulation presented we have compensated receivers of social pensions such that this group does not contribute to the financing of the earned income tax credit.

Only employees pay the labor market contribution tax. However, social transfers are indexed to the wage rate net of the labor market contribution tax, which implies that also social transfers are increased by the reduction.

Gross income (1000 DKK)	Net gain from tax credit
80	4,998
150	4,542
200	3,792
300	1,504
400	-987
500	-3,478

Table 5: Net gains to wage earners from the tax credit.

For given personal income the net gains to wage earners at different income levels are shown in table 5. Wage earners with a personal income less than approximately DKK 360.000 gain from the earned income tax credit (for a given per tax income). Wage earners with higher personal income are net financiers of the tax credit.

5. Results

In this section we present the main results of the introduction of the EITC scheme. As the effect is highly sensitive to the initial level of unemployment we present results of scenarios with both a high and a low level of unemployment. The scenario with a high level of unemployment has an initial level unemployment of 10.5 percent which is equal to the recorded level of unemployment in Denmark in 1995. This scenario is considered the base scenario. The low initial unemployment scenario has an initial level of unemployment of 5.0 percent.

The sensitivity of the EITC scheme to the initial level of unemployment is due to the non-linearity of the wage distribution functions depicted in figure 1. From this figure one observes that the distributions share the common feature that they are relatively steep for low incomes, relatively flat for middle incomes and relatively steep for high incomes. A high level of initial unemployment implies that the distributions initially are cut off at a relatively flat part. Therefore a high level of initial unemployment implies that a given decrease in the minimum wage will generate a relatively large increase in employment. Similarly a low level of initial unemployment will imply that the distributions are cut off at a relatively steep part. Therefore a given reduction in the minimum wage will generate a relative low increase in employment.

As unemployment is exclusively due to too high minimum wages in the present modelling one may argue that the simulation based on the actual level of unemployment tends to give a biased evaluation of the effectiveness of the EITC scheme as e.g. unemployment due to frictions in the labor market may not be reduced as effectively by the EITC scheme. One the other hand the scenario based on an initial level of unemployment of 5 percent may represent a too low measure of the part of the unemployment caused by high minimum wages and high levels of unemployment benefits. Therefore the two scenarios may be considered as a measure of the upper and lower bound on the effect of the EITC scheme.

Finally a third scenario is presented. This scenario represents a situation where the domestic country is faced with higher price elasticities in the foreign trade. With high levels of the price elasticities the terms of trade become approximately fixed and therefore the output price of domestic firm is given from world market. With an interest rate given from the world market this implies that the capital labor ratio and therefore the real product wage is given in the long run equilibrium. This scenario has an initial level of unemployment of 10.5 percent and should therefore be compared to the base scenario.

5.1. Macroeconomic effects

Table 6 shows the macroeconomic results of the introduction of the EITC. In the base scenario the rate of unemployment is reduced by 2.5 percentage points. As unemployment is concentrated among persons with low productivity the reduction in unemployment is higher than the increase in productivity adjusted employment, which is increased by 1.6 percent both in the short and in the long run.

The capital stock is gradually increased so that the long run increase is 1.1 percent. Long run real GDP increase by 1.4 percent. Private consumption increases by approximately 1.3 percent in the long run.

In the low unemployment scenario the unemployment rate is reduced by 1.4 percentage points or approximately 1.1 percentage point less than in the base scenario. The reduction in the productivity corrected employment index is even more pronounced. For this variable the increase is 0.6 percentage point in the low unemployment scenario, which should be compared to the 1.5 - 1.6 percentage point increase in the base scenario. The fact that the reduction in the effect on employment is higher is explained by the fact that with the lower level of unemployment the level of productivity of the marginally employed is lower. Therefore the increase in employment is made up of individuals who have a lower level of productivity than in the base scenario. The effects on the rest of the macroeconomic variables in the low unemployment scenario are scaled proportionally to the effects on employment.

The scenario with the high price elasticities in the foreign trade reveals that the bargaining solution is not affected by these elasticities. This is due to the assumption of a Nash equilibrium in wages, where each set of bargainers takes the outcome of the bargains in other labor market segments and the aggregate price level as given. In a more centralized bargaining these effects would have been internalized and the outcome would have been more sensitive to the change in elasticities. The main difference between the base scenario and the scenario with high price elasticities in foreign trade is that the expansion of the domestic production that follows from the introduction of the EITC scheme can is made without deteriorating the terms of trade. Therefore the expansive effect become slightly more pronounced in the case of high price elasticities in foreign trade.

Figure 3 shows the increase in employment in each of the 8 labor market segments in the base scenario and the low unemployment scenario respectively. Observe that the impact effect differs from on segment to another. This difference is a consequence of the different distributions of productivity and the different bargaining powers of the unions. Unskilled workers are concentrated in the segment of tradesmen which includes unskilled workers in the building and construction sector and in the segment of unskilled workers that consist of female unskilled workers and lorry drivers etc. In the base scenarios these segments have an increase in employment of more than 3.5 percent on impact. The relatively low-paid group of public employees experience an increase in employment of more than 3 per cent. Finally the competitive segment has by far the lowest increase in employment (less than 1 percentage point) which is due to the fact the distribution of productivity of this group is "thin" close to the reservation wage.

In the low unemployment scenario the distribution of effects is similar but not identical to the base scenario but all effects are scaled down to approximately 3/5 of the size in high unemployment scenario.

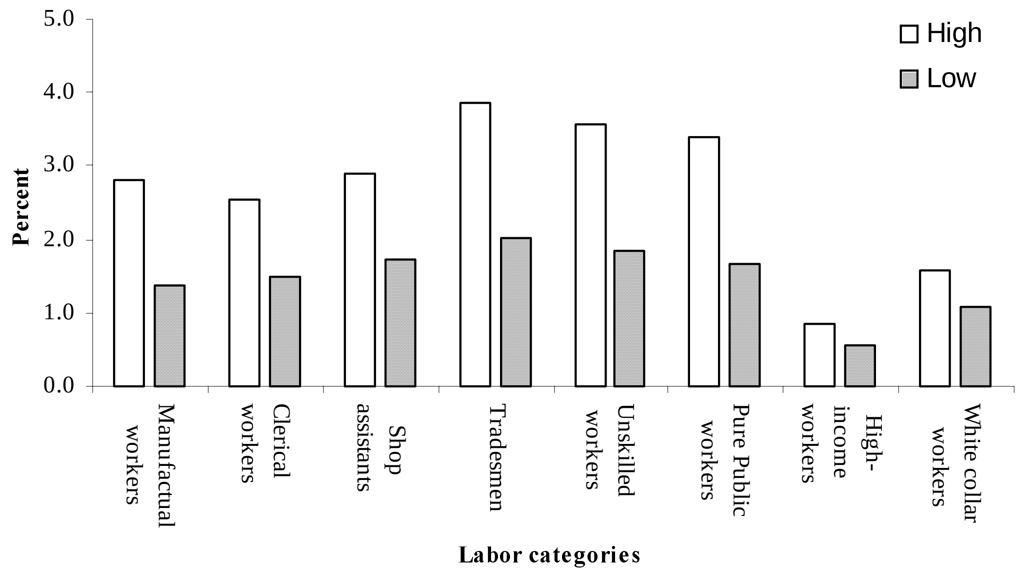


Figure 3. Sectoral distribution of employment effects. Year 2000.

Billion DKK	Initial stationary state	Year 2000	2005	2020	2050
Real private consumption	416	417	418	419	420
Base case		(0.3)	(0.4)	(0.7)	(0.9)
Low initial unemployment		(0.0)	(0.1)	(0.3)	(0.5)
High foreign price elasticities		(0.5)	(0.6)	(0.9)	(1.3)
Real GDP (factor prices)	823	829	830	832	833
Base case		(0.7)	(0.9)	(1.1)	(1.3)
Low initial unemployment		(0.3)	(0.4)	(0.4)	(0.4)
High foreign price elasticities		(0.8)	(1.0)	(1.3)	(1.5)
Wage level (index)	100.0	98.3	98.3	98.5	98.7
Base case		(-1.7)	(-1.7)	(-1.5)	(-1.3)
Low initial unemployment		(-0.7)	(-0.7)	(-0.6)	(-0.4)
High foreign price elasticities		(-1.4)	(-1.1)	(-0.8)	(-0.6)
Employment (index)	100.0	101.5	101.5	101.6	101.6
Base case		(1.5)	(1.5)	(1.6)	(1.6)
Low initial unemployment		(0.6)	(0.6)	(0.6)	(0.6)
High foreign price elasticities		(1.5)	(1.6)	(1.6)	(1.6)
Unemployment rate (percent)	10.5	7.9	7.9	7.9	7.9
Low initial unemployment	5.0	3.6	3.6	3.6	3.6
High foreign price elasticities	10.5	7.9	7.9	7.9	7.9
Capital stock	2755	2763	2769	2778	2784
Base case		(0.3)	(0.5)	(0.8)	(1.0)
Low initial unemployment		(0.1)	(0.2)	(0.4)	(0.4)
High foreign price elasticities		(0.4)	(0.7)	(1.1)	(1.3)
Value of firms	1269	1281	1280	1278	1277
Base case		(0.9)	(0.8)	(0.7)	(0.7)
Low initial unemployment		(0.3)	(0.3)	(0.3)	(0.3)
High foreign price elasticities		(1.5)	(1.4)	(1.2)	(1.2)
Foreign assets	-266	-269	-269	-267	-260
Low initial unemployment		-266	-264	-260	-251
High foreign price elasticities		-274	-276	-273	-259

Note: the numbers in parantheses are the percentage change compared to the calibration year

Table 6. Macro results.

5.1.1. Supply side effects

The macroeconomic effects summarized in table 6 may traced back to supply and demand shocks to the economy from the introduction of the EITC scheme. The supply shock is the dominating and is due to the direct effects of the EITC on the wage formation in the economy.

The impact effect of the supply shock is a outward shift in the (minimum) wage curves for all segments of the labor market. The obvious reason for this is that the gain from becoming employed is increased for the low income groups. Furthermore the EITC increases the sum of the after tax wages. This is due to the fact that the tax increase does not generate the same revenue for a given level of activity as is spend on the EITC scheme. The two reasons for this are that public expenditures are reduced from the lower level of unemployment and that the tax credit is not fully financed on impact.

The increased employment increases the marginal product of capital, which will further initiate a rise in investments that gradually lead to a higher stock of capital. Observe however that the increased capital stock does not lead to an increase in total employment but to an increase in the overall wage rate . In fact total employment in the private sector is reduced over time. This implies that the minimum wage curves of the labor market segments in the private sector are almost vertical but slightly decreasing in employment.

Pedersen *et al.* (1998) shows that bargaining over minimum wages tends to generate very steep wage curves and that these curves may become negatively sloped depending on the curvature of the distribution of productivity. In the present case the minimum wage curves of the private sector are practically vertical.

In the public sector the employment is slightly increasing. This is due to the assumption that public spending is a constant fraction of GDP. Therefore public spending increases slightly over time and this pulls up public employment as well.

5.1.2. Demand side effects

On the demand side the tax credit implies an increase in the stock of human capital for those generations who have a sufficiently long remaining period in the labor force, since both average wages after tax and employment are increasing due to the tax credit. The fact that initially the tax credit is less than fully financed by the increase in the marginal tax rates implies that the government increases the lump sum tax to balance the budget. This affects human capital negatively but the effect from the wage sum is dominating. For the retired generations the increase in the lump sum tax is almost off set by the increase in social pension (that follows from the reduction in the labor market contribution tax) leaving the human capital practically unaffected.

The value of firms jump on impact by 0.75 percent due to increased future dividends that follows from the reduction in the producer wage and the derived increase in employment, capital stock and production.

The total aggregate non-human and human wealth in the economy are slightly increased on impact so that aggregate consumption increases initially. This rise is largest for the youngest generations. This tend to make consumption slightly growing over time.

5.2. Distributional effects

As already mentioned the overall effect of the EITC is to reduce the tax burden of the low-income group and to increase the tax burden of the high-income group. Although there are some shifts in the distribution of employment between the labor market segments and therefore also some shifts in the distribution of pre-tax income they cannot reverse the impact effect from the restructuring of the tax system. Therefore the overall effect is a compression in the distribution of income in the economy.

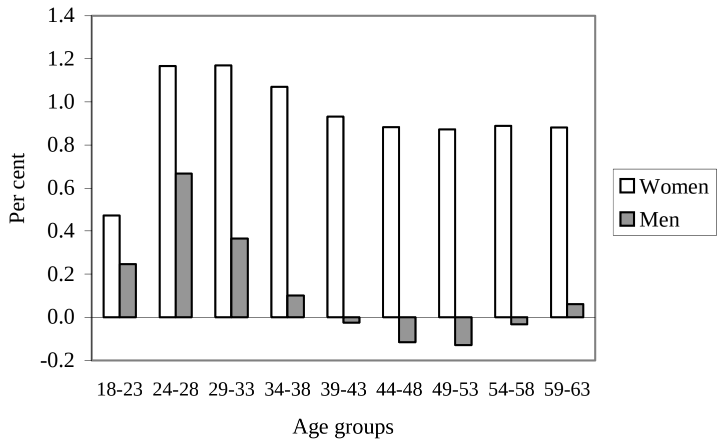


Figure 4: Increase in average labor market income after tax. Year 2000.

This overall distributional effect may be allocated across gender and generations according to the distributions of productivity across these groups. Figure 4 shows the net average labor market income gains in the base scenario for men and women respectively for each of the 9 age groups who are active on the labor market.[15] The conclusion from the figure is conspicuous: Women gain in net income for all age groups and the gain is higher than for men in the corresponding age groups. Young women gain the most. The men in the forties and the fifties experience a reduction in the average net income.

This picture of gains and losses reflects the fact that on average women have a lower salary than men in the same age groups. Furthermore, the high income group is concentrated among men in the forties and fifties, who therefore are the net contributors to the financing of the tax credit.

6. Concluding remarks

In this paper the effects of a general EITC scheme financed through progressively increased marginal tax rate of the Danish income tax system were analyzed in a model with heterogeneous labor and bargaining over minimum wages. We showed that the introduction of the scheme generates a reduction in unemployment and a smaller increase in employment measured in productivity corrected units. The effect of the EITC depends crucially on the level of structural unemployment. In the present analysis an upper and a lower bound of the level of unemployment were analyzed. The measured effect at the upper bound was approximately twice as large as the effect at the lower bound, but even in the latter case a significant reduction in unemployment was found.

With regard to the distribution of income in society the EITC tends to compress the income distribution by reducing the wage income after tax of the highest income groups and increasing the wages after tax for the low income groups. Individuals who remain unemployed are not worse of after the introduction.

Two different kinds of extensions to the analysis may affect the generally positive conclusion of the present paper in a negative direction. First, introduction of endogenous labor supply (or bargaining over the length of the working day) may imply that individuals with high and middle incomes reduce their labor supply due to the higher marginal tax rate. The fact that also the average tax rate increases may dampen the effect. Overall the total effect of the EITC may become less positive. Second the fact that the distribution of income become more compressed

[15] The average net labor market income is defined in Appendix 1.

may reduce the incentive to education and therefore introduce a negative effect on average productivity in the long run. However this effect may not be too large as the compression in the distribution of income is only marginal. Introduction of the later two effects are obvious topics for further research.

REFERENCES

Binmore, K., A. Rubinstein and A. Wolinsky (1986): "The Nash Bargaining Solution in Economic Modelling", *Rand Journal of Economics*, vol. 17 pp. 176-88

Bovenberg, L. and F. Van der Ploeg (1994): "Effects of the Tax and Benefit System on Wage Formation and Unemployment", Working paper 94 - 08, CentER, Tilburg University and University of Amsterdam

Bovenberg, L. and F. Van der Ploeg (1998): "Consequences of Environmental Tax Reform for Unemployment and Welfare", *Environmental and Resorce Economics* vol. 12 pp. 137-50

Danish Employers' Confederation (1998): "Labor Market Report", Copenhagen (in Danish)

Economic Council (1997): *The Danish Economy, Autumn 1997.* Copenhagen: Economic Council. (in Danish containing an English summary)

European Commission (1994): *Growth, Competitiveness, Employment: The Challenges and Ways forward into the 21st. Century*, European Commission.

Hansen, C. T., L. H. Pedersen and T. Sløk (1995): "Progressive Taxation, Wages and Activity in a Small, Open Economy", EPRU Working Paper Series 1995-21, Copenhagen

Hansen, C. T., L. H. Pedersen and T. Sløk (1999): "Ambiguous Effects of Tax Progressivity - Theory and Danish Evidence", *Journal of Labour Economics* (forthcoming)

Holmlund, B. and A. S. Kolm (1999): "Environmental Tax Reform in a Small, Open Economy with Structural Unemployment, *International Tax and Public Finance*, (forthcomming)

Knudsen, M. B., L. H. Pedersen, T. W. Petersen, P. Stephensen and P. Trier (1998): "A Prototype of a DREAM (Danish Rational Economic Agents Model)" Version 1.2, Statistics Denmark

Lockwood, B and A. Manning (1993): "Wage Setting and the Tax System. Theory and Evidence for the United Kingdom", *Journal of Public Economics,* vol. 52 pp 1-29

Ministry of Business, Ministry of Economic Affairs, Ministry of Finance, Ministry of Taxation, the Prime Minister's Department (1996): "Effects of Earned Income Tax Credit Proposals" (in Danish), Copenhagen

Nicolaisen, S. (1997): "Increased Coverage of Individual Wage Contracts", (in Danish) *Arbejdsmarkedspolitisk Agenda*, Copenhagen: Dansk Arbejdsgiverforening.

Nielsen, S. B., L. H. Pedersen, P. B. Sørensen (1995): "Environmental Policy, Pollution, Unemployment, and Endogenous Growth", *International Tax and Public Finance*, vol. 2, pp 185-205

OECD (1996): *Employment Outlook*, Paris

Pedersen, L. H., N. Smith and P. Stephensen (1998): "Wage Formation and Minimum Wage Contracts: Theory and Evidence from Danish Panel Data". Economic Modelling Working Paper Series 1998:5, Statistics Denmark

Pisauro, G. (1991): "The Effect of Taxes in Efficiency Wage Models", *Journal of Public Economics*, 46, 254-79.

Pissarides, C. A. (1998): "The Impact of Employment Tax Cuts on Unemployment and Wages; The Role of Unemployment Benefits and Tax Structure", *European Economic Review*, vol 42, pp 155-83

Statistics Denmark (1991): "IDA - an integrated data base for labour market research" Main report. (in Danish), Copenhagen

Statistics Denmark (1996): "DISCO-88 Classification of Occupations of Statistics Denmark" (in Danish), Copenhagen

Statistics Denmark (1997): "Collective Bargaining Agreements 1995" (in Danish) News from Statistics Denmark no. 225, Copenhagen

Sørensen, P. B. (1997): "Public Finance Solutions to the European Unemployment Problem", *Economic Policy*, vol. 25 pp. 223-264

Van der Ploeg, F. (1987): "Trade Unions, Investment, and Employment: A Noncooperative Approach", *European Economic Review* vol. 31 pp 1465-92

APPENDIX 1: Calibration

This section describes the procedure used to calibrate the labor market part of the model. The general idea of this calibration is to reproduce the distributions found in the 10 percent sample of the register of Danish workplaces and persons (IDA). Given these distributions the aggregated levels are calibrated to data of National Accounts by scaling the sample.

The calibration of parameters in the rest of the model follows a fairly standard procedure of calibrating the model to a stationary state and is not reported here. The interested reader is referred to Knudsen *et al.* (1998) where both the procedure and the results of the calibration are reported.

The starting point of the calibration of the labor market is the distributions of wages for the 8 labor market segments that are reported in figure 1. In this figure the persons belonging to a specific segment are arranged according to hourly wage. The wage is decreased with the number of persons. Therefore the figure shows a monotone depiction from number of persons to the wage rate. Each of these discrete depictions are fitted to continuous functions using polynomia of order 5.

$$w_{95}^{j}(i) = \sum_{v=0}^{5} a_v^j \left(\frac{i}{m^j}\right)^v \tag{A1.1}$$

where i is person number i in the specified ranking of persons of labor category j, and a_i $i \in \{1, ..., 5\}$ is a parameter.

Using the fact that the wage of individual i of category j is given as the individual productivity times the productivity corrected wage of category j

$$w_{95}^{j}(i) = \rho^j(i)\, W_{95}^j$$

and normalizing the productivity corrected wage, W_{95}^j to one in the base year, implies that the polynomium in (6.1) yields a calibration of $\rho^j(i)$.

The rate of unemployment within each labor market segment is reported in the IDA-sample. Defining the rate of unemployment of the labor of category j as u_t^j where

$$u_t^j \equiv \frac{U_t^j}{m_t^j \bar{l}} \equiv \frac{\int_0^{m_t^j} \left(\bar{l} - l_t^j(i)\right) di}{m_t^j \bar{l}} = \frac{\int_{r_t^j}^{m_t^j} \bar{l}\, di}{\bar{l}} = \frac{\bar{l}\left(m_t^j - r_t^j\right)}{m_t^j \bar{l}} = \frac{m_t^j - r_t^j}{m_t^j} \tag{A1.2}$$

where U_t^j is the number of unemployed hours. $m_t^j \bar{l}$ is the potential number of working hours of members of union j. The third equation follows from the definition of r_t^j in (2.14) and the fact that individuals are either fully employed or fully unemployed. This implies that r_t^j is the number of employed in segment j.

Inserting the calibrated value of r_t^j into the polynomium that is fitted to the distribution of wages in segment j and using the fact that the marginal employed receives the minimum wage of the segment yields a calibration of the minimum wage.[16]

$$\rho^j\left(r_{95}^j\right) = \rho^j\left(r_{95}^j\right) W_{95}^j = w_{95}^j\left(r_{95}^j\right) = w_{95}^{\min,j}$$

This yields a set of minimum wages as shown in table 4. Observe that these minimum wages are calibrated such that all unemployment within a specific labor market segment is caused by too low productivity. If unemployment is caused by other reasons (e.g. temporary lay off due to weather conditions in the construction sector), then this tends to imply that the calibrated minimum wage becomes too high. Except for tradesmen, where the mentioned temporary lay off is particularly large, the calibrated minimum wages are in same range as the observed minimum wages, which is between DKK 70 and 90 per hour. To quote Danish Employers' Confederation (1998): "In 1996 the minimum wages were between DKK 70 and 90 per hour, but more than 3 out of 4 minimum wages were between 70 and 75 DKK per hour."(our translation).

Labor market segment	Minimum wage (DDK)	Size		Private wage sum	Public wage sum	Unemployment rate
Manufactural workers	90	454	20%	97.2	4.0	10.6
Clerical workers	85	387	17%	57.1	22.1	9.9
Shop assistants	73	227	10%	36.3	8.5	11.2
Tradesmen	105	140	6%	29.0	0.7	16.7
Unskilled workers	91	213	9%	31.3	9.0	14.6
Pure public workers	89	493	22%	0.0	104.7	9.0
High-income workers	107	273	12%	66.7	23.4	8.1
White collar workers	80	93	4%	23.8	1.2	5.1
Total		2281	100%			10.4

Note: Wage sums are measured in billion DKK, size is measured in 1000 persons

Table 7: Properties of labor market segments

The observed unemployment rates in the IDA sample exclude persons who have been unemployed for a whole year (or have been unemployed for part of the year

[16]The same kind of procedure is used to find the reservation wage of the competitive segment of the labor market.

and on a leave scheme for the rest of the period). This is due to the fact that we do not have wage observations regarding this type of persons. This however reduces the measured unemployments rates in the sample significantly below the official unemployment rate. Therefore all unemployment rates of the sample are scaled proportionally, so that the average unemployment rate is identical to the official rate. This introduces a bias in the unemployment rates as measured by the sample compared to actual unemployment rates as the long term unemployment is very biased towards unskilled workers in the actual data.

Integrating the polynomium (6.1) over workers from 0 to r_t^j (i.e. the employed part of workers of category j) yields a calculated wage sum for workers of category j. This wage sum for segment j is calibrated to the wage sum for this group as reported in the IDA sample. This is done by scaling the number of persons in the segment. This scaling reduces the number of persons in each segment by 2 to 4 percent.

The wage sums of all labor market segments are added together to yield the total wage sum of employed from the IDA sample. The full scale economy is obtained by scaling the number of employed in the IDA sample so that the total wage sum is equal to the wage sum of the National Accounts for 1995. The resulting percentage increase in employment is used to increase the number of employed of each category proportionally. The resulting number of members and the wage sum in each labor segment is shown in table 7.

By comparing to table 1 observe that the adjustments in the relative size of the labor market segments between the original sample and calibrated number in the model are less than one percentage point for every group. Observe that the calculated number of persons is significantly below the official labor force which is 2.8 billion persons. This difference is due to the fact that the wages of the persons in the sample is higher than for the persons in the economy. This is due to the fact that the sample is taken from persons who are registered as having a main occupation which is included in the labor force. This leaves out low income groups as students, assisting wives etc.

For each segment of labor market (except the pure public employed) we need a split between public and private employment. To find these values the public wage sum is calculated for each segment of the labor market in the IDA-sample. From this calculation the percentage share of total public wage sum is calculated for each labor market segment. The calibrated public wage sums used in the CGE-model is found by splitting total public wage sum from National Account between the labor market segments according to the calculated shares in the IDA-sample.

Normalizing the wage rates to 1 in the base year we now have sufficient information to calibrate the CES functions of employment in each of the categories $\left(L_t^{P1},...,L_t^{P7}\right)$. The share parameters of the CES function are calibrated in the standard way using the demand equations (2.4) and the similar demand equations for the public sector. The elasticity of substitution between any two categories of labor is set to 0.5.

Finally we need to calibrate the bargaining power of the union of workers of category j, λ^j for all j such that the model-generated wage sum of each bargaining segment becomes equal to wage sums reported in table 4. This is done by using the first order condition to the Nash bargaining problem (2.26).

6.1. Age and gender specific distributions

To find the income of specific generation of households we need wage income data distributed according to both age and gender, since the construction of households implies that men are distributed to households that, in general, will have a different age than their own.

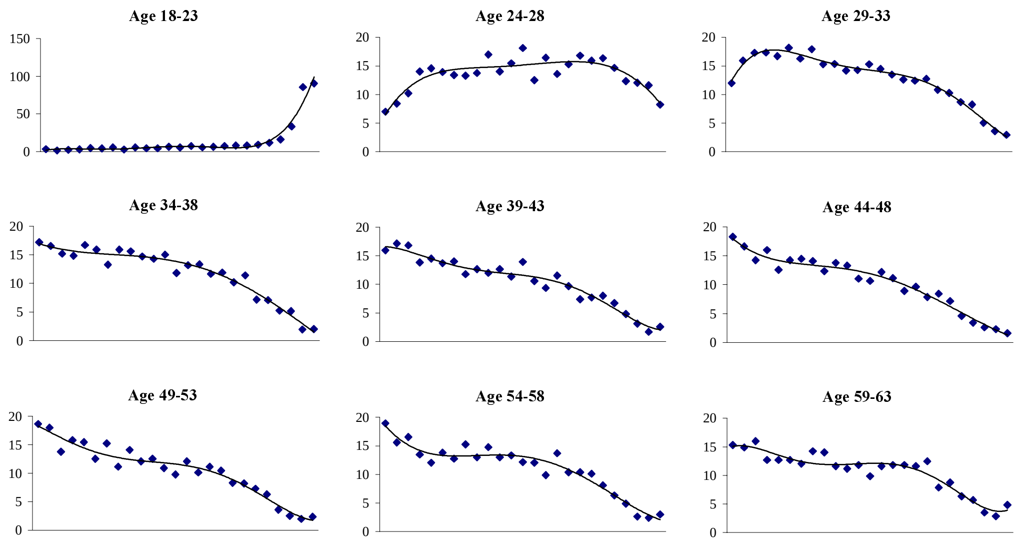
Data from the IDA-sample are divided into sub-categories according to age (in 9 5-year intervals) and gender for each category of workers. In this way the material is divided into 144 (=9*2*8) sub-categories.[17] The persons are grouped into 25 groups for each labor segment according to hourly wage. The number of persons in each of these groups is divided by the total number of persons of the specific age and gender. This yields an empirical counterpart of the function $\phi_{a,,j}^{J}(s)$ that measures the fraction of persons with gender J, age a, labor category j who are of productivity type s in labor category j. Following the same procedure as in case of the wage distributions in labor market segments we estimate a polynomium of order 5 to fit the empirical distribution of $\phi_{a,,j}^{J}(s)$. Figure 6 shows the empirical distributions and the fitted polynomia for the nine age-groups of males in the labor market segment of manufactural workers.

The distribution of persons of a given age and gender is used to calculate the average net-income of the age and gender group. Using the specification of the tax function in (2.21) and the fact that the wage of an employed person with productivity i may be written as $w_t^j(i) = \rho^j(i) W_t^j$ we may write the expression for the average net-income of gender J, who is age a at time t as

[17] This large amount of sub-categories is the primary reason why we have chosen to aggregate the number of labor market segments into 8, since a larger number would require a larger IDA-sample.

$$
\begin{aligned}
NI_{a,t}^{J} \;=\; & \sum_{j=1}^{8}\left[W_t^j\bar{l}\int_0^{r_t^j}\rho^j\left(s\right)\phi_{a,j}^{J}\left(s\right)ds - a_t\left(W_t^j\bar{l}\right)^{b_t}\int_0^{r_t^j}\rho^j\left(s\right)^{b_t}\phi_{i,j}^{J}\left(s\right)ds\right. \\
& +\left(b_t\bar{\bar{l}} - a_t\left(b_t\bar{l}\right)^{b_t}\right)\left(\int_0^{m_j}\phi_{i,j}^{J}\left(s\right)ds - \int_0^{r_{t,j}}\phi_{i,j}^{J}\left(s\right)ds\right) \\
& \left.+D_t^T\int_0^{m_j}\phi_{i,j}^{J}\left(s\right)ds\right]
\end{aligned}
$$

This definition of age and gender specific net labor market income is used to depict the distributional consequences of the earned income tax credit scheme in section 5.



Note: 1. axis: Productivity categories, 2. axis: Per mille of all males in the specific age group

Figur 6: Example of age/gender distributions. Males, organized in manufactual workers union

APPENDIX 2:
Definition of labor market segments by DISCO codes

The ten major labor market segments for blue collar workers in Denmark defined by bargaining agreements are shown in the table below. The column coverage measures the approximative number of employed persons covered by the specific bargaining agreement. These 10 bargaining agreements covers approximately 60 per cent of the total employment in firms that are members of the Danish Employers' Confederation

Employers	Union	Line of business	Contract type	Coverage
DI	CO-industri	Manufacturing	Minimum wage	150.000
DI	TL/HK	Manufacturing	Fully decentralized	55.000
DHS	HK	Service	Minimum wage	40.000
Contractors	SiD	Construction	Minimum wage	20.000
DI	TIB	Manufacturing	Minimum wage	20.000
HORESTA	RBF	Service	Minimum wage	15.000
DV	SiD	Service	Standard wage	15.000
AHTS	KAD/DF	Service	Standard wage	15.000
BYG	TIB	Construction	Minimum wage	10.000
ELFO	El-union	Construction	Minimum wage	10.000
Total				350.000

Source: Danish Employers' Confederation, unpublished material.

The coverage of agreements are approximative numbers for 1997

Table 8: The 10 major private labor market segments for blue collar workers

The employers' associations abbreviated in the table are the following: DI, Danish Federation of Manufactural Employers ("Dansk Industri" in Danish); DHS, Danish Federation of Employers in the retail sector ("Dansk Handel og Service" in Danish); Contractors assoc. is the Federation of Danish Contractors ("Entrepenørforeningen" in Danish); HORESTA is the Federation of Employers in Hotels Restaurants and Tourism ("Hotel og Restaurations- og Turisterhvervets Arbejdsgiverforening" in Danish); DV is the Federation of Employers in the Road-transport sector ("Danske Vognmænd" in Danish); AHTS is the Federation of

Employers in Retail, Transport and Service ("Arbejdsgiverforeningen for Handel, Transport og Service" in Danish); BYG is the Federation of Employers in the Construction Sector ("Byggeriets Arbejdsgivere" in Danish); ELFO is the Federation of Employers of Electricians ("Elinstallatørernes Landsforening" in Danish)

The unions abbreviated in the table are the following: CO-industri is a cartel of unions in manufacturing, this is the major exception to the fact that bargaining in Denmark is performed according to crafts. TL is the union of Technicians, ("Teknisk Landsforbund" in Danish); HK is the union of clerks and shopassistants ("Handels- og Kontorfunktionærernes forbund" in Danish); SiD is the union of Unskilled Workers ("Specialarbejderforbundet i Danmark" in Danish); TIB is the union of wood workers ("Forbundet Træ-Industri-Byg" in Danish); RBF is union of workers in Hotels, Restaurants and Breweries ("Restaurations- and Bryggeriarbejderforbundet" in Danish); KAD is the union of unskilled female workers ("Kvindeligt Arbejderforbund i Danmark" in Danish); DF is the union of caretakers ("Dansk Funktionærforbund" in Danish); El-union is the union of electricians (Dansk El-forbund in Danish).

In DREAM, the net wage distribution for each labor market segment is disaggregated according to age (9 different age groups) and gender. As explained above this generates 144 different distributions. Furthermore, the empirical wage distributions are based on a 10 percent sample of the Danish population. Therefore, we need to restrict the number of labor market segments, so that the number of observations for a given age group of a given gender belonging to a specific labor market segment is sufficiently large.

To obtain this we aggregate labor market segments where the wage aggregates are expected to be highly correlated. This leads to the following rules

1. The three bargaining agreements covering employment in the construction sector are aggregated into one labor market segment

2. The two bargaining agreements with a standard wage contracts are aggregated into one labor market segment

3. The bargaining agreement of the hotel and restaurant sector (between HORESTA and RBF) and the bargaining agreement for shopassistants (between DHS and HK) are aggregated into one labor market segment.

This procedure leaves 5 major labor market segments for blue collar workers:

1. Manufactural workers (both skilled and unskilled)

2. Clerical workers
3. Shopassistants and employees in restaurants etc.
4. Tradesmen or construction workers (both skilled and unskilled)
5. Unskilled workers primarily employed in the transport sector

In addition to these labor market segments we define three labor market segments for white collar workers:

6. Pure public employees (teachers, nurses, priests etc.)
7. High income workers (academic personnel and managers)
8. White collar workers primarily employed in the financial sector (bankers etc.)

To construct the basis of the labor market in the model we assume that theses 8 labor market segments are a partitioning of the labor market, i.e., all employed persons belong to one of the 8 labor market segments specified above and no employed person belong to more than one (at a given point in time). Given this definition the observations in the sample are distributed across the 8 segments according to the DISCO code attached to the observation. The DISCO code is a four digit code identifying the type of job (by e.g. the level of education and the line of business) for any given employed.

The 8 segments are identified by the DISCO codes. The labor market segment called "Manufactural workers" are defined as containing the following DISCO codes:

31, 311, 313, 61, 611-615, 721-742, 72-74, 81, 811-828, 921, 3111-3115, 3117-3119, 3131-3139, 3141, 3145, 6000-6999, 7111-7113, 7211-7442, 8111-8290, 9211-9213, 9320

The labor market segment called "Clerical workers" contains the following DISCO codes:

41,42, 411-414, 422, 3431-3439, 4111-4211, 4213-4223

The labor market segment called "Shop Assistants" contains the following DISCO codes:

51, 52, 511-513, 3414-3416, 3422, 5111-5131, 5139-5149, 5210-5220

The labor market segment called "Tradesmen" contains the following DISCO codes:

71, 74, 712, 742, 93, 931, 7121-7124, 7129, 7131-7143, 9311-9313

The labor market segment called "Unskilled workers" contains the following DISCO codes:

83, 832-833, 91, 913, 8311-8340, 9113-9162, 9330

The labor market segment called "Pure public workers" contains the following DISCO codes:

0110, 22, 221, 23, 24, 315, 32, 321-322, 33, 344, 2230, 2320-2359, 2431, 2432, 2460, 2470, 3116, 3151-3223, 3231-3340, 3423-3428, 3441-3460, 3480, 5132, 5133, 5161-5169,

The labor market segment called "High-income workers" contains the following DISCO codes:

11, 114, 12, 122, 123, 131, 211-214, 230-242, 245, 314, 341-343, 1000-1319, 2000-2229, 2310, 2411, 2421-2429, 2441-2446, 2451-2455, 3121-3123, 3142-3144, 3224, 3471-3475,

The labor market segment called "White collar workers" contains the following DISCO codes:

2412-2419, 3225-3229, 3411-3413, 3417-3421, 3429, 4212,

APPENDIX 3:
Derivation of the demand for the aggregate labor index in the private sector

In the bargaining the employers' association and the union consider the price of the produced good and the price of materials as given. Given perfect knowledge of the production technology this implies that the index of value added, p_t^{PH}, is given. This follows from the fact that the output price is given as

$$p_t^P = \left[\left(\mu_{YM}^P \right)^{\sigma_Y^P} \left(P_t^{PM} \right)^{1-\sigma_Y^P} + \left(\mu_{YH}^P \right)^{\sigma_Y^P} \left(P_t^{PH} \right)^{1-\sigma_Y^P} \right]^{\frac{1}{1-\sigma_Y^P}}$$

where p_t^P is the output price of the private sector indicated by the superscript P, elasticities of substitution has the sign σ. Superscript refers to sector, whereas subscript refers to the CES nest where the elasticity appears. This implies that σ_Y^P is the elasticity between the index of materials and the index of value added in the production technology (which is the $Y-nest$ of the private sector). Weight parameters in the CES functions are called μ. Again superscript refers to sector, whereas subscript refers to nest and the variable in question. $\mu_{YM}^P > 0$ and $\mu_{YH}^P > 0$. $p_t^{PM} = p_t^{PM} \left(p_t^P, p_t^G, 1 \right)$ is the price index of materials, which is a function of the prices of materials from the domestic private and public sector and from abroad. Therefore the relation above determines the price of value added given the mentioned prices.

The capital stock, K_{t-1}^P is pre-determined, i.e. we assume that wage negotiations take place after the capital stock has been up-dated but before employment decisions are made.

The part of the productions structure that is relevant with respect to the bargaining is therefore given by the following standard CES factor demand relations

$$\bar{K}_{t-1}^P = \left(\mu_{HK}^P \right)^{\sigma_H^P} \left(\frac{MPK_t^P}{\bar{P}_t^{PH}} \right)^{-\sigma_H^P} H_t^P$$

$$L_t^P = \left(\mu_{HL}^P \right)^{\sigma_H^P} \left(\frac{W_t^P}{\bar{P}_t^{PH}} \right)^{-\sigma_H^P} H_t^P$$

where σ_H^P is the elasticity of substitution between capital and aggregate labor input in the private sector. $\mu_{HL}^P > 0$, $\mu_{HK}^P > 0$ are constants. $\bar{K}_{t-1}^P$, $\bar{P}_t^{PH}$ indicate that the capital stock and the price index of value added are considered given in the wage negotiations. MPK_t^P is the marginal product of capital in the private sector. W_t^P is the overall wage index of the private sector. The wage index is defined as

$$W_t^P = \left(\sum_{j=1}^{m} \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(W_t^j\right)^{1-\sigma_L^P} \right)^{\frac{1}{1-\sigma_L^P}}$$

where $\mu_{Lj}^P > 0$ is a constant. W_t^j is the wage index for labor of category j. The price index of value added, P_t^{PH} is defined as

$$\bar{P}_t^{PH} = \left(\left(\mu_{HL}^P\right)^{\sigma_H^P} \left(W_t^P\right)^{1-\sigma_H^P} + \left(\mu_{HK}^P\right)^{\sigma_H^P} \left(MPK_t^P\right)^{1-\sigma_H^P} \right)^{\frac{1}{1-\sigma_H^P}}$$

Using the demand relationships for capital and labor we may write labor demand as a function of the capital stock and the relative price

$$L_t^P = \left(\frac{\mu_{HL}^P}{\mu_{HK}^P} \frac{MPK_t^P}{W_t^P} \right)^{\sigma_H^P} \bar{K}_{t-1}^P$$

To see that the relative price is a function of the wage we rewrite the price index of value added to yield

$$\frac{\mu_{HL}^P}{\mu_{HK}^P} \frac{MPK_t^P}{W_t^P} = \left(\frac{1}{\mu_{HK}^P} \left(\frac{\mu_{HL}^P \bar{P}_t^{PH}}{W_t^P} \right)^{1-\sigma_{PYH}} - \frac{\mu_{HL}^P}{\mu_{HK}^P} \right)^{\frac{1}{1-\sigma_H^P}}$$

Observing that the right-hand side is a function of the wage index of aggregate labor we define

$$\eta^P \left(W_t^P\right) = \left(\frac{1}{\mu_{HK}^P} \left(\frac{\mu_{HL}^P \bar{P}_t^{PH}}{W_t^P} \right)^{1-\sigma_H^P} - \frac{\mu_{HL}^P}{\mu_{HK}^P} \right)^{\frac{1}{1-\sigma_H^P}} \quad , \quad \eta^{P\prime} \left(W_t^P\right) < 0$$

Inserting the latter to relations into the demand for aggregate labor yields

$$L_t^P = \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P$$

APPENDIX 4: Derivation of the demand for the aggregate labor index in the public sector

The public managers choose the optimal input of materials and the different categories of labor to solve the following maximization problem

$$\max \quad F\left(M_t^G, H^G\left(\bar{K}_{t-1}^G, L_t^G\right)\right)$$

subject to

$$P_t^{GM} M_t^G + W_t^G L_t^G = \bar{R}_t$$

The first order conditions are:

$$\begin{aligned} \frac{\partial F^G}{\partial M_t^G} - \lambda P_t^{GM} &= 0 \\ \frac{\partial F^G}{\partial H_t^G}\frac{\partial H_t^G}{\partial L_t^G} - \lambda W_t^G &= 0 \end{aligned}$$

such that

$$\frac{\frac{\partial F^G}{\partial M_t^G}}{\frac{\partial F^G}{\partial H_t^G}\frac{\partial H_t^G}{\partial L_t^G}} = \frac{P_t^{GM}}{W_t^G}$$

or

$$\frac{\frac{\partial F^G}{\partial M_t^G}}{\frac{\partial F^G}{\partial H_t^G}} = \frac{P_t^{GM}}{W_t}\frac{\partial H_t^G}{\partial L_t^G}$$

Using the specified nest structure of the production technology the derivatives may be written as

$$\begin{aligned} \frac{\partial F^G}{\partial M_t^G} &= \frac{\mu_{YM}^G \left(M_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}{\mu_{YM}^G \left(M_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}} + \mu_{YH}^G \left(H_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}} \frac{Y_t^G}{M_t^G} \\ \frac{\partial F^G}{\partial H_t^G} &= \frac{\mu_{YH}^G \left(H_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}{\mu_{YM}^G \left(M_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}} + \mu_{YH}^G \left(H_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}} \frac{Y_t^G}{H_t^G} \\ \frac{\partial H_t^G}{\partial L_t^G} &= \frac{\mu_{HL}^G \left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}{\mu_{HK}^G \left(\bar{K}_{t-1}^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}} + \mu_{HL}^G L^{\frac{\sigma_H^G-1}{\sigma_H^G}}} \frac{H_t^G}{L_t^G} \end{aligned}$$

where the symbol σ indicates an elasticity of substitution. Superscript G indicates production technology of the public sector, whereas subscript indicates the nest in question. This implies that σ_Y^G is the elasticity of substitution between materials $\left(M_t^G\right)$ and the index of value added $\left(H_t^G\right)$ (which is the $Y-nest$) in the public production, σ_H^G is the elasticity of substitution between capital $\left(\bar{K}_{t-1}^G\right)$ and labor $\left(L_t^G\right)$ in the public sector (which is the $H-nest$). The symbol μ indicates a weight parameter in the CES nest. Again superscript indicates the production sector, whereas the subscript indicates the nest and the input factor in question. Thus e.g. μ_{YM}^G is the weight of materials in the $Y-nest$ of the public sector. Finally Y_t^G is the production in the public sector.

Inserting these derivatives into the reduced first order condition above yields

$$\frac{\mu_{YM}^G\left(M_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}{\mu_{YH}^G\left(H_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}\frac{H_t^G}{M_t^G}=\frac{P_t^{GM}}{W_t^G}\frac{\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}{\mu_{HK}^G\left(\bar{K}_{t-1}^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}+\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}\frac{H_t^G}{L_t^G}$$

$\Leftrightarrow$

$$\frac{\mu_{YM}^G\left(M_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}{\mu_{YH}^G H_t^G\left(\bar{K}_{t-1}^G,L_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}\frac{L_t^G}{M_t^G}=\frac{P_t^{GM}}{W_t^G}\frac{\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}{\mu_{HK}^G\left(\bar{K}_{t-1}^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}+\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}$$

substitution of budget constraint

$$P_t^{GM}M_t^G+W_t^G L_t^G=\bar{R}_t$$

into the expression implies

$$\frac{\mu_{YM}^G\left(\frac{\bar{R}_t-W_t^G L_t^G}{P_t^{GM}}\right)}{\mu_{YH}^G H_t^G\left(\bar{K}_{t-1}^G,L_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}}\frac{L_t^G}{\bar{R}_t-W_t^G L_t^G}=\frac{1}{W_t^G}\frac{\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}{\mu_{HK}^G\left(\bar{K}_{t-1}^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}+\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}}$$

$\Leftrightarrow$

$$\mu_{YM}^G\left(P_t^{GM}\right)^{\frac{1-\sigma_Y^G}{\sigma_Y^G}}\left(\bar{R}_t-W_t^G L_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}W_t^G L_t^G\left(\mu_{HK}^G\left(\bar{K}_{t-1}^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}\right)+$$

$$\mu_{YM}^G\left(P_t^{GM}\right)^{\frac{1-\sigma_Y^G}{\sigma_Y^G}}\left(\bar{R}_t-W_t^G L_t^G\right)^{\frac{\sigma_Y^G-1}{\sigma_Y^G}}W_t^G L_t^G\mu_{HK}^G\left(\mu_{HL}^G\left(L_t^G\right)^{\frac{\sigma_H^G-1}{\sigma_H^G}}\right)$$

$$= \quad \mu^G_{HL} \left(L^G_t\right)^{\frac{\sigma^G_H - 1}{\sigma^G_H}} \left(\bar{R}_t - W^G_t L^G_t\right) \mu^G_{YH} H^G_t \left(\bar{K}^G_{t-1}, L^G_t\right)^{\frac{\sigma^G_Y - 1}{\sigma^G_Y}}$$

$\Leftrightarrow$

$$\mu^G_{YM} \left(P^{GM}_t\right)^{\frac{1-\sigma^G_Y}{\sigma^G_Y}} \left(\bar{R}_t - W^G_t L^G_t\right)^{\frac{-1}{\sigma^G_Y}} W^G_t L^G_t \; H^G_t \left(\bar{K}^G_{t-1}, L^G_t\right)^{\frac{\sigma^G_H - 1}{\sigma^G_H}}$$
$$= \quad \mu^G_{HL} \left(L^G_t\right)^{\frac{\sigma^G_H - 1}{\sigma^G_H}} \mu^G_{YH} H^G_t \left(\bar{K}^G_{t-1}, L^G_t\right)^{\frac{\sigma^G_Y - 1}{\sigma^G_Y}}$$

$\Leftrightarrow$

$$\left(P^{GM}_t\right)^{\frac{1-\sigma^G_Y}{\sigma^G_Y}} \left(\bar{R}_t - W^G_t L^G_t\right)^{\frac{-1}{\sigma^G_Y}} W^G_t \left(L^G_t\right)^{\frac{1}{\sigma^G_H}} \;\; H^G_t \left(\bar{K}^G_{t-1}, L^G_t\right)^{\frac{\sigma^G_H - 1}{\sigma^G_H} - \frac{\sigma^G_Y - 1}{\sigma^G_Y}}$$
$$= \quad \frac{\mu^G_{HL} \mu^G_{YH}}{\mu^G_{YM}}$$

$\Leftrightarrow$

$$\left(\bar{R}_t - W^G_t L^G_t\right)^{-\frac{1}{\sigma^G_Y}} W^G_t \left(L^G_t\right)^{\frac{1}{\sigma^G_H}} \;\; H^G_t \bar{K}^G_{t-1}, L^G_t = \frac{\mu^G_{HL} \mu^G_{YH}}{\mu^G_{YM}} \left(P^{GM}_t\right)^{\frac{\sigma^G_{Y-1}}{\sigma^G_Y}}$$

where

$$\beta \equiv \frac{\sigma^G_H - 1}{\sigma^G_H} - \frac{\sigma^G_Y - 1}{\sigma^G_Y}$$

APPENDIX 5: Derivation of the total demand for the aggregate labor index

The demand for aggregate labor in the two sectors are given by:

$$\int_0^{m_j} w_t^{Pj}(i)\, l_t^{Pj}(i)\, di = W_t^{Pj} \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{W_t^{Pj}}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P$$

$$\int_0^{m_j} w_t^{Gj}(i)\, l_t^{Gj}(i)\, di = W_t^{Gj} \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{W_t^{Gj}}{W_t^G}\right)^{-\sigma_L^G} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right)$$

Using the fact that $\frac{w^{sj}(i)}{W^{sj}} = \rho^j(i)$ the sum the demand for category j in all sectors becomes

$$\begin{aligned} & \int_0^{m_j} \rho^j(i)\, l_t^{Pj}(i)\, di + \int_0^{m_j} \rho^j(i)\, l_t^{Gj}(i)\, di \\ = & \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{W_t^{Pj}}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P \\ & + \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{W_t^{Gj}}{W_t^G}\right)^{-\sigma_L^G} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \end{aligned}$$

From the fact that workers are employed up to productivity category r^{sj}

$$\begin{aligned} & \int_0^{r^{Pj}} \rho^j(i)\, l_t^{Pj}\, di + \int_0^{r^{Gj}} \rho^j(i)\, l_t^{Gj}\, di \\ = & \left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{W_t^{Pj}}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_{PYH}} \bar{K}_{t-1}^P \\ & + \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{W_t^{Gj}}{W_t^G}\right)^{-\sigma_L^G} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \end{aligned}$$

Using the fact that

$$\begin{aligned} W^{Pj} &= W^{Gj} \equiv W^j \\ r^{Pj} &= r^{Gj} \equiv r^j \end{aligned}$$

we may rewrite the sum as

$$\int_0^{r_j} \rho^j(i)\, l_t^{Pj}\, di + \int_0^{r_j} \rho^j(i)\, l_t^{Gj}\, di$$

$$=$$

$$\left[\left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{W_t^j}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P \right.$$

$$\left. \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{W_t^j}{W_t^G}\right)^{-\sigma_L^G} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \right]$$

$\Leftrightarrow$

$$\left(l_t^{Pj} + l_t^{Gj}\right) \int_0^{r^j} \rho^j(i)\, di$$

$$=$$

$$\left[\left(\mu_{Lj}^P\right)^{\sigma_L^P} \left(\frac{W_t^j}{W_t^P}\right)^{-\sigma_L^P} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_H^P} \bar{K}_{t-1}^P \right.$$

$$\left. \left(\mu_{Lj}^G\right)^{\sigma_L^G} \left(\frac{W_t^j}{W_t^G}\right)^{-\sigma_L^G} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \right]$$

Inserting the fact that there is full employment for all employed productivity types yields

$$\bar{l}^j \int_0^{r^j} \rho^j(i)\, di$$

$$=$$

$$\left[\left(\mu_j^{PL}\right)^{\sigma_{PL}} \left(\frac{W_t^j}{W_t^P}\right)^{-\sigma_{PL}} \left(\eta^P \left(W_t^P\right)\right)^{\sigma_{PYH}} \bar{K}_{t-1}^P \right.$$

$$\left. \left(\mu_j^{GL}\right)^{\sigma_{GL}} \left(\frac{W_t^j}{W_t^G}\right)^{-\sigma_{GL}} \eta^G \left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \right]$$

Finally inserting

$$w_t^{\min,j} \equiv \rho^j\left(r^j\right) W_t^j$$

We find the demand relation

$$
\begin{aligned}
\bar{l}^{j}\int_{0}^{r^{j}}\rho^{j}\left(i\right)\,di &= \left(\mu_{Lj}^{P}\right)^{\sigma_{L}^{P}}\left(\frac{\frac{w_{t}^{\min,j}}{\rho^{j}\left(r^{j}\right)}}{W_{t}^{P}}\right)^{-\sigma_{L}^{P}}\left(\eta^{P}\left(W_{t}^{P}\right)\right)^{\sigma_{H}^{P}}\bar{K}_{t-1}^{P} \\
&\quad+\left(\mu_{Lj}^{G}\right)^{\sigma_{L}^{G}}\left(\frac{\frac{w_{t}^{\min,j}}{\rho^{j}\left(r^{j}\right)}}{W_{t}^{G}}\right)^{-\sigma_{L}^{G}}\eta^{G}\left(W_{t}^{G},\bar{K}_{t}^{G},\bar{R}_{t},P_{t}^{GM}\right)
\end{aligned}
$$

APPENDIX 6:
Derivation of the objective function of the union

$$
\begin{aligned}
V_t^j &= \int_0^{m_t^j} \left(\frac{NI_t\left[\left(1-t_t^l\right) w_t^j(i)\, l_t^j(i) + b_t\left(\bar{l} - l_t^j(i)\right)\right]}{P_t^C} - \gamma_1 \frac{\gamma}{1+\gamma} l_t^j(i)^{\frac{1+\gamma}{\gamma}} \right) di \\
&= \int_0^{r_t^j} \left(\frac{NI_t\left(\left(1-t_t^l\right)\rho_j(i)\, W_t^j \bar{l}\right)}{P_t^C} - \gamma_1 \frac{\gamma}{1+\gamma} \bar{l}^{\frac{1+\gamma}{\gamma}} \right) di + \int_{r_t^j}^{m_t^j} \frac{NI_t\left(b_t \bar{l}\right)}{P_t^C} di \\
&= \int_0^{r_t^j} \frac{NI_t\left(\left(1-t_t^l\right)\rho_j(i)\, W_t^j \bar{l}\right)}{P_t^C} di - r_t^j \gamma_1 \frac{\gamma}{1+\gamma} \bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t \bar{l}\right)}{P_t^C}\left(m_t^j - r_t^j\right)
\end{aligned}
$$

where the first equality follows from the fact that all persons with a productivity higher than (or equal to) r_t^j is fully employed whereas the rest of the members are fully unemployed. Inserting the definition of the net-income function yields

$$
\begin{aligned}
V_t^j &= \int_0^{r_t^j} \frac{\left(1-t_t^l\right)\rho_j(i)\, W_t^j \bar{l}_j}{P_t^C} di - a_t \int_0^{r_t^j} \frac{\left(\left(1-t_t^l\right)\rho_j(i)\, W_t^j \bar{l}\right)^{b_t}}{P_t^C} di + \int_0^{r_t^j} \frac{d_t}{P_t^C} di \\
&\quad - r_t^j \gamma_1 \frac{\gamma}{1+\gamma} \bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t \bar{l}\right)}{P_t^C}\left(m_t^j - r_t^j\right) \\
&= \frac{\left(1-t_t^l\right) W_t^j \bar{l}}{P_t^C} \int_0^{r_t^j} \rho_j(i)\, di - a_t \frac{\left[\left(1-t_t^l\right) W_t^j \bar{l}\right]^{b_t}}{P_t^C} \int_0^{r_t^j} \rho_j(i)^{b_t} di + \frac{d_t}{P_t^C} r_t^j \\
&\quad - r_t^j \gamma_1 \frac{\gamma}{1+\gamma} \bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t \bar{l}\right)}{P_t^C}\left(m_t^j - r_t^j\right) \\
&= \frac{\left(1-t_t^l\right) W_t^j \bar{l}}{P_t^C} I_t^{2j} - a_t \frac{\left[\left(1-t_t^l\right) W_t^j \bar{l}\right]^{b_t}}{P_t^C} I_t^{3j} + \frac{d_t}{P_t^C} r_t^j \\
&\quad - r_t^j \gamma_1 \frac{\gamma}{1+\gamma} \bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t \bar{l}\right)}{P_t^C}\left(m_t^j - r_t^j\right)
\end{aligned}
$$

where

$$
I_t^{2j} \equiv \int_0^{r_t^j} \rho_j(i)\, di
$$

$$
I_t^{3j} \equiv \int_0^{r_t^j} \rho_j(i)^{b_t}\, di
$$

APPENDIX 7:
Derivation of the objective function of the employers' association

$$\frac{\pi_t^P}{P_t^C} = \frac{(1-t_t^c)}{P_t^C}\left(p_t\left(F\left(M_t^P, H_t^P\right) - \Phi(\bar{I}_t, \bar{K}_{t-1})\right) - p_t^{PM} M_t^P - (1+t_t^a)\, W_t^P L_t^P\right)$$

where π_t^P is the gross profits after corporate taxation of the representative firm in the private sector.

$$\tilde{Y}_t^P = F\left(M_t^P, H_t^P\right)$$

$$\begin{aligned} M_t^P &= \left(\mu_{YM}^P\right)^{\sigma_Y^P}\left(\frac{\bar{P}_t^{PM}}{\bar{p}_t}\right)^{-\sigma_Y^P}\tilde{Y}_t^P \\ H_t^P &= \left(\mu_{YH}^P\right)^{\sigma_Y^P}\left(\frac{\bar{P}_t^{PH}}{\bar{p}_t}\right)^{-\sigma_Y^P}\tilde{Y}_t^P \end{aligned}$$

Dividing the demand for materials with the demand for value added yields

$$\frac{M_t^P}{H_t^P} = \left(\frac{\mu_{YM}^P}{\mu_{YH}^P}\frac{\bar{P}_t^{PH}}{\bar{P}_t^{PM}}\right)^{\sigma_Y^P} \equiv \kappa_1$$

where κ_1 is a constant. Using the fact that the gross production function has constant returns to scale implies

$$\tilde{Y}_t^P = F\left(\frac{M_t^P}{H_t^P}, 1\right) H_t^P = F\left(\kappa_1, 1\right) H_t^P \equiv \kappa_2 H_t^P$$

Inserting these relations into the real gross profits yields

$$\frac{\pi_t^P}{P_t^C} = \frac{(1-t_t^c)}{P_t^C}\left(p_t\left(\kappa_2 H_t^P - \Phi(\bar{I}_t, \bar{K}_{t-1})\right) - P_t^{PM}\kappa_1 H_t^P - (1+t_t^a)\, W_t^P L_t^P\right)$$

The CES relationship between aggregate labor and value added is given as

$$H_t^P = \left(\mu_{HL}^P\right)^{-\sigma_H^P}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H^P} L_t^P$$

Substituting for H_t^P yields

$$\frac{\pi_t^P}{P_t^C} = \frac{(1-t_t^c)}{P_t^C}\left(p_t\left(\kappa_2 H_t^P - \Phi(\bar{I}_t, \bar{K}_{t-1})\right) - P_t^{PM}\kappa_1 H_t^P - (1+t_t^a)\, W_t^P L_t^P\right)$$

$\Leftrightarrow$

$$\frac{\pi_t^P}{P_t^C} = \frac{(1-t_t^c)}{P_t^C}\left(\bar{p}_t\kappa_2 - P_t^{PM}\kappa_1\right)\left(\mu_{HL}^P\right)^{-\sigma_H^P}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H^P} L_t^P -$$

$$= \frac{(1-t_t^c)}{P_t^C}\left(\left(1+t_t^a\right)W_t^P L_t^P - p_t\Phi(\bar{I}_t^P, \bar{K}_{t-1}^P)\right)$$

$\Leftrightarrow$

$$= \frac{(1-t_t^c)}{P_t^C}\left(\left(\bar{p}_t\kappa_2 - P_t^{PM}\kappa_1\right)\left(\mu_{HL}^P\right)^{-\sigma_H^P}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H^P} - \left(1+t_t^a\right)W_t^P\right) L_t^P -$$

$$\frac{(1-t_t^c)}{P_t^c} p_t\Phi(\bar{I}_t^P, \bar{K}_{t-1}^P)$$

and from Appendix 3

$$L_t^P = \eta^P\left(W_t^P\right)^{\sigma_H^P}\bar{K}_{t-1}^P$$

so that

$$\frac{\pi_t^P}{P_t^C} = \frac{(1-t_t^c)}{P_t^C}\left(\left(\bar{p}_t\kappa_2 - P_t^{PM}\kappa_1\right)\left(\mu_{HL}^P\right)^{-\sigma_H^P}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H^P}\right)\eta^P\left(W_t^P\right)^{\sigma_H^P}\bar{K}_{t-1}^P$$

$$- \left(1+t_t^a\right)\eta^P\left(W_t^P\right)^{\left(\sigma_H^P+1\right)}\bar{K}_{t-1}^P - \frac{(1-t_t^c)}{P_t^c}p_t\Phi(\bar{I}_t^P, \bar{K}_{t-1}^P)$$

APPENDIX 8: The first order condition of the Nash bargaining in the private sector

$$
\begin{aligned}
& \arg\max_{w^{\min,j}} \left\{ \left(V_t^j - \bar{V}\right)^{\lambda} \left(\frac{\pi_t^P}{P_t^C}\right)^{(1-\lambda)} \right\} \\
= \; & \arg\max_{w^{\min,j}} \left\{ \lambda \log\left(V_t^j - \bar{V}\right) + (1-\lambda)\log\left(\frac{\pi_t^P}{P_t^C}\right) \right\}
\end{aligned}
$$

The first order condition

$$
\frac{\partial\left(V_t^j - \bar{V}\right)}{\partial w^{\min,j}} \frac{1}{\left(V_t^j - \bar{V}\right)} + \frac{\partial\left(\frac{\pi_t^P}{P_t^C}\right)}{\partial w^{\min,j}} \frac{1}{\frac{\pi_t^P}{P_t^C}} = 0
$$

In the following we consider each of the 2 parts of the first order condition separately. We begin with the union. The j'th union chooses the minimum wage $w_j^{\min}$ given the minimum wages of the other unions, and has the following utility function:

$$
V_t^j - \bar{V} = \int_0^{r_t^j} \frac{NI_t\left(\left(1-t_t^l\right)\rho_j(i)\,W_t^j\bar{l}\right)}{P_t^C} di - r_t^j\left(\gamma_1\frac{\gamma}{1+\gamma}\bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t\bar{l}\right)}{P_t^C}\right)
$$

where the following conditions are given

$$
\begin{aligned}
a) \quad \bar{l}\int_0^{r_t^j} \rho_j(i)\,di &= \left(\mu_{Lj}^P\right)^{\sigma_L^P}\left(\frac{W_t^j}{W_t^P}\right)^{-\sigma_L^P}\left(\eta^P\left(W_t^P\right)\right)^{\sigma_H^P}\bar{K}_{t-1}^P \\
&\quad + \left(\mu_{Lj}^G\right)^{\sigma_L^G}\left(\frac{W_t^j}{W_t^G}\right)^{-\sigma_L^G}\eta^G\left(W_t^G, \bar{K}_t^G, \bar{R}_t, P_t^{GM}\right) \\
b) \quad W_t^s &= \left(\sum_{j=1}^{m^s}\left(\mu_{Lj}^s\right)^{\sigma_L^s}\left(W_t^j\right)^{1-\sigma_L^s}\right)^{\frac{1}{1-\sigma_L^s}}, \quad s = P,\, G \\
c) \quad \rho_i\left(r_t^i\right)W_t^i &= \bar{w}_t^{\min,i} \quad for\ i \neq j
\end{aligned}
$$

Note the difference between, W_t^s, where s specifies the sector in which the wage index is defined, and, $W_t^j = W_t^{Pj} = W_t^{Gj}$, where the index j indicates that the wage index is the productivity corrected wage index for labor market segment j. Since the productivity corrected wage for a given labor market segment is identical across sectors, the sector indicator is omitted.

The solution the maximization problem with the minimum wage $w_t^{\min,j}$ as control variable is identical to the solution of the maximization problem with the marginal employment category r_t^j,as control variable.

Direct differentiation of $\left(V_t^j - \bar{V}\right)$ yields

$$\begin{aligned} &\frac{\partial\left(V_t^j - \bar{V}\right)}{\partial r_t^j} \\ = & \frac{NI_t\left(\left(1-t_t^l\right)\rho_j\left(i\right)W_t^j\bar{l}\right)}{P_t^C} - \left(\gamma_1\frac{\gamma}{1+\gamma}\bar{l}^{\frac{1+\gamma}{\gamma}} + \frac{NI_t\left(b_t\bar{l}\right)}{P_t^C}\right) \\ & + \frac{\partial W_t^j}{\partial r_t^j}\bar{l}_j\int_0^{r_t^j}\frac{NI_t'\left(\left(1-t_t^l\right)\rho_j\left(i\right)W_t^j\bar{l}\right)}{P_t^C}\left(1-t_t^l\right)\rho_j\left(i\right)di \qquad \text{(A8.1)} \end{aligned}$$

Observe from the definition of the NI_t function

$$\begin{aligned} NI_t\left(X_t^j\right) &= X_t^j - a_t\left(X_t^j\right)^{b_t} + d_t \quad \Leftrightarrow \\ NI_t'\left(X_t^j\right) &= 1 - b_t a_t\left(X_t^j\right)^{b_t-1} \end{aligned}$$

Therefore the integral in $\frac{\partial\left(V_t^j-\bar{V}\right)}{\partial r_t^j}$ becomes

$$\begin{aligned} & \int_0^{r_t^j}\frac{NI_t'\left(\left(1-t_t^l\right)\rho_j\left(i\right)W_t^j\bar{l}\right)}{P_t^C}\left(1-t_t^l\right)\rho_j\left(i\right)di \\ = & \int_0^{r_t^j}\frac{1-ba\left(\left(1-t_t^l\right)\rho_j\left(i\right)W_t^j\bar{l}\right)^{b-1}}{P_t^C}\rho_j\left(i\right)di \\ = & \frac{\left(1-t_t^l\right)}{P_t^C}\left(I_t^{2j} - b_t a_t\left(W_t^j\bar{l}\right)^{b_t-1}\int_0^{r_t^j}\rho_j\left(i\right)^{b_t}di\right) \\ = & \frac{\left(1-t_t^l\right)}{P_t^C}\left(I_t^{2j} - b_t a_t\left(W_t^j\bar{l}\right)^{b_t-1}I_t^{3j}\right) \qquad \text{(A8.2)} \end{aligned}$$

where I_t^{2j}, I_t^{3j} are defined in appendix 6. Since $NI_t\left(\left(1-t_t^l\right)\rho_j\left(i\right)W_t^j\bar{l}\right)$ and $NI_t\left(b_t\bar{l}\right)$ are given by the definition of the NI_t- function, it remains to find $\frac{\partial W_t^j}{\partial r_j}$.

Total differentiation of condition a) yields

$$\begin{aligned}
&\bar{l}\rho^{i}\left(r_t^i\right)\frac{\partial r_t^i}{\partial r_t^j} \\
= \; & L_t^{Pi}\left(-\sigma_L^P\frac{\partial W_t^i}{\partial r_t^j}\frac{1}{W_t^i}+\left(\sigma_L^P+\sigma_H^P\frac{\eta^{P\prime}\left(W_t^P\right)W_t^P}{\eta^P\left(W_t^P\right)}\right)\frac{\partial W_t^P}{\partial r_t^j}\frac{1}{W_t^P}\right)+ \\
& L_t^{Gi}\left(-\sigma_L^G\frac{\partial W_t^i}{\partial r_t^j}\frac{1}{W_t^i}+\left(\sigma_L^G+\frac{\eta^{G\prime}\left(W_t^G\right)W_t^G}{\eta^G\left(W_t^G\right)}\right)\frac{\partial W_t^G}{\partial r_t^j}\frac{1}{W_t^G}\right)
\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}
&\left(L_t^{Pi}\sigma_L^P+L_t^{Gi}\sigma_L^G\right)\frac{\partial W_t^i}{\partial r_t^j}\frac{1}{W_t^i}+\bar{l}\rho^i\left(r_t^i\right)\frac{\partial r_i}{\partial r_j} \\
= \; & L_t^{Pi}\left(\sigma_L^P+\sigma_H^P\frac{\eta^{P\prime}\left(W_t^P\right)W_t^P}{\eta^P\left(W_t^P\right)}\right)\frac{\partial W_t^P}{\partial r_t^j}\frac{1}{W_t^P} \\
& +L_t^{Gi}\left(\sigma_L^G+\frac{\eta^{G\prime}\left(W_t^G\right)W_t^G}{\eta^G\left(W_t^G\right)}\right)\frac{\partial W_t^G}{\partial r_t^j}\frac{1}{W_t^G}
\end{aligned}$$

Total differentiation of c) yields

$$\rho^{i\prime}\left(r_t^i\right)W_t^i\frac{\partial r_t^i}{\partial r_t^j}=-\rho^i\left(r_t^i\right)\frac{\partial W_t^i}{\partial r_t^j}$$

$\Leftrightarrow$

$$\frac{\partial r_t^i}{\partial r_t^j}=-\frac{\rho^i\left(r_t^i\right)}{\rho^{i\prime}\left(r_t^i\right)}\frac{\partial W_t^i}{\partial r_t^j}\frac{1}{W_t^i},\quad i\neq j$$

Substitution this expression into the expression above yields

$$\begin{aligned}
\frac{\partial W_t^i}{\partial r_t^j} \; = \; & \frac{L_t^{Pi}\left(\sigma_L^P+\sigma_H^P\frac{\eta^{P\prime}\left(W_t^P\right)W_t^P}{\eta^P\left(W_t^P\right)}\right)\frac{\partial W_t^P}{\partial r_t^j}\frac{W_t^i}{W_t^P}}{\left(\left(L_t^{Pi}\sigma_L^P+L_t^{Gi}\sigma_L^G\right)-\bar{l}\rho^i\left(r_t^i\right)\frac{\rho^i\left(r_t^i\right)}{\rho^{i\prime}\left(r_t^i\right)}\right)}+ \\
& \frac{L_t^{Gi}\left(\sigma_L^G+\frac{\eta^{G\prime}\left(W_t^G\right)W_t^G}{\eta^G\left(W_t^G\right)}\right)\frac{\partial W_t^G}{\partial r_t^j}\frac{W_t^i}{W_t^G}}{\left(\left(L_t^{Pi}\sigma_L^P+L_t^{Gi}\sigma_L^G\right)-\bar{l}\rho^i\left(r_t^i\right)\frac{\rho^i\left(r_t^i\right)}{\rho^{i\prime}\left(r_t^i\right)}\right)}
\end{aligned}$$

for $i\neq j$

It follows directly from the total differentiation of a) that

$$\frac{\partial W_t^j}{\partial r_t^j} = \frac{L_t^{Pi}\left(\sigma_L^P + \sigma_H^P \frac{\eta^{P\prime}(W_t^P)W_t^P}{\eta^P(W_t^P)}\right)\frac{\partial W_t^P}{\partial r_t^j}\frac{W_t^j}{W_t^P}}{(L_t^{Pi}\sigma_L^P + L_t^{Gi}\sigma_L^G)} + \frac{L_t^{Gi}\left(\sigma_L^G + \frac{\eta^{G\prime}(W_t^G)W_t^G}{\eta^G(W_t^G)}\right)\frac{\partial W_t^G}{\partial r_t^j}\frac{W_t^j}{W_t^G} - \bar{l}\rho^i\left(r_t^i\right)W_t^j}{(L_t^{Pi}\sigma_L^P + L_t^{Gi}\sigma_L^G)} \tag{A8.3}$$

Total differentiation of b) yields:

$$\frac{\partial W_t^s}{\partial r_t^j}\frac{1}{W_t^s} = \sum_{i=1}^{m^s}\frac{W_t^i L_t^{si}}{W_t^s L_t^s}\frac{\partial W_t^i}{\partial r_t^j}\frac{1}{W_t^i}$$

Inserting the two expressions for $\frac{\partial W_t^i}{\partial r_t^j}\frac{1}{W_t^i}$ (for $i = j$ and for $i \neq j$) from above yields

$$\begin{aligned}\frac{\partial W_t^s}{\partial r_t^j} = {} & \frac{L_t^{sj}}{L_t^s}\frac{L_t^{Pi}\left(\sigma_L^P + \sigma_H^P \frac{\eta^{P\prime}(W_t^P)W_t^P}{\eta^P(W_t^P)}\right)\frac{\partial W_t^P}{\partial r_t^j}\frac{W_t^j}{W_t^P}}{(L_t^{Pi}\sigma_L^P + L_t^{Gi}\sigma_L^G)} + \\ & \frac{L_t^{sj}}{L_t^s}\frac{L_t^{Gi}\left(\sigma_L^G + \frac{\eta^{G\prime}(W_t^G)W_t^G}{\eta^G(W_t^G)}\right)\frac{\partial W_t^G}{\partial r_t^j}\frac{W_t^j}{W_t^G} - \bar{l}\rho^i\left(r_t^i\right)W_t^j}{(L_t^{Pi}\sigma_L^P + L_t^{Gi}\sigma_L^G)} + \\ & \sum_{i\neq j}^{m^s}\frac{L_t^{si}}{L_t^s}\frac{L_t^{Pi}\left(\sigma_L^P + \sigma_H^P \frac{\eta^{P\prime}(W_t^P)W_t^P}{\eta^P(W_t^P)}\right)\frac{\partial W_t^P}{\partial r_t^j}\frac{W_t^i}{W_t^P}}{\left((L_t^{Pi}\sigma_L^P + L_t^{Gi}\sigma_L^G) - \bar{l}\rho^i\left(r_t^i\right)\frac{\rho^i(r_t^i)}{\rho^{i\prime}(r_t^i)}\right)} + \\ & \sum_{i\neq j}^{m^s}\frac{L_t^{si}}{L_t^s}\frac{L_t^{Gi}\left(\sigma_L^G + \frac{\eta^{G\prime}(W_t^G)W_t^G}{\eta^G(W_t^G)}\right)\frac{\partial W_t^G}{\partial r_t^j}\frac{W_t^i}{W_t^G}}{\left((L_t^{Pi}\sigma_L^P + L_t^{Gi}\sigma_L^G) - \bar{l}\rho^i\left(r_t^i\right)\frac{\rho^i(r_t^i)}{\rho^{i\prime}(r_t^i)}\right)}\end{aligned} \tag{A8.4}$$

$$\text{for} \quad s = P, G$$

$\frac{\partial\left(V_t^j - \bar{V}\right)}{\partial r_t^j}$ is found by inserting (A8.4) in (A8.3) and inserting the result in (A8.1) along with (A8.2). This yields the expression for the union part of the first order condition to the first order condition of the Nash bargaining problem used in the model. The expression cannot be solved analytically but is solved numerically in the computer program of the model.

The final part of this appendix is concerned with the employers' part of the first order condition to the Nash bargaining problem.

Direct differentiation of the profit, π implies

$$\frac{1}{1-t^c}\frac{\partial \pi_t^P}{\partial r_t^j}$$

$$= \left(\sigma_H \left(p_t\kappa_2 - p_t^{PM}\kappa_1\right)(\mu_{HL})^{-\sigma_H}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H-1}\frac{1}{\bar{P}_t^{PH}}\right)\eta\left(W_t^P\right)^{\sigma_H}\bar{K}_{t-1}^P\frac{\partial W_t^P}{\partial r_t^j} -$$

$$(1+t^a)\,\eta\left(W_t^P\right)^{\sigma_H}\bar{K}_{t-1}^P\frac{\partial W_t^P}{\partial r_t^j}$$

$$+\left(p_t\kappa_2 - p_t^{PM}\kappa_1\right)(\mu_{HL})^{-\sigma_H}\left(\frac{W_t^P}{\bar{P}_t^{PH}}\right)^{\sigma_H}\sigma_H\eta\left(W_t^P\right)^{\sigma_H-1}\eta'\left(W_t^P\right)\bar{K}_{t-1}^P\frac{\partial W_t^P}{\partial r_t^j}$$

$$-(1+t^a)\,W_t^P\sigma_H\eta\left(W_t^P\right)^{\sigma_H-1}\eta'\left(W_t^P\right)\bar{K}_{t-1}^P\frac{\partial W_t^P}{\partial r_t^j}$$

Rearranging and using the definition of π, implies that

$$\frac{\partial \pi_t^P}{\partial r_t^j}$$

$$= \sigma_H\left(\frac{\pi_t^P}{1-t^c} + p_t\Phi(\bar{I}_t^P, \bar{K}_{t-1}^P)\right)\left(1+\frac{\eta'\left(W_t^P\right)W_t^P}{\eta\left(W_t^P\right)}\right)\frac{\partial W_t^P}{\partial r_t^P}\frac{(1-t^c)}{W_t^P}$$

$$-(1-\sigma_H)(1+t^a)\,W_t^P L_t^P\frac{\partial W_t^P}{\partial r_t^P}\frac{(1-t^c)}{W_t^P}$$

This is the final part that is inserted into the expression for the first order condition of the Nash bargaining problem.

The Working Paper Series

The Working Paper Series of the Economic Modelling Unit of Statistics Denmark documents the development of the two models, DREAM and ADAM. DREAM (Danish Rational Economic Agents Model) is a relatively new computable general equilibrium model, whereas ADAM (Aggregate Danish Annual Model) is a Danish macroeconometric model used by e.g. government agencies.

The Working Paper Series contains documentation of parts of the models, topic booklets, and examples of using the models for specific policy analyses. Furthermore, the series contains analyses of relevant macroeconomic problems – analyses of both theoretical and empirical nature. Some of the papers discuss topics of common interest for both modelling traditions.

The papers are written in either English or Danish, but papers in Danish will contain an abstract in English. If you are interested in back numbers or in receiving the Working Paper Series, phone the Economic Modelling Unit at (+45) 39 17 32 02, fax us at (+45) 39 17 39 99, or e-mail us at dream@dst.dk or adam@dst.dk. Alternatively, you can also visit our Internet home pages at http://www.dst.dk/adam or http://www.dst.dk/dream and download the Working Paper Series from there.

The following titles have been published previously in the Working Paper Series, beginning in January 1998.

****

1998:1 Thomas Thomsen: Faktorblokkens udviklingshistorie, 1991-1995. (The development history of the factor demand system, 1991-1995). [ADAM]

1998:2 Thomas Thomsen: Links between short- and long-run factor demand. [ADAM]

1998:3 Toke Ward Petersen: Introduktion til CGE-modeller. (An introduction to CGE-modelling). [DREAM]

1998:4 Toke Ward Petersen: An introduction to CGE-modelling and an illustrative application to Eastern European Integration with the EU. [DREAM]

1998:5 Lars Haagen Pedersen, Nina Smith and Peter Stephensen: Wage Formation and Minimum Wage Contracts: Theory and Evidence from Danish Panel Data. [DREAM]

1998:6 Martin B. Knudsen, Lars Haagen Pedersen, Toke Ward Petersen, Peter Stephensen and Peter Trier: A CGE Analysis of the Danish 1993 Tax Reform. [DREAM]

* * * * *

1999:1 Thomas Thomsen: Efterspørgslen efter produktionsfaktorer i Danmark. (The demand for production factors in Denmark). [ADAM]

1999:2 Asger Olsen: Aggregation in Macroeconomic Models: An Empirical Input-Output Approach. [ADAM]

1999:3 Lars Haagen Pedersen and Peter Stephensen: Earned Income Tax Credit in a Disaggregated Labor Market with Minimum Wage Contracts. [DREAM]