Chapter 8

Miscellaneous

8.1 The Foreign Sector

The domestic economy is integrated in the world economy through trade and capital flows.

We assume that domestic corporate and government bonds are perfect substitutes for foreign bonds in a perfect world bond market. This means that the uncovered interest parity must hold in the model: Given a fixed exchange rate, the domestic after-tax interest rate must equal the foreign after-tax interest rate. Furthermore, residence-based taxation of interest income implies that the domestic pre-tax interest rate is equal to the exogenous foreign pre-tax interest rate.

The modelling of relations with the foreign sector includes three key equations describing exports, the trade balance and the current account. In addition to that, a number of equations keep track of aggregate transfers to and from the foreign sector, which are used in the determination of the current account. Note that imports are not modelled independently here as they are already given from the solutions to the intratemporal optimization problems of firms, households and the government sector.

As to the modelling of goods trading we make use of the Armington assumption[1], in which goods are assumed to be qualitatively different if they are produced in different countries. This means that we specify domestic goods to be imperfect substitutes for foreign goods with the result that domestic producers face a downward-sloping export demand function. This implies that the economy has endogenous terms of trade with the rest of the world. As for

[1] Armington, P.S. (1969)

imports, any import demanded at the world price P_t^F is supplied.

[As is well known from classical trade theory, the introduction of endogenous terms of trade implies that there exists a positive optimal tariff. In the present version of the model, any policy that introduces a marginal cut back on domestic production will generate a positive terms of trade effect that ceteris paribus tends to increase the utility of the representative consumer at the expense of consumers abroad. Thus the endogenous terms of trade introduces the possibility of using "beggar thy neighbour policies" to increase domestic welfare.]

The foreign demand for domestically produced goods can be thought of as demand functions derived from intertemporal optimization by foreigners. For simplicity, it is assumed that the foreign demand functions are isoelastic and that the position of the demand curves are fixed through time. (8.1) consequently determines exports as a function of the price of domestic goods of sector k, $P_{k,t}^Y$ (which is measured relative to the price of the foreign good):

$$X_{k,t} = \mu_k^X \left[\left(1 + t_t^{X,DutyV} + t_{k,t}^{X,DutyQ} - s_t^{G,X,Spe} - s_t^{EU,X,Exp} - s_t^{EU,X,Spe} \right) P_{k,t}^Y \right]^{\varepsilon_k}, \quad k \in \{C, P, G\}, \tag{8.1}$$

where $P_{k,t}^Y$ is the domestic good price, $\mu_k^X \geq 0$ is a scale parameter, and $\varepsilon_k = -5$ is the price elasticity of export demand. The term $\left(1 + t_t^{X,DutyV} + t_{k,t}^{X,DutyQ} - s_t^{G,X,Spe} - s_t^{EU,X,Exp} - s_t^{EU,X,Spe} \right)$ represents taxes and subsidies that affect the domestic price faced by foreigners. Thus $t_t^{X,DutyV}$ and $t_{k,t}^{X,DutyQ}$ represent product-specific value and quantity tax rates on exports respectively, while $s_t^{G,X,Spe}$ denotes product-specific government subsidy rates on exports. $s_t^{EU,X,Exp}$ represents the EU export subsidy rate, and $s_t^{EU,X,Spe}$ finally denotes the rate of other EU product-specific subsidies on exports.

The trade balance is defined as the value of exports minus the value of imports:

$$
\begin{aligned}
TB_t \;=\; & \sum_{k\in\{C,P,G\}} \left(1+t_t^{X,DutyV}+t_{k,t}^{X,DutyQ}-s_t^{G,X,Spe}-s_t^{EU,Exp}-s_t^{EU,X,Spe}\right) P_{k,t}^{Y} X_{k,t} && (8.2a)\\
& -P_t^F \times \Big[\sum_{e\in\{D,R,G,P\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} C_{e,k,c,t}^{H,2} && (8.2b)\\
& \quad + \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} C_{k,c,t}^{G,2} && (8.2c)\\
& \quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} M_{j,k,c,t}^{2} && (8.2d)\\
& \quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P\}} \sum_{c\in\{F\}} I_{j,k,c,t}^{P,B,2} && (8.2e)\\
& \quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{P\}} \sum_{c\in\{F\}} I_{j,k,c,t}^{P,M,2} && (8.2f)\\
& \quad + \sum_{j\in\{C,P\}} I_{j,t}^{F,I} \Big]. && (8.2g)
\end{aligned}
$$

Thus imports consist of goods for household consumption in $(8.2b)$, goods for government consumption $(8.2c)$, goods for material inputs into production $(8.2d)$ and finally goods for building and machinery investments $(8.2e)$ and $(8.2f)$. The term $I_{j,t}^{FI}$ captures inventory investments.

Note that the above representation formally sums over imports of all non-dwelling sectors, even though the standard version of DREAM treats imports as coming solely from the foreign manufacturing sector.

The current account (8.3) is defined as the trade balance TB_t plus transfers from foreigners to households $o_t^{F,H} \sum_{a\in Ax0} N_{a,t}^{Adult}$, net transfers from foreigners to the government sector $OR_t^{F,G,Net}$ and the two last tems which represent interest payments on foreign assets. $i_t A_{t-1}^F$ represents the interest payments on total foreign assets when the interest rate is the ordinary bond rate i_t. The last term represents the supplement to the interest rate paid by the government sector (the supplement originates from the calibration process and is explained in chapter 7 on the government sector). As it is assumed that all of this supplement is paid to foreign holders of government bonds, it enters the definition of the current account.

$$
CA_t = TB_t + \sum_{a\in Ax0} N_{a,t}^{Adult} o_t^{F,H} + OR_t^{F,G,Net} + i_t A_{t-1}^F - i_t^{G,Add} D_{t-1}^G. \tag{8.3}
$$

$OR_t^{F,G,Net}$ in (8.3) is the net sum of all transfers between foreigners and the government sector and given by:

$$\begin{aligned} OR_t^{F,G,Net} &= OR_t^{EU,G} + OR_t^{F,G,Res} + OR_t^{F,G,cap} - OR_t^{G,F,Res} - OR_t^{G,FI} - OR_t^{G,GR} \\ &- OR_t^{G,F,cap} - OR_t^{G,EU,GNI} - OR_t^{G,EU,Res} - OR_t^{G,EU,VAT} + SR_t^{EU,G} - OR_t^{G,EU,Cus}. \end{aligned} \quad (8.4)$$

Net transfers from foreigners to the government consist of transfers from the EU $OR_t^{EU,G}$, remaining transfers from foreign countries $OR_t^{F,G,Res}$, capital transfers from foreign countries $OR_t^{F,G,cap}$, transfers from the government to foreign countries (mainly development aid) $OR_t^{G,F,Res}$, transfers to the Faroe Islands and Greenland $OR_t^{G,FI}$ and $OR_t^{G,GR}$, capital transfers from the government to foreigners $OR_t^{G,F,cap}$, GNI contributions to the EU $OR_t^{G,EU,GNI}$, other contributions to the EU $OR_t^{G,EU,Res}$, VAT contributions to the EU $OR_t^{G,EU,VAT}$, total subsidies from the EU $SR_t^{EU,G}$ and finally total customs tax revenues paid to the EU $OR_t^{G,EU,Cus}$.

Transfers and subsidies

A number of these transfer/subsidy variables are aggregates or calculated on the basis of other variables. These will be explained in the following.

The GNI contribution $OR_t^{G,EU,GNI}$ to the EU in (8.4) is a fraction of Gross National Income and given by

$$OR_t^{G,EU,GNI} = o_t^{G,EU,GNI} Y_t^{GNI}, \quad (8.5)$$

where the rate $o_t^{G,EU,GNI}$ is calibrated to match aggregate contributions. Y_t^{GNI} is defined later on page 236.

Total VAT contributions to the EU, $OR_t^{G,EU,VAT}$ defined in (8.6a) - (8.6f) below, is found as a fraction $o_t^{G,EU,VAT}$ of all VAT collected by the government. Thus (8.6b) gives the amount of collected VAT from household consumption of domestic and foreign goods, where $t_{h,t}^{H,Cus}$ represent the customs tax rate on household demand. The following lines represent the collection of VAT on government demand (8.6c), materials demand for inputs (8.6d) , investment demand for machinery (8.6e) and finally investment demand for buildings (8.6f) .

$$
\begin{aligned}
OR_t^{G,EU,VAT} &= o_t^{G,EU,VAT} \\
&\times \Bigg[\sum_{e\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} \left(\sum_{c\in\{F\}} \left(1+t_{e,t}^{H,Cus}\right) P_t^F C_{e,k,c,t}^{H,2} + \sum_{c\in\{D\}} P_{k,t}^Y C_{e,k,c,t}^{H,2} \right) \\
&\quad + \sum_{k\in\{C,P,G\}} \left(\sum_{c\in\{F\}} \left(1+t_t^{G,Cus}\right) P_t^F C_{k,c,t}^{G,2} + \sum_{c\in\{D\}} P_{k,t}^Y C_{k,c,t}^{G,2} \right) \\
&\quad + \sum_{j\in\{C,P,G\}} \left(\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{M,Cus}\right) P_t^F M_{j,k,c,t}^2 + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y M_{j,k,c,t}^2 \right) \\
&\quad + \sum_{j\in\{C,P,G\}} \left(\sum_{k\in\{P\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{IM,Cus}\right) P_t^F I_{j,k,c,t}^{P,M,2} + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,M,2} \right) \\
&\quad + \sum_{j\in\{C,P,G,D\}} \left(\sum_{k\in\{C,P\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{IB,Cus}\right) P_t^F I_{j,k,c,t}^{P,B,2} + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,B,2} \right) \Bigg]
\end{aligned}
$$

Total transfers from the EU $SR_t^{EU,G}$ are given in $(8.7a)$ - $(8.7h)$. They are composed of subsidies related to rural set-aside schemes $SR_{i,t}^{EU,P,SetAside}$ and other subsidies related to rural land $SR_{i,t}^{EU,P,Rural}$ in $(8.7a)$. Furthermore, a range of product-specific subsidies from the EU apply to the different components of aggregate demand. These include subsidies on materials input in production at rate $s_{i,t}^{EU,Res} + s_{i,t}^{EU,P,Spe}$ in $(8.7b)$ and $(8.7c)$, subsidies on government consumption at rate $s_t^{EU,G,Spe}$ in $(8.7d)$, export subsidies at rate $s_t^{EU,X,Exp} + s_t^{EU,X,Spe}$ in $(8.7e)$, subsidies on household consumption at rate $s_{e,t}^{EU,H,Spe}$ in $(8.7f)$ and subsidies on investment demand in machinery and buildings at rates $s_{i,t}^{EU,IM,Spe}$ and $s_{i,t}^{EU,IB,Spe}$ in $(8.7g)$ and $(8.7h)$, respectively.

$$
\begin{aligned}
SR_t^{EU,G} = & \sum_{j\in\{C,P\}} \left(s_{j,t}^{EU,P,SetAside} + s_{j,t}^{EU,P,Rural} \right) && (8.7a)\\
& + \sum_{j\in\{C,P,G\}} \Bigg[\left(s_{j,t}^{EU,P,Res} + s_{j,t}^{EU,P,Spe} \right) && (8.7b)\\
& \times \left(\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(1 + t_{j,t}^{M,Cus}\right) P_t^F M_{j,k,c,t}^2 + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y M_{j,k,c,t}^2 \right) \Bigg] && (8.7c)\\
& + s_t^{EU,G,Spe} \left(\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(1 + t_t^{G,Cus}\right) P_t^F C_{k,c,t}^{G,2} + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y C_{k,c,t}^{G,2} \right) && (8.7d)\\
& + \left(s_t^{EU,X,Exp} + s_t^{EU,X,Spe} \right) \sum_{k\in\{C,P,G\}} P_{k,t}^Y X_{k,t} && (8.7e)\\
& + \sum_{e\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} s_{e,t}^{EU,H,Spe} \left(\sum_{c\in\{F\}} \left(1 + t_{e,t}^{H,Cus}\right) P_t^F C_{e,k,c,t}^{H,2} + \sum_{c\in\{D\}} P_{k,t}^Y C_{e,k,c,t}^{H,2} \right) && (8.7f)\\
& + \sum_{j\in\{C,P,G\}} s_{j,t}^{EU,IM,Spe} \left(\sum_{k\in\{P\}} \sum_{c\in\{F\}} \left(1 + t_{j,t}^{IM,Cus}\right) P_t^F I_{j,k,c,t}^{P,M,2} + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,M,2} \right) && (8.7g)\\
& + \sum_{j\in\{C,P,G,D\}} s_{j,t}^{EU,IB,Spe} \left(\sum_{k\in\{C,P\}} \sum_{c\in\{F\}} \left(1 + t_{j,t}^{IB,Cus}\right) P_t^F I_{j,k,c,t}^{P,B,2} + \sum_{k\in\{C,P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,B,2} \right) && (8.7h)
\end{aligned}
$$

$OR_t^{G,EU,Cus}$ is the total customs tax revenue paid to the EU by the government. As can be seen from $(8.8a)$ - $(8.8f)$, this amounts to the revenue from all customs taxes collected by the government. I.e. customs taxes on imports of household consumption goods in $(8.8a)$, imports for government sector consumption $(8.8c)$, imports of materials for input in production in $(8.8d)$, imports for investments in machinery and buildings in production in $(8.8e)$ and $(8.8f)$, imports of inventory investment goods in $(8.8g)$ and finally residual taxes to the EU in $(8.8h)$.

$$
\begin{aligned}
OR_t^{G,EU,Cus} &= P_t^F && \text{(8.8a)}\\
&\times \Bigg[\sum_{e\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_{d,t}^{H,Cus} C_{d,k,c,t}^{H,2} && \text{(8.8b)}\\
&\quad + \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_t^{G,Cus} C_{k,c,t}^{G,2} && \text{(8.8c)}\\
&\quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_{j,t}^{M,Cus} M_{j,k,c,t}^{2} && \text{(8.8d)}\\
&\quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P\}} \sum_{c\in\{F\}} t_{j,t}^{IB,Cus} I_{j,k,c,t}^{P,B,2} && \text{(8.8e)}\\
&\quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{P\}} \sum_{c\in\{F\}} t_{j,t}^{IM,Cus} I_{j,k,c,t}^{P,M,2} && \text{(8.8f)}\\
&\quad + \sum_{j\in\{C,P\}} t_t^{II,Cus} I_{j,t}^{FI} \Bigg] && \text{(8.8g)}\\
&+OR_t^{G,EU,Cus,Res}. && \text{(8.8h)}
\end{aligned}
$$

Total net transfers from foreigners to each adult-equivalent is given by (8.9).

$$
o_t^{F,H} = \frac{OR_t^{F,H,Exo} + OR_t^{F,H,Wage}}{\sum_{a\in Ax0} N_{a,t}^{Adult}}, \tag{8.9}
$$

where $OR_t^{F,H,Exo}$ are total residual net transfers from foreigners and $OR_t^{F,H,Wage}$ is the total net wage earned abroad by domestic inhabitants.

8.2 Equilibrium conditions

Goods markets

(8.10) states the equilibrium conditions for the goods markets

$$
\begin{aligned}
Y_{j,t} + Y_{j,t}^{NorthSea} \quad = \quad & \sum_{k \in \{C,P,G\},k=j} \Bigg[\sum_{e \in \{D,R,P,G\}} \sum_{c \in \{D\}} C_{e,k,c,t}^{H,2} \\
& + \sum_{c \in \{D\}} C_{k,c,t}^{G,2} \\
& + \sum_{j \in \{C,P,G\}} \sum_{c \in \{D\}} M_{j,k,c,t}^{2} \\
& + \sum_{j \in \{C,P,G,D\}} \sum_{c \in \{D\}} I_{j,k,c,t}^{PB,2} \\
& + \sum_{j \in \{C,P,G\}} \sum_{c \in \{D\}} I_{j,k,c,t}^{PM,2} \\
& + X_{k,t} \Bigg] + I_{j,t}^{PI}, \quad j \in \{C,P,G\}.
\end{aligned} \tag{8.10}
$$

Supply in each non-dwelling production sector must equal the total demand for the sector's output. Thus the left hand side of (8.10) represents production in sector j, where $Y_{j,t}^{NorthSea}$ is the additional resource rent from the North Sea, with $Y_{j,t}^{NorthSea} > 0$ for $j \in \{P\}$ only. The right-hand side is the sum of demands from each of the components of aggregate demand: Household consumption $C_{e,k,c,t}^{H,2}$, government sector consumption $C_{k,c,t}^{G,2}$, materials used for input in production $M_{j,k,c,t}^{2}$, investments in buildings $I_{j,k,c,t}^{PB,2}$, investments in machinery $I_{j,k,c,t}^{PM,2}$ and finally demand from exports $X_{k,t}$. The last term $I_{j,t}^{P,I}$ represents inventory investments.

In the dwelling production sector, the equilibrium condition states that total supply must equal the sum of demand of residential capital by all generations:

$$
Y_{j,t}^{Dwe} = \sum_{a} N_{a-1,t-1}^{Adult,Eq} K_{a-1,t-1}^{H}, \qquad j = D. \tag{8.11}
$$

Note that demand (and consequently, in equilibrium, supply) is modelled as the actual residential capital stock. It might be more natural to model demand as the flow of services from this stock. However, these are proportional so that

Labour market

(8.12) is the equilibrium condition for the labour market and states that aggregate effective labour supply (i.e. employment) must equal aggregate labour demand.

$$\sum_{j\in\{C,P,G\}} L_{j,t}^{D} = \sum_{o\in oLab}\sum_{s\in\{m,f\}}\sum_{a\in Ax0} N_{o,s,a,t}^{Ind} adj_t^{Hours} r_{o,s,a,t}^{LabFull}\left(1+adj_t^{LS}\right) L_{o,s,a,t}^{S}\rho_{o,s,a,t} + \frac{OR_t^{C,S,Imp}}{W_t}. \quad (8.12)$$

The term $\frac{OR_t^{C,S,Imp}}{W_t}$ in (8.12) is included to correct for the modelling of civil servants pension in the National Accounts. The NA treats the civil servants pension scheme as a funded scheme in the sense that the government sector is assumed to pay (to itself) contributions to the scheme in anticipation of future pension payouts. In the NA these contributions appear as increases in the total wage sum of the government sector. Because DREAM is calibrated to replicate the government sector wage sum, this means that the wage sum in DREAM is also increased by these fictitious contributions. This necessarily affects either the wage or the labour quantity (or both). For technical reasons, it is the quantity, i.e. government sector labour demand which is affected. In order to obtain the government sector labour demand net of this effect, we could subtract the real part of the increase in the wage sum $\frac{OR_t^{C,S,Imp}}{W_t}$ from government labour demand. Equivalently, we may add it to labour supply as it is done in (8.12).

Financial assets

(8.13) is an equilibrium condition for the allocation of financial assets. It says that the supply of financial assets from the foreign sector A_t^F, the government debt D_t^G and the sum of value and equity of domestic firms $\sum_{j\in\{C,P\}}\left(V_{j,t}+D_{j,t}^P\right)$ must equal the total demand for such assets by private households and pension funds including ATP, SP and LD: $A_t^{HZ}+A_t^{LD}+A_t^{SP,Tot}+A_t^{ATP,Tot}$.

$$A_t^F = A_t^{HZ} - \sum_{j\in\{C,P\}}\left(V_{j,t}+D_{j,t}^P\right) - D_t^G + A_t^{LD} + A_t^{SP,Tot} + A_t^{ATP,Tot}. \quad (8.13)$$

Residential land market

(8.14) represents the equilibrium condition for the residential land market. The condition is that that the size of land demanded per adult equivalent by households $K_{a,t}^{H,L,Ind}$ must be equal to the total supply of land in the economy, which is the exogenous variable $K_{D,t}^{H,L,Tot}$ (which is constant over time in the base-line projection).

$$\sum_{j\in\{D\}} K_{j,t}^{H,L,Tot} = \sum_{a} K_{a,t}^{H,L,Ind} N_{a,t}^{AdultEq}. \tag{8.14}$$

8.3 National Account Measures

In the following we will describe DREAM's representation of three central figures in the National Accounts: Gross Domestic Product at factor prices (BFI), Gross Domestic Product at market prices (BNP) and Gross National Income (BNI). Apart from being of interest in their own right, these measures are used in the model for various purposes. Thus a number of transfers to and from the government as well as collective government consumption are indexed to GDP (at market prices). Furthermore, Gross National Income is used to model the GNI contributions made by the government to the EU (see page 228 of this chapter).

(8.15) defines the gross domestic product at factor prices Y_t^{GDPF}:

$$\begin{aligned}
Y_t^{GDPF} = & \sum_{j\in\{C,P,G,D\}} P_{j,t}^{Y}\left(Y_{j,t} + Y_{j,t}^{NorthSea}\right) \\
& - \sum_{j\in\{C,P,G\}} P_{j,t}^{M} M_{j,t} - \sum_{j\in\{C,P,G\}} \left(t_{j,t}^{Emp} + t_{j,t}^{W}\right) W_t L_{j,t}^{D} \\
& - \sum_{j\in\{C,P,G\}} \left(t_{j,t}^{P,Weight} K_{j,t-1}^{P,M} + t_{j,t}^{P,Land} P_{j,t-1}^{I,P,B} K_{j,t-1}^{P,B}\right) \\
& - t_t^{H,Land} \sum_{j\in\{D\}} P_{j,t-1}^{KH,L} \sum_{a} K_{a-1,t-1}^{H,L,Ind} N_{a-1,t-1}^{AdultEq} \\
& + \sum_{j\in\{C,P\}} \left(s_{j,t}^{EU,P,SetAside} + s_{j,t}^{EU,P,Rural}\right).
\end{aligned} \tag{8.15}$$

The starting point is the sum of the values of output of each sector j measured at basic prices: $P_{j,t}^{Y}(Y_{j,t} + Y_{j,t}^{NorthSea})$. From this, we subtract intermediate consumption $P_{j,t}^{M} M_{j,t}$, which gives GDP measured at basic prices (i.e. excluding production taxes and subsidies, but including other taxes less subsidies on production - equal to the Danish term BVT). To get GDP at factor prices, we consequently add other subsidies on production (rural subsidies from the EU $s_{j,t}^{EU,P,Setaside} + s_{j,t}^{EU,P,Rural}$) and subtract other taxes on production. These include firstly wage sum taxes and other taxes related to labour in production paid by employers:

$$\sum_{j\in\{C,P,G\}} \left(t_{j,t}^{Emp} + t_{j,t}^{W}\right) W_t L_{j,t}^{D}$$

Secondly vehicle registration taxes and property taxes in the non-dwelling production sectors :

$$\sum_{j\in\{C,P,G\}} \left(t_{j,t}^{P,Weight} K_{j,t-1}^{P,M} + t_{j,t}^{P,Land} P_{j,t-1}^{I,P,B} K_{j,t-1}^{P,B} \right)$$

and thirdly property taxes (grundskyld) paid by households

$$t_t^{H,Land} \left(\sum_{j\in\{D\}} P_{j,t-1}^{KH,L} \right) \left(\sum_a K_{a-1,t-1}^{H,L} N_{a-1,t-1}^{AdultEq} \right).$$

As DREAM follows the NA convention of modelling dwelling as a production sector, property taxes paid by households are treated as a tax on production and must consequently be subtracted here in line with other taxes on production.

(8.16) shows the expression for Gross Domestic Product evaluated at market prices (BNP). As usual, the distinction between GDP at market prices and GDP at factor prices is that all taxes less subsidies on production are included when evaluating output at market prices. Hence, in (8.16) we again add the taxes on production which were subtracted in (8.15). In addition, we add all product taxes which consist of VAT (TR_t^{VAT}), motor vehicle registration duties (TR_t^{Reg}), excise duties (TR_t^{Duty}), customs taxes and the residual term TR_t^{Res} (for a definition of these collecting terms, cf. chapter 7 on the government sector) and subtract all subsidies (SR_t). Note that customs taxes TR_t^{Cus} as defined on page 175 in chapter 7 is net of the transfer of the customs tax revenue to EU ($OR_t^{G,EU,Cus}$). As this transfer does not affect GDP, it is added in (8.16). In the same way, from SR_t is subtracted the amount of EU-subsidized subsidies $SR_t^{EU,G}$.

$$\begin{aligned} Y_t^{GDP} = {} & Y_t^{GDPF} \\ & + \sum_{j\in\{C,P,G\}} \left(t_{j,t}^{Emp} + t_{j,t}^{W} \right) W_t L_{j,t}^D \\ & + \sum_{j\in\{C,P,G\}} t_{j,t}^{P,Weight} K_{j,t-1}^{P,M} + t_{j,t}^{P,Land} P_{j,t-1}^{I,P,B} K_{j,t-1}^{P,B} \\ & - \sum_{j\in\{C,P\}} \left(s_{j,t}^{EU,P,SetAside} + s_{j,t}^{EU,P,Rural} \right) \\ & + TR_t^{VAT} + TR_t^{Reg} + TR_t^{Duty} + TR_t^{Cus} + OR_t^{G,EU,Cus} + TR_t^{Res} - SR_t - SR_t^{EU,G}. \end{aligned} \tag{8.16}$$

(8.17) defines gross national income as GDP corrected for receipts of interest on net foreign assets $i_t A_{t-1}^F$ (which initially are negative), wage income received from abroad $OR_t^{F,H,Wage}$

and the EU-related transfers $SR_t^{EU,G}$ and $OR_t^{G,EU,Cus}$. The supplement to the interest rate of government debt is supposed to be paid exclusively to foreign creditors. The calibration correction factor k_t^{iYGNI} ensures that net interest payments on the foreign debt and GNI itself are calibrated to the correct NA values.

$$Y_t^{GNI} = Y_t^{GDP} + OR_t^{F,H,Wage} + SR_t^{EU,G} - OR_t^{G,EU,Cus} + k_t^{iYGNI} i_t A_{t-1}^F - i_t^{G,add} D_{t-1}^G. \quad (8.17)$$

8.4 Financial assets

The following equations are related to holdings of financial assets in households, the pension fund and in private pensions.

Recall that the composition of household financial wealth is fixed exogenously such that firm shares account for the fixed share w^{Assets} of household financial assets, while bonds account for the remaining share. In the standard version, w^{Assets} is equal to 1/3.

For a given value of household financial wealth $A_{a,t}^{H,Fin,Ind}$ of household a, we then have the following expression for the share of aggregate firm value owned per adult-equivalent by household a:

$$\frac{w_{a,t}^{H,Shares}}{k^{shares}} \sum_{j \in \{C,P\}} V_{j,t} = w^{Assets} A_{a,t}^{H,Fin,Ind}, \quad (8.18)$$

where $w_{a,t}^{H,Shares}$ is the share of aggregate firm value owned by household a, and k^{shares} is a scale parameter. Similarly, since the bequest in any period consists of household assets (left by the parent household), the share of aggregate firm value tied to the bequest is subject to the same relationship:

$$\frac{w_t^{Beq,Shares}}{k^{shares}} \sum_{j \in \{C,P\}} V_{j,t} = w^{Assets} A_t^{H,Beq}, \quad (8.19)$$

where $w_t^{Beq,Shares}$ is the share of aggregate firm value tied to the bequest left by the representative parent household. Total financial wealth in the households is found by summing over households, and noting that $A_{a,t}^{H,Fin,Ind}$ and $A_t^{H,Beq}$ are given per adult-equivalent, we get:

$$A_t^{H,Fin} = \sum_a A_{a,t}^{H,Fin,Ind} N_{a,t}^{Adult,Eq} + \sum_{a \in \{Apu\}} A_t^{H,Beq} N_{a,t}^{Adult,Eq}.$$

Using (8.18) and (8.19), we can write total household financial wealth in period t including the bequest left in period t as:

$$A_t^{H,Fin} = \frac{1}{w^{Assets}} \frac{\left(\sum_a w_{a,t}^{H,Shares} N_{a,t}^{Adult,Eq} + \sum_{a\in\{Apu\}} w_t^{Beq,Shares} N_{a,t}^{Adult,Eq}\right)}{k^{shares}} \sum_{j\in\{C,P\}} V_{j,t} \quad (8.20)$$

As a result of the exogenously fixed composition of household wealth, (8.20) expresses total household financial assets as a function of aggregate firm value only with no explicit representation of bond holdings.

Total financial assets held in relation to pensions is given as the sum of financial assets held by the Pension Fund and assets held in private pension arrangements:

$$A_t^Z = A_t^{Z,F} + A_t^{Z,P} \quad (8.21)$$

Using (8.20) and (8.21), we can write the aggregate value of financial assets in the economy as the sum of financial assets held by households and financial assets held by the Pension Fund and in private pension arrangements:

$$A_t^{H,Z} = A_t^{H,Fin} + A_t^Z. \quad (8.22)$$

8.5 Steady state

We assume that the DREAM model is saddle-path stable and has a unique steady state[2]. We will now explain the equations that bring the model to the steady state, namely the terminal conditions. To understand the role and formulation of these terminal conditions, consider the following abstract reduced-form representation of DREAM.

Suppose we have 3 vectors of unknown variables j_t , s_t and x_t in each period t, where j_t is an n-vector, s_t is an m vector and x_t is a d vector, such that we have $n + m + d$ variables each period. Furthermore, suppose there exists an intial value for each of the variables in s_t, m in total.

The $n + m + d$ variables are determined each period in a system of $n + m + d$ equations

[2]For an earlier version of DREAM, numerical simulations have confirmed the belief that the system has a unique steady state. See chapter 6 in Knudsen et al. (1998).

$$j_{t+1} = f(j_t, s_t, x_t) \qquad n \text{ equations}, \tag{8.23}$$

$$0 = h(j_t, s_t, x_t) \qquad d \text{ equations}, \tag{8.24}$$

$$s_t = g(j_t, s_{t-1}, x_t) \qquad m \text{ equations}. \tag{8.25}$$

Thus, the DREAM model is a system of non-linear equations with a number of unknown variables. As mentioned above, we assume that the system is saddle-path stable with a unique steady state.

In principle, this steady state is reached after an infinite number of periods, given a set of initial values on the unique saddle path. However, since the model simulates on a computer, it cannot run for an infinite number of periods so in practice the simulation must stop at some predefined date, which we will call period T. The assumption is that T is sufficiently large for the model to be very close to steady state in period T.

The fact that we have to cut off the simulation after a finite number of periods T is the reason why we include the terminal conditions below. To see this, note first that since we simulate the model for T periods, $t = 1, ..., T$, we have a total of $(n + m + d)\, T$ equations over the entire simulation. Obviously, to solve the model we need to have just as many unknown variables. So counting the variables, we see from (8.25) that defining the m equations for $t = 1, ..., T$ means defining $m(T + 1)$ variables of type s_t, but as mentioned there exist m initial values s_0 (consider equation (8.25) for $t = 1$) so the number of unknown variables of type s_t equals mT. From (8.24) we see that defining the d equations for $t = 1, ..., T$ means defining dT variables of type x_t. Finally, from (8.23) we have that defining the n equations for $t = 1, ..., T$ means defining $n(\,T + 1)$ variables of type j_t. In total we have $(n + m + d)\, T + n$ unknown variables, but only $(n + m + d)\, T$ equations.

Hence, we include n terminal conditions such that the number of variables equals the number of equations.

In the context of the above reduced-form system, it should be noted that in DREAM the n equations of the type (8.23) are not defined in the last solution period T. In the above framework, this just means that the n period $T + 1$ variables of type j_t are not defined either, giving the same net result as above.

Introducing an additional set of equations in the form of terminal conditions means putting further restrictions on the variables of the model, which requires knowledge of plausible restrictions. This knowledge comes from the fact that we have assumed above that T is so large that the model is in steady state (or very close to steady state). This assumption is utilized in the terminal conditions since they impose a steady state on the variables in question.

The choice of which variables to subject to a terminal condition is free in a simultaneous system. In DREAM a terminal condition is defined for the following variables:

The household real estate $K_{a,t}^{H}$, Household residential buildings $K_{a,t}^{H,B}$, household residential land $K_{a,t}^{H,L}$, residential land price $P_{i,t}^{K,H,L}$, household financial assets $A_{a,t}^{H,Fin,Ind}$, bequests $A_{t}^{H,Beq}$, the value of firms $V_{j,t_{j\in\{C,P\}}}$, the shadow price of a marginal unit of machinery capital $Q_{j,t_{j\in\{C,P\}}}^{K,P,M}$, the shadow price for a marginal unit of buildings capital $Q_{j,t_{j\in\{C,P\}}}^{K,P,B}$, the shadow price of a marginal unit of machinery book capital $Q_{j,t_{j\in\{C,P\}}}^{K,P,M,Book}$ and finally the shadow price of a marginal unit of buildings book capital $Q_{j,t_{j\in\{C,P\}}}^{K,P,B,Book}$.

The terminal conditions for the stock variables $K_{a,t}^{H}$ and $K_{a,t}^{H,B}$ have the following functional form:

$$K_{a,T}^{H} = (1+g)\, K_{a,T-1}^{H}. \tag{8.26}$$

I.e. along the balanced growth path the economy is growing at the rate of exogenous technological progress g.

The terminal conditions for the shadow prices on capital $Q_{j,t_{j\in\{C,P\}}}^{K,P,M}$ and $Q_{j,t_{j\in\{C,P\}}}^{K,P,B}$ have the following functional form:

$$Q_{j,T}^{K,P,M} = \left(1+g^{P}\right) Q_{j,T-1}^{K,P,M}, \quad j \in \{C,P\}. \tag{8.27}$$

I.e. in steady state, shadow prices on capital grow at the rate of exogenous foreign inflation.

Shadow prices on the book value of capital are constant as is the amount of land per adult-equivalent of a certain age $K_{a,t}^{H,L,Ind}$. Hence, the terminal conditions of these variables are given by

$$Q_{j,T}^{K,P,M,Book} = Q_{j,T-1}^{K,P,M,Book}, \quad j \in \{C,P\}. \tag{8.28}$$

Terminal conditions for $A_{a,t}^{H,Fin,Ind}$, $A_{t}^{H,Beq}$, $V_{j,t_{j\in\{C,P\}}}$ and the price on land $P_{i,t}^{K,H,L}$ have the following form:

$$A_{a,T}^{H,Fin,Ind} = (1+g)\left(1+g^{P}\right) A_{a,T-1}^{H,Fin,Ind}. \tag{8.29}$$

The reason that the terminal condition for the price on land $P_{i,t}^{K,H,L}$ is of the form (8.29) is that the productivity of land grows at the rate of exogenous technological progress, which is propagated onto the price on land.

Given the dimensions of the sets over which the relevant variables are defined, these 11 equations represent 90 terminal conditions in total.

8.6 Appendix: Equations describing miscellaneous behaviour in growth- and inflation-corrected terms

In this section, the equations from the preceding sections which are actually used in the computer version of the model are written in growth- and inflation-corrected terms.

Foreign Sector

Exports (8.1), the trade balance (8.2a) and the current account (8.3):

$$X_{k,t} = \mu_k^X \left[\left(1 + t_t^{X,DutyV} + t_{k,t}^{X,DutyQ} - s_t^{G,X,Spe} - s_t^{EU,X,Exp} - s_t^{EU,X,Spe}\right)\frac{P_{k,t}^{Y}}{P_t^{F}}\right]^{\varepsilon_k}, k \in \{C, P, G\}.$$

$$
\begin{aligned}
TB_t \;=\; & \sum_{k\in\{C,P,G\}} \left(1+t_t^{X,DutyV}+t_{k,t}^{X,DutyQ}-s_t^{G,X,Spe}-s_t^{EU,X,Exp}-s_t^{EU,X,Spe}\right) P_{k,t}^{Y} X_{k,t} \\
& -P_t^F \times \Big[\sum_{d\in\{D,R,G,P\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} C_{d,k,c,t}^{H,2} \\
& \quad + \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} C_{k,c,t}^{G,2} \\
& \quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} M_{j,k,c,t}^{2} \\
& \quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P\}} \sum_{c\in\{F\}} I_{j,k,c,t}^{P,B,2} \\
& \quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{P\}} \sum_{c\in\{F\}} I_{j,k,c,t}^{P,M,2} \\
& \quad + \sum_{j\in\{C,P\}} I_{j,t}^{F,I} \Big].
\end{aligned}
$$

$$
CA_t = TB_t + \sum_{a\in Ax0} N_{a,t}^{Adult} o_t^{F,H} + OR_t^{F,G,Net} + i_t \frac{A_{t-1}^F}{(1+g_t)\left(1+g_t^P\right)} - i_t^{G,Add} \frac{D_{t-1}^G}{(1+g_t)\left(1+g_t^P\right)}.
$$

Definitions of transfers (8.4), (8.5), (8.6a), (8.7a), (8.8a) and (8.9):

$$
\begin{aligned}
OR_t^{F,G,Net} \;=\; & OR_t^{EU,G} + OR_t^{F,G,Res} + OR_t^{F,G,cap} - OR_t^{G,F,Res} - OR_t^{G,FI} - OR_t^{G,GR} \\
& -OR_t^{G,F,cap} - OR_t^{G,EU,GNI} - OR_t^{G,EU,Res} - OR_t^{G,EU,VAT} + SR_t^{EU,G} - OR_t^{G,EU,Cus},
\end{aligned}
$$

$$
OR_t^{G,EU,GNI} = o_t^{G,EU,GNI} Y_t^{GNI},
$$

$$
\begin{aligned}
OR_t^{G,EU,VAT} &= o_t^{G,EU,VAT} \\
&\times \Bigg[\sum_{d\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} \left(\sum_{c\in\{F\}} \left(1+t_{d,t}^{H,Cus}\right) P_t^F C_{d,k,c,t}^{H,2} + \sum_{c\in\{D\}} P_{k,t}^Y C_{d,k,c,t}^{H,2} \right) \\
&\quad + \sum_{k\in\{C,P,G\}} \left(\sum_{c\in\{F\}} \left(1+t_t^{G,Cus}\right) P_t^F C_{k,c,t}^{G,2} + \sum_{c\in\{D\}} P_{k,t}^Y C_{k,c,t}^{G,2} \right) \\
&\quad + \sum_{j\in\{C,P,G\}} \left(\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{M,Cus}\right) P_t^F M_{j,k,c,t}^2 + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y M_{j,k,c,t}^2 \right) \\
&\quad + \sum_{j\in\{C,P,G\}} \left(\sum_{k\in\{P\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{IM,Cus}\right) P_t^F I_{j,k,c,t}^{P,M,2} + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,M,2} \right) \\
&\quad + \sum_{j\in\{C,P,G,D\}} \left(\sum_{k\in\{C,P\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{IB,Cus}\right) P_t^F I_{j,k,c,t}^{P,B,2} + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,B,2} \right) \Bigg] \quad ,
\end{aligned}
$$

$$
\begin{aligned}
SR_t^{EU,G} &= \sum_{j\in\{C,P\}} \left(s_{j,t}^{EU,P,SetAside} + s_{j,t}^{EU,P,Rural} \right) \\
&+ \sum_{j\in\{C,P,G\}} \Bigg[\left(s_{j,t}^{EU,P,Res} + s_{j,t}^{EU,P,Spe} \right) \\
&\times \left(\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{M,Cus}\right) P_t^F M_{j,k,c,t}^2 + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y M_{j,k,c,t}^2 \right) \Bigg] \\
&+ s_t^{EU,G,Spe} \left(\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(1+t_t^{G,Cus}\right) P_t^F C_{k,c,t}^{G,2} + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y C_{k,c,t}^{G,2} \right) \\
&+ \left(s_t^{EU,X,Exp} + s_t^{EU,X,Spe} \right) \sum_{k\in\{C,P,G\}} P_{k,t}^Y X_{k,t} \\
&+ \sum_{d\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} s_{d,t}^{EU,H,Spe} \left(\sum_{c\in\{F\}} \left(1+t_{d,t}^{H,Cus}\right) P_t^F C_{d,k,c,t}^{H,2} + \sum_{c\in\{D\}} P_{k,t}^Y C_{d,k,c,t}^{H,2} \right) \\
&+ \sum_{j\in\{C,P,G\}} s_{j,t}^{EU,IM,Spe} \left(\sum_{k\in\{P\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{IM,Cus}\right) P_t^F I_{j,k,c,t}^{P,M,2} + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,M,2} \right) \\
&+ \sum_{j\in\{C,P,G,D\}} s_{j,t}^{EU,IB,Spe} \left(\sum_{k\in\{C,P\}} \sum_{c\in\{F\}} \left(1+t_{j,t}^{IB,Cus}\right) P_t^F I_{j,k,c,t}^{P,B,2} + \sum_{k\in\{C,P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,B,2} \right),
\end{aligned}
$$

$$
\begin{aligned}
OR_t^{G,EU,Cus} &= P_t^F \\
&\times \Bigg[\sum_{e\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_{e,t}^{H,Cus} C_{e,k,c,t}^{H,2} \\
&\quad + \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_t^{G,Cus} C_{k,c,t}^{G,2} \\
&\quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_{j,t}^{M,Cus} M_{j,k,c,t}^{2} \\
&\quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P\}} \sum_{c\in\{F\}} t_{j,t}^{IB,Cus} I_{j,k,c,t}^{P,B,2} \\
&\quad + \sum_{j\in\{C,P,G\}} \sum_{k\in\{P\}} \sum_{c\in\{F\}} t_{j,t}^{IM,Cus} I_{j,k,c,t}^{P,M,2} \\
&\quad + \sum_{j\in\{C,P\}} t_t^{II,Cus} I_{j,t}^{FI} \Bigg] \\
&+OR_t^{G,EU,Cus,Res},
\end{aligned}
$$

$$
o_t^{F,H} = \frac{OR_t^{F,H,Exo} + OR_t^{F,H,Wage}}{\sum_{a\in Ax0} N_{a,t}^{Adult}}.
$$

Equilibrium conditions

Equilibrium conditions for the goods markets (8.10) and (8.11), the labour market (8.12) , the financial market (8.13) and the market for residential land (8.14):

:

$$\begin{aligned} Y_{j,t} + Y_{j,t}^{NorthSea} \quad = \quad & \sum_{k \in \{C,P,G,D\}} \Bigg[\sum_{e \in \{D,R,P,G\}} \sum_{c \in \{D\}} C_{e,k,c,t}^{H,2} \\ & + \sum_{c \in \{D\}} C_{k,c,t}^{G,2} \\ & + \sum_{j \in \{C,P,G\}} \sum_{c \in \{D\}} M_{j,k,c,t}^{2} \\ & + \sum_{j \in \{C,P,G,D\}} \sum_{c \in \{D\}} I_{j,k,c,t}^{PB,2} \\ & + \sum_{j \in \{C,P,G\}} \sum_{c \in \{D\}} I_{j,k,c,t}^{PM,2} \\ & + X_{k,t} \Bigg] + I_{j,t}^{PI} \qquad , j \in \{C,P,G\}. \end{aligned}$$

$$Y_{j,t}^{Dwe} = \sum_{a} N_{a-1,t-1}^{Adult,Eq} \frac{K_{a-1,t-1}^{H}}{1+g_t} \qquad , j = D.$$

$$\begin{aligned} \sum_{j \in \{C,P,G\}} L_{j,t}^{D} \quad = \quad & \sum_{o \in oLab} \sum_{s \in \{m,f\}} \sum_{a \in Ax0} N_{o,s,a,t}^{Ind} adj_t^{Hours} r_{o,s,a,t}^{LabFull} \left(1 + adj_t^{LS}\right) L_{o,s,a,t}^{S} \rho_{o,s,a,t} \\ & + \frac{OR_t^{C,S,Imp}}{W_t}. \end{aligned}$$

$$A_t^F = A_t^{HZ} - \sum_{j \in \{C,P\}} \left(V_{j,t} + D_{j,t}^P\right) - D_t^G + A_t^{LD} + A_t^{SP,Tot} + A_t^{ATP,Tot} \; .$$

$$\sum_{j \in \{D\}} K_{j,t}^{H,L,Tot} = \sum_{a} K_{a,t}^{H,L,Ind} N_{a,t}^{AdultEq}.$$

National Account Measures

Gross domestic product at factor prices (8.15):

$$
\begin{aligned}
Y_t^{GDPF} = & \sum_{k\in\{C,P,G,D\}} \sum_{j\in\{C,P,G,D\}} P_{k,t}^{Y}\left(Y_{j,t}+Y_{j,t}^{NorthSea}\right) \\
& - \sum_{j\in\{C,P,G\}} P_{j,t}^{M} M_{j,t} - \sum_{j\in\{C,P,G\}} \left(t_{j,t}^{Emp}+t_{j,t}^{W}\right) W_t L_{j,t}^{D} \\
& - \sum_{j\in\{C,P,G\}} \left(t_{j,t}^{P,Weight}\frac{K_{j,t-1}^{P,M}}{1+g_t}+t_{j,t}^{P,Land}\frac{P_{j,t-1}^{I,P,B}}{\left(1+g_t^{P}\right)}\frac{K_{j,t-1}^{P,B}}{\left(1+g_t\right)}\right) \\
& - t_t^{H,Land}\sum_{j\in\{D\}} P_{j,t-1}^{KH,L}\sum_{a}\frac{K_{a-1,t-1}^{H,L,Ind}N_{a-1,t-1}^{AdultEq}}{\left(1+g_t\right)\left(1+g_t^{P}\right)} \\
& + \sum_{j\in\{C,P\}} \left(s_{j,t}^{EU,P,SetAside}+s_{j,t}^{EU,P,Rural}\right).
\end{aligned}
$$

Next we have the expression for Gross Domestic Product (BNP) evaluated at market prices (8.16):

$$
\begin{aligned}
Y_t^{GDP} = & Y_t^{GDPF} \\
& + \sum_{j\in\{C,P,G\}} \left(t_{j,t}^{Emp}+t_{j,t}^{W}\right) W_t L_{j,t}^{D} \\
& + \sum_{j\in\{C,P,G\}} t_{j,t}^{P,Weight}\frac{K_{j,t-1}^{P,M}}{1+g_t}+t_{j,t}^{P,Land}\frac{P_{j,t-1}^{I,P,B}}{\left(1+g_t^{P}\right)}\frac{K_{j,t-1}^{P,B}}{\left(1+g_t\right)} \\
& - \sum_{j\in\{C,P\}} \left(s_{j,t}^{EU,P,SetAside}+s_{j,t}^{EU,P,Rural}\right) \\
& + \text{Indirect Taxes} + \text{Customs Taxes} - \text{Subsidies},
\end{aligned}
$$

where Indirect Taxes, Customs Taxes and Subsidies are given below:

Indirect Taxes =

$$\sum_{d\in\{D,R,P,G\}}\sum_{k\in\{C,P,G\}}\sum_{c\in\{F\}}\left[\left(t^{H,Reg}_{d,t}+t^{H,VAT}_{d,t}+t^{H,DutyV}_{d,t}+t^{H,DutyQ}_{d,k,c,t}\right)\left(1+t^{H,Cus}_{d,t}\right)P^F_t C^{H,2}_{d,k,c,t}\right]$$
$$+\sum_{d\in\{D,R,P,G\}}\sum_{k\in\{C,P,G\}}\sum_{c\in\{D\}}\left[\left(t^{H,Reg}_{d,t}+t^{H,VAT}_{d,t}+t^{H,DutyV}_{d,t}+t^{H,DutyQ}_{d,k,c,t}\right)P^Y_{k,t}C^{H,2}_{d,k,c,t}\right]$$
$$+\sum_{k\in\{C,P,G\}}\sum_{c\in\{F\}}\left[\left(t^{G,Reg}_t+t^{G,VAT}_t+t^{G,DutyV}_t+t^{G,DutyQ}_{k,c,t}\right)\left(1+t^{G,Cus}_t\right)P^F_t C^{G,2}_{k,c,t}\right]$$
$$+\sum_{k\in\{C,P,G\}}\sum_{c\in\{D\}}\left[\left(t^{G,Reg}_t+t^{G,VAT}_t+t^{G,DutyV}_t+t^{G,DutyQ}_{k,c,t}\right)P^Y_{k,t}C^{G,2}_{k,c,t}\right]$$
$$+\sum_{j\in\{C,P,G\}}\sum_{k\in\{C,P,G\}}\sum_{c\in\{F\}}\left[\left(t^{Res}_{j,t}+t^{M,VAT}_{j,t}+t^{M,DutyV}_{j,t}+t^{M,DutyQ}_{j,k,c,t}\right)\left(1+t^{M,Cus}_{j,t}\right)P^F_t M^2_{j,k,c,t}\right]$$
$$+\sum_{j\in\{C,P,G\}}\sum_{k\in\{C,P,G\}}\sum_{c\in\{D\}}\left[\left(t^{Res}_{j,t}+t^{M,VAT}_{j,t}+t^{M,DutyV}_{j,t}+t^{M,DutyQ}_{j,k,c,t}\right)P^Y_{k,t}M^2_{j,k,c,t}\right]$$
$$+\sum_{j\in\{C,P,G\}}\sum_{k\in\{P\}}\sum_{c\in\{F\}}\left[\left(t^{P,Reg}_{j,t}+t^{IM,VAT}_{j,t}+t^{IM,DutyV}_{j,t}+t^{IM,DutyQ}_{j,k,c,t}\right)\left(1+t^{IM,Cus}_{j,t}\right)P^F_t I^{P,M,2}_{j,k,c,t}\right]$$
$$+\sum_{j\in\{C,P,G\}}\sum_{k\in\{P\}}\sum_{c\in\{D\}}\left[\left(t^{P,Reg}_{j,t}+t^{IM,VAT}_{j,t}+t^{IM,DutyV}_{j,t}+t^{IM,DutyQ}_{j,k,c,t}\right)P^Y_{k,t}I^{P,M,2}_{j,k,c,t}\right]$$
$$+\sum_{j\in\{C,P,G,D\}}\sum_{k\in\{C,P\}}\sum_{c\in\{F\}}\left[\left(t^{IB,VAT}_{j,t}+t^{IB,DutyV}_{j,t}+t^{IB,DutyQ}_{j,k,c,t}\right)\left(1+t^{IB,Cus}_{j,t}\right)P^F_t I^{P,B,2}_{j,k,c,t}\right]$$
$$+\sum_{j\in\{C,P,G,D\}}\sum_{k\in\{C,P\}}\sum_{c\in\{D\}}\left[\left(t^{IB,VAT}_{j,t}+t^{IB,DutyV}_{j,t}+t^{IB,DutyQ}_{j,k,c,t}\right)P^Y_{k,t}I^{P,B,2}_{j,k,c,t}\right]$$
$$+t^{H,Land}_t\sum_{j\in\{D\}}P^{KH,L}_{j,t-1}\sum_a\frac{K^{H,L,Ind}_{a-1,t-1}N^{AdultEq}_{a-1,t-1}}{(1+g_t)\left(1+g^p_t\right)}$$
$$+\sum_{k\in\{C,P,G\}}\left[\left(t^{X,DutyV}_t+t^{X,DutyQ}_{k,t}\right)P^Y_{k,t}X_{k,t}\right]$$
$$+\sum_{k\in\{D,P,G,C\}}\sum_{j\in\{C,P\}}\sum_{c\in\{D\}}\left[\left(t^{II,DutyV}_t+t^{II,DutyQ}_{j,c,t}\right)P^Y_{k,t}I^{PI}_{j,t}\right]$$
$$+\sum_{j\in\{C,P\}}\sum_{c\in\{F\}}\left[\left(t^{II,DutyV}_t+t^{II,DutyQ}_{j,c,t}\right)\left(1+t^{II,Cus}_t\right)P^F_t I^{FI}_{j,t}\right]$$

$$
\begin{aligned}
\text{Customs Taxes} = P_t^F \times \Big[& \sum_{d\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_{d,t}^{H,Cus} C_{d,k,c,t}^{H,2} \\
& + \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_t^{G,Cus} C_{k,c,t}^{G,2} \\
& + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} t_{j,t}^{M,Cus} M_{j,k,c,t}^2 \\
& + \sum_{j\in\{C,P,G\}} \sum_{k\in\{C,P\}} \sum_{c\in\{F\}} t_{j,t}^{IB,Cus} I_{j,k,c,t}^{P,B,2} \\
& + \sum_{j\in\{C,P,G\}} \sum_{k\in\{P\}} \sum_{c\in\{F\}} t_{j,t}^{IM,Cus} I_{j,k,c,t}^{P,M,2} \\
& + \sum_{j\in\{C,P\}} t_t^{II,Cus} I_{j,t}^{F,l} \Big]
\end{aligned}
$$

$$
\begin{aligned}
\text{Subsidies} = & \sum_{j\in\{C,P,G\}} \Big[\left(s_{j,t}^{G,P,Dwe} + s_{j,t}^{G,P,Res} + s_{j,t}^{EU,P,Res} + s_{j,t}^{G,P,Spe} + s_{j,t}^{EU,P,Spe}\right) \\
& \times \Big[\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(\left(1 + t_{j,t}^{M,Cus}\right) P_t^F M_{j,k,c,t}^2\right) + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y M_{j,k,c,t}^2 \Big] \Big] \\
& + \left(s_t^{G,G,Dwe} + s_t^{G,G,Spe} + s_t^{EU,G,Spe}\right) \\
& \times \Big[\sum_{k\in\{C,P,G\}} \sum_{c\in\{F\}} \left(\left(1 + t_t^{G,Cus}\right) P_t^F C_{k,c,t}^{G,2}\right) + \sum_{k\in\{C,P,G\}} \sum_{c\in\{D\}} P_{k,t}^Y C_{k,c,t}^{G,2} \Big] \\
& + \left(s_t^{G,X,Spe} + s_t^{EU,X,Exp} + s_t^{EU,X,Spe}\right) \sum_{k\in\{C,P,G\}} P_{k,t}^Y X_{k,t} \\
& + \sum_{d\in\{D,R,P,G\}} \sum_{k\in\{C,P,G\}} \Big[\left(s_{d,t}^{G,H,Dwe} + s_{d,t}^{G,H,Spe} + s_{d,t}^{EU,H,Spe}\right) \\
& \times \Big[\sum_{c\in\{F\}} \left(\left(1 + t_{d,t}^{H,Cus}\right) P_t^F C_{d,k,c,t}^{H,2}\right) + \sum_{c\in\{D\}} P_{k,t}^Y C_{d,k,c,t}^{H,2} \Big] \Big] \\
& + \sum_{j\in\{C,P,G\}} \Big[\left(s_{j,t}^{G,IM,Spe} + s_{j,t}^{EU,IM,Spe}\right) \\
& \times \Big[\sum_{k\in\{P\}} \sum_{c\in\{F\}} \left(\left(1 + t_{j,t}^{IM,Cus}\right) P_t^F I_{j,k,c,t}^{P,M,2}\right) + \sum_{k\in\{P\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,M,2} \Big] \Big] \\
& + \sum_{j\in\{C,P,G,D\}} \Big[\left(s_{j,t}^{G,IB,Dwe} + s_{j,t}^{G,IB,Spe} + s_{j,t}^{EU,IB,Spe}\right) \\
& \times \Big[\sum_{k\in\{P,C\}} \sum_{c\in\{F\}} \left(\left(1 + t_{j,t}^{IB,Cus}\right) P_t^F I_{j,k,c,t}^{P,B,2}\right) + \sum_{k\in\{P,C\}} \sum_{c\in\{D\}} P_{k,t}^Y I_{j,k,c,t}^{P,B,2} \Big] \Big]
\end{aligned}
$$

Finally gross national income is given by equation (8.17) :

$$
\begin{aligned}
Y_t^{GNI} &= Y_t^{GDP} + OR_t^{F,H,Wage} + SR_t^{EU,G} - OR_t^{G,EU,Cus} \\
&\quad + kiY_t^{GNI} i_t \frac{A_{t-1}^F}{(1+g_t)(1+g_t^P)} - i_t^{G,add} \frac{D_{t-1}^G}{(1+g_t)(1+g_t^P)}.
\end{aligned}
$$

Financial assets

Equation (8.18) gives an expression for the share of aggregate firm value owned by household a, and equation (8.19) gives the share of aggregate firm value:

$$
\begin{aligned}
\frac{w_{a,t}^{H,Shares}}{k^{shares}} \sum_{j\in\{C,P\}} V_{j,t} &= w^{Assets} A_{a,t}^{H,Fin,Ind}, \\
\frac{w_t^{Beq,Shares}}{k^{shares}} \sum_{j\in\{C,P\}} V_{j,t} &= w^{Assets} A_t^{H,Beq}.
\end{aligned}
$$

Total household finansial wealth, total finansial assets held in relation to pensions and the aggregate value of financial assets in the economy are given in equation (8.20), (8.21) and (8.22), respectively:

$$
\begin{aligned}
A_t^{H,Fin} &= \frac{1}{w^{Assets}} \frac{\left(\sum_a w_{a,t}^{H,Shares} N_{a,t}^{Adult,Eq} + \sum_{a\in\{Apu\}} w_t^{Beq,Shares} N_{a,t}^{Adult,Eq}\right)}{k^{shares}} \sum_{j\in\{C,P\}} V_{j,t}, \\
A_t^Z &= A_t^{Z,F} + A_t^{Z,P}, \\
A_t^{H,Z} &= A_t^{H,Fin} + A_t^Z.
\end{aligned}
$$

Steady state

For the last solution period T we have the following 11 equation representing 90 terminal conditions:

$$
\begin{aligned}
K_{a,T}^{H} &= K_{a,T-1}^{H}, \\
K_{a,T}^{HB} &= K_{a,T-1}^{HB}, \\
K_{a,T}^{HL,Ind} &= K_{a,T-1}^{HL,Ind}, \\
P_{j,T}^{KHL} &= P_{j,T-1}^{KHL} && , j \in \{D\}, \\
A_{a,T}^{H,Fin,Ind} &= A_{a,T-1}^{H,Fin,Ind}, \\
A_{T}^{H,Beq} &= A_{T-1}^{H,Beq}, \\
V_{j,T} &= V_{j,T-1} && , j \in \{P, C\}, \\
Q_{j,T}^{K,P,M} &= Q_{j,T-1}^{K,P,M} && , j \in \{P, C\}, \\
Q_{j,T}^{K,P,B} &= Q_{j,T-1}^{K,P,B} && , j \in \{P, C\}, \\
Q_{j,T}^{K,P,M,book} &= Q_{j,T-1}^{K,P,M,book} && , j \in \{P, C\}, \\
Q_{j,T}^{K,P,B,book} &= Q_{j,T-1}^{K,P,B,book} && , j \in \{P, C\}.
\end{aligned}
$$