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# Chapter 1

## Preface

Several present and former employees of DREAM have participated during the years in refining, improving and extending the model and its various satellite modules, among which are Michael Andersen, Ninette Pilegaard Glud, Cathrine Marie Gruno, Marianne Frank Hansen, Andreas Koch, Martin B. Knudsen, Anders Due Madsen, Lars Haagen Pedersen, Toke Ward Petersen, Poul Schou, Peter Stephensen, Morten Lobedanz Sørensen, Peter Trier and Benn Vestergaard.

The present documentation is the result of the work of all those and inspiration and advice from many other economists in Denmark and abroad. The actual text has been written by Michael Andersen and Poul Schou.



# Chapter 2

## Introduction (Instructions for reading, principles for dating etc.)

The first version of DREAM was made in 1997, and since then the model has been continually refined and expanded - a process which is still continuing. The latest previous full documentation of DREAM appeared in 1998. Since then, several important extensions and other changes to the model have taken place. The aim of the present documentation is to provide an up-to-date description of the DREAM model. Among the most important changes since 1998 are:

- 1) Introduction of imperfect competition in goods markets
- 2) Introduction of trade union behaviour on the labour market
- 3) Introduction of housing as a separate consumption (and capital) good
- 4) Introduction of funded pensions arrangements (labour-market pensions funds as well as individual pensions arrangements)
- 5) A considerably extended modelling of the government sector, following the classification of the national accounts in greater detail
- 6) Subdivision of the population into various ethnic groups (immigrants/descendants of immigrants/remaining population)
- 7) A change of the run-time intervals of the model from 5-year to 1-year periods.

Some of these changes have been described in various single-standing papers, whereas others have not been publicly documented until now. The present study covers all these features as well as other model changes since 1998 and provides a new listing of DREAM's data sources, a presentation of DREAM's present base-line projection of the Danish economy and a collection of simulation results.

One of the particular objectives is to present and explain every single equation in the main DREAM model to make the documentation useful for the actual users who work hands-on

with the model. As the model is very complex, a thorough reading-through of all chapters may not be the most suitable way of acquiring a sound over-all knowledge of the model. For readers who are interested in getting this, it is recommended to read chapter 2, which presents a non-technical overview of the main characteristics of DREAM, and chapter 9 which presents a number of simulation experiments. Chapters 3-7 presents the details of the model. Chapter 8 describes the calibration process and the background sources for the data and determination of individual parameters which are employed in DREAM.

The documentation is written in English to facilitate communication with DREAM's international CGE modelling colleagues. In the case of particular Danish institutions (like names of specific transfers, tax or pensions arrangements, etc.) where an official English translation may either not exist or not be well-known to all readers, the Danish name may be mentioned in brackets after the English term.

It is recommended to consult the appendix on principles for naming variables when reading the main chapters of the documentation.

## 2.1 A note on dating conventions

The general rule in DREAM is that variables are dated according to end of time convention. Stock variables which are active in a period, e.g.  $t+1$ , are consequently nominated  $t$ , since stocks are updated at the end of the period. Flow variables of period  $t$  are naturally nominated  $t$ .

As an exception to this rule it is assumed that people are born in the beginning of the period.

In the case of a surprise shock, the announcement of the shock itself leads to instantaneous adjustments in various assets of the economy (shares, residential buildings and land), which are supposed to be realized at the end of the period before the shock is dated. In this situation, an extra round of asset trade is assumed to take place, allowing agents to re-optimize their asset portfolios in the face of the new set of prices.

The order of events in a given period  $t$  can be summed up as follows (where steps 4 and 5 take place only in the case of a surprise shock in period  $t+1$ ):

1. At the very start of the period, all generations which existed at the end of period  $t-1$  grow one year older. The premature generation of period  $t-1$  thus becomes 17 years old and forms the youngest genuine generation of the economy.
2. During the period, all ordinary flow transactions are executed. These include production, reception of wage, capital and transfer income and of the bequest left over from last period, consumption, investment, depreciation of capital stocks, etc.
3. At the end of the period, all wealth stocks are updated as the result of the flow transactions,



and the various assets of the economy are traded. Households adjust their residential dwelling stocks (and consequently their financial assets) by buying or selling. Pension funds adjust their portfolio of bonds and assets. During this transaction, also the premature generation (the 16-year-old people) become active. They start out with a total wealth of zero and consequently takes a financial loan which exactly equals the value of the residential building they acquire. The last planning generation (the 76-year-old people) sell their residential buildings and transfer their total wealth to a bequest which is transferred to their children during period  $t+1$ . After this, the 76-year-olds like the premature generation has zero wealth left, and - as they still need a dwelling in future periods - like the premature generation, they take a financial loan which equals the value of the new residential building which they buy. The same is true for all the non-planning generations. The very oldest generation (the 101-year-old people) also sell their residential buildings, use the money to pay their financial debt, and leave the economy completely at this point.

4. In the case of a shock being announced in period  $t+1$ , the shock is assumed to be known to all agents at this point, i.e. after ordinary capital transactions have taken place. The shock generally leads to new equilibrium prices for shares in firms and for residential buildings and land. All generations hence experience an immediate (and possibly negative) capital gain on their share holdings and their residential stocks. This includes the premature generation, the - temporarily ownerless - assets of the bequest and all the non-planning generations except the very oldest one (the 101-year-old people who are no longer present in the economy). Also the pension funds experience a capital gain or loss on their share holdings.

5. All households now have the opportunity to re-optimize by adjusting their holdings of residential assets - and their financial assets accordingly - and the pension funds re-adjust their financial portfolios.

Step 5 consequently does not change the total wealth of either individual households or pension funds, but only its composition. In reality, steps 4 and 5 should be thought of as taking place simultaneously (as it is really the transactions of step 5 which create the new equilibrium prices mentioned in step 4), but conceptually it may be easier to think of them as two separate steps to make clear to whom the capital gains are allocated.

[Graphical description of timing as in old documentation is placed here]



# Chapter 3

## An Overview of the Model

### 3.1 Introduction

DREAM (Danish Rational Economic Agents Model) belongs to the class of applied economic models called computable general equilibrium (CGE) models. CGE models emphasize a very close correspondance with modern state-of-the-art economic theory. In particular, CGE models emphasize the following features:

- a) The important agents of the economy like households and firms have explicitly optimizing behaviour,
- b) Prices are flexible so that the markets of the economy adjust instantaneously to new situations (this does not imply that supply and demand necessarily equal each other. For instance, the labour market in DREAM is characterized by structural unemployment).
- c) The models are GENERAL which signifies that they contain all the most important markets of the economy, making it possible to follow consequences of events in one market for other parts of the economy.

Price flexibility as well as other assumptions inherited from the underlying general equilibrium theory is generally considered to be more realistic in a long-run than in a short-run perspective. Consistently with this, DREAM is intended to describe long-run developments in the

Danish economy and long-run effects of various policy experiments and exogenous shocks to the economy. Short-run properties of the model should be interpreted very cautiously, and the model is unsuited to analyze e.g. consequences of business cycle-related phenomena.

The most important constituent parts of the model - producers, private households, pension funds, the government sector, asset markets and international relations - are described in detail in the following chapters. Here, a brief overview of the main principles and mechanisms of the model is provided together with a description of the characteristics of DREAM's present base-line projection of the Danish economy.

### 3.1.1 Producers

In the standard version of DREAM (which is the subject of the present documentation) there are two private production sectors: a construction sector producing dwellings for households and building investments for firms and the government sector, and a manufacturing sector producing all other privately produced commodities. Additionally, the government sector acts as a producer of goods which are mainly used for government consumption, though the government also sells a minor fraction of its produce to households, private firms and the rest of the world on market terms. Formally, the standard version even contains a fourth sector: The production sector for dwelling services. This sector, however, employs no labour, but uses capital as its only production factor (i.e. the housing stock, which is owned by the households: In DREAM, the entire housing stock is treated as owner-occupied dwellings) and produces a single output: dwelling services, i.e. the service given by a dwelling to its owner. Hence, the output of this sector is exclusively used for domestic consumption. Consequently, the returns to capital for this housing stock is also calculated and exists as an imputed income to households in the model. However, when referring to production sectors in the following, this sector is normally not considered included.

DREAM also exists in a multi-sector version which has been used for analyses where the industrial structure is particularly important. An example is an analysis of the consequences for the Danish economy of the enlargement of the EU from 15 to 25 countries. The multi-sector version is not covered in this documentation, but a more thorough description may be found in Madsen and Sørensen (2002).

The two private sectors are treated symmetrically. In each sector, the producers are represented by corporate firms, the shares of which are owned by the households and pension funds. The fundamental behavioural assumption of each firm is that it strives to maximize the value of its stock of shares, which can be shown to be equal to the discounted stream of future dividends. Firms like other agents in DREAM have rational expectations. Combined with the absence of any risk and uncertainty, this implies that firms - and other agents - have perfect foresight, except in the case of an unannounced change in a policy variable. Even after such an event agents are assumed to again form expectations with certainty.

Each firm decides upon the optimal building and machinery investment in each period (i.e. it optimizes intertemporally) and employs labour and buys materials from various domestic and foreign producers to use as inputs. The production function is a nested CES production function with elasticities below unity between materials and the labour-capital composite, and between capital and labour.

The firms produce goods which are assumed to be imperfect substitutes to goods produced by other firms in the same sector. The particular modelling of imperfect competition is the so-called "large group imperfect competition assumption". Each firm faces a downward-sloping demand curve and consequently possesses some market power: It sets its own price optimally and above marginal costs so that the firm earns positive profits. Whereas in some imperfectly

competitive set-ups, free entry of new firms drive down profits to zero in the long run, in DREAM the number of firms is assumed to be constant, allowing for positive profits both in the short and long run.

### **Equity structure**

Firms can finance investment by borrowing or by enlarging its equity (in practice by retaining some of the earnings of the stockholders. We assume that firms abstain from financing activities by issuing new shares). Ideally, the finance decision should be made optimally. However, in a model without uncertainty, differentiated tax rules would lead to a corner solution. To avoid this, it is assumed that corporate debt is always a constant fraction (60 per cent) of the capital stock.

In symmetric equilibrium, it can be shown that the aggregate behaviour of all the firms in one sector is equivalent to the behaviour of one "representative" firm, so that in the actual computer version of the model, each sector is represented by just one aggregate firm.

### **Investment**

There are two kinds of investment goods: building and machinery investments, and both are necessary production inputs in all three production sectors. Installation of both kinds of capital is costly, and installation costs are increasing and strictly convex in the investment level, so that the costs depend positively on the amount of investments relative to the existing capital stocks. *Ceteris paribus* this implies that it is optimal for the firm to spread desired investments over time rather than to make instantaneous adjustments of the capital stock.

### **Production of government services**

As mentioned production of government services takes place in a separate sector. The production technology is similar to that of the private sectors, except that we assume that there are no installation costs for the government producer. Also, the producer in this sector (we assume that there is only one producer) does not maximize the value of shares intertemporally, but simply undertakes investment in each period according to a mechanical rule stating that the capital-output ratio in the government sector must always be constant. Given the capital stock, other production inputs are chosen subject to cost minimization in each period. As profits are not maximized, it is not immediately clear how the output price should be chosen, but in DREAM it is assumed that the price is chosen so that it covers average costs associated with production. Most of the output of this sector is used for government consumption, but some produce is sold to private consumers, to the three domestic production sectors as production inputs, or for exports.

### 3.1.2 Households

#### Population

As one of the chief purposes of DREAM is to evaluate the effects of demographic changes, the population and household structure is of particular importance in the model. One of the most basic inputs to the DREAM model is the independent population projection made annually by DREAM. The projection is made for each year until 2100 and projects the whole population on groups distributed according to gender, age and 5 different origin groups (immigrants from so-called more developed countries, immigrants from less developed countries, descendants of immigrants from more developed countries, descendants of immigrants from less developed countries, and the remaining population). This distribution on groups enables the determination of long-run aggregate economic developments in areas where these groups differ in behaviour empirically. For a more detailed description of the method used to project the population, cf. Koch et al. (2004).

To be able to perform analyses of intergenerational questions, DREAM is an overlapping generations model. All adults of the same age (divided into one-year intervals) form a generation, and each generation is represented in the model by a representative household. One implication of this is that women are considered to be married to men of the same age. The youngest household in the model consists of all 17-year old people, and the oldest household is made up of all people who are 101 years, yielding altogether 85 households. Children younger than 17 years belong to the household of their mothers; they do not work or perform any other independent economic actions, but they do count when determining the optimal consumption level of the household, and the household receives various government transfers applicable to children.

Households derive utility from consumption and from leaving a bequest to their children and disutility from working. Consequently, besides the intertemporal decision of how to divide consumption (and consequently savings) between the different periods of their time horizon and the bequest, the households also choose the composition of their consumption bundle inside each period and their labour supply.

The time horizon of the households for intertemporal optimization (that is, making savings decisions) is finite and deterministic and smaller than the life-span: It ends at the age of 76 years. Households from 17 until 76 years have rational expectations and determine their consumption/savings decision optimally according to a life-cycle perspective. At the end of the year when the household has turned 76 years, it no longer saves or dissaves. At this point in time, it leaves its remaining assets as a bequest to its heirs (the generations which are the children of the generation in question). The households who are 77-101 years old (referred to as "old" households) no longer optimize over time, but still perform some economic actions: In each period, they receive a certain income from funded pensions and government transfers and spend it on consumption. In doing this, they still optimize intratemporally, choosing their consumption basket in each period depending on the current relative prices of consumption

goods.

**Income and expenditure** The household derives income from various sources: Labour income for the generations with a positive labour supply (generations aged 17 until 74 years), capital income (which may be negative) from the stock and bond holdings of all generations who are not more than 76 years old, imputed income from the possession of owner-occupied dwellings, government transfers of many kinds - DREAM distinguishes altogether 20-odd different kinds of potential transfers from the government to the private households - and pensions from funded pensions arrangements (cf. section 3.1.7) for generations which are at least 60 years old. In addition, households receive a small amount of transfers from abroad and an inheritance from their parents' generation.

**The consumption bundle** A major distinction when determining private consumption of the household is between housing and non-housing goods. Non-housing goods are produced either by the government sector or by the private manufacturing sector; in the last instance, they may be either imported or produced at home. Note that only government-produced goods which are bought on market conditions enter the utility function. Goods used for individual or collective government consumption (by far the largest proportion of government-produced goods) are assumed not to affect the utility of households.

Housing goods consist of dwellings (the predominant part) and consumption of housing repair, a service delivered by the domestic construction sector, which also forms part of private consumption. The ownership of dwellings is modelled so that the household during each period owns a certain amount of building capital and a certain amount of land, which together form the dwelling, the services of which yield utility. Whereas the amount of building capital can vary from period to period both as a result of depreciation, investment in the construction of new buildings, and buying from or selling existing buildings to other households, the amount of land owned by each representative household can only change through sales: The total amount of land available in the economy for housing purposes is fixed.

**Savings** Each household maximizes its utility over its entire time horizon from 17 to 76 years. Normally, the income stream for the household over this period deviates from the desired consumption stream, and the household consequently chooses to save or dissave in each period in order to finance its optimal consumption stream. The time profile of consumption over the time horizon can be expressed as a Keynes-Ramsey rule, i.e. it depends upon the real after-tax rate of return to household assets as well as on household preferences (impatience and the strength of the desire for a smooth consumption path over time). Typically, household consumption develops more smoothly over time than income.

Financial savings take place in shares and bonds. In order to avoid corner solutions, the household does not determine its portfolio optimally, but instead follows a fixed rule stating that one third of savings (or dissavings) are in shares and two thirds in bonds. This implies

that in the present version of DREAM, households with negative net financial savings also hold negative stocks of shares. One may think of this as households "going short" in shares.

### 3.1.3 The labour market

The labour market is characterized by imperfect competition in the form of labour unions who determine the effective labour supply as a function of the difference between marginal after-tax wages and marginal after-tax unemployment benefits. The hypothetical concept of full employment corresponds to the labour supply which each worker would supply in the absence of the union and does not depend on unemployment benefits, but solely on a trade-off between the marginal net wage and disutility of labour. The difference between this hypothetical labour supply and the effective labour supply determined by the union is regarded as unemployment, and the model is calibrated so that it corresponds to the official unemployment rate in the calibration year. The equilibrium wage is the wage rate which equals labour demand by the cost-minimizing producers to effective union labour supply.

### 3.1.4 The government sector

The government acts as a producer, collects taxes and pays out subsidies to firms and transfers to households and various foreign recipients, and provides government consumption. Government consumption is subdivided into individual government consumption, consisting of all government consumption expenditure which can be assigned to specific individuals, mainly education, health and social care, and collective government consumption which cannot be attributed to any specific persons, mainly expenditure for administration, defence, the judicial system, research and infrastructure.

DREAM models the following taxes: VAT, excise duties, motor vehicle weight duties, labour market contributions from employers, customs taxes, a residual tax on production, bottom-bracket, middle-bracket and top-bracket central government income taxes, municipal, county and church income taxes, property (= land) taxes, the tax on owner-occupied dwellings, taxes on personal income from stock holdings (dividends and capital gains), payroll taxes, taxes on once-and-for-all pensions, corporate taxes, taxes on yields of pension scheme assets and taxes on bequests. Besides, the government receives some revenue from various social contributions, transfers from other sectors and from the rest of the world and various rents and the technical accounting revenue term of gross operating surplus (which is identical to government consumption of fixed capital and consequently appears indirectly on the expenditure balance sheet as well as on the revenue side).

Besides government consumption, transfers to households and subsidies to firms, the government spends resources for investment and various other transfers to diverse recipients.

Despite the detailed modelling of the government sector, the behaviour of the government is



rather automatical: Practically all tax and subsidy rates are kept constant during the projection, though effects from the tax freeze which reduces the effective tax on owner-occupied dwellings as well as some excise duties have been implemented for the years 2003-2010. Expenditure for most transfers are indexed according to the Danish wage regulation law so that they follow the average hourly wage net of payroll taxes and labour-market pensions contributions. Expenditure for collective government consumption is indexed to GDP, whereas expenditure for individual government consumption per person follows the rate of exogenous productivity growth, which is 2 per cent annually.

### **Fiscal sustainability**

To ensure consistency in a model with rational expectations among agents, the government must always be able to service its debt, i.e. government debt cannot grow explosively. When the growth-adjusted interest rate is positive (as it is assumed in DREAM), this is equivalent to assuming that the government must fulfill its intertemporal budget constraint, i.e. the initial government debt may not exceed the present discounted value of all future primary budget surpluses. When this requirement is fulfilled, fiscal policy is said to be sustainable. In DREAM, a sustainable fiscal policy must be secured by some endogenous policy rule which adjusts either some revenues or some expenditure at some point in time if fiscal policy is not exactly sustainable at the outset. Such a policy rule can take many different forms, and the choice of which one to use can be considered to be arbitrary. Often, projections using the DREAM model measure the sustainability condition by adjusting either collective government consumption or the bottom-bracket state income tax rate, but also other instruments are possible. In the same way, the timing of the adjustment(s) is arbitrary: It is possible to make a one-time constant change in some tax rate or to make continuous adjustments over time parallelly to the changes in exogenous expenditure demands, for instance.

### **3.1.5 International relations**

The economy is integrated in the world economy through trade and capital flows. Materials and foreign consumer goods are imported while domestic products are exported. Domestic production is an imperfect substitute for imported goods (this is the so-called Armington assumption), implying that the terms of trade are endogenous. Financial capital is assumed to be perfectly mobile internationally, and the exchange rate is fixed. These assumptions, the absence of uncertainty and the presence of residence-based taxation of interest income imply that the domestic and the foreign pre-tax interest rate are equal.

Foreign demand for the domestic good can be thought of as demand functions derived from intertemporal optimization of foreigners. For simplicity, it is assumed that the foreign demand function for the domestic good is isoelastic, i.e. it depends on the domestic price index relative to the foreign price with a constant elasticity.

The imported foreign good is used as numeraire in the economy. The assumptions of the model imply that the foreign inflation rate which is assumed to be constant at 1.75 per cent annually, is imported to the economy, even though the endogenous price movements in DREAM may cause Danish inflation to deviate from foreign inflation when the economy is not in a steady state.

The rest of the world is considered to be in steady state during the entire projection. This also means that the foreign inflation, productivity growth and interest rates are constant as is the position of the export demand curve. This assumption means that for instance demographic changes in the rest of the world which may potentially affect these variables are ignored<sup>1</sup>.

### 3.1.6 Financial markets and equilibrium

#### Real assets

In DREAM, the following real assets exist: 1) Machinery capital, which is exclusively owned and used by firms including the government producer. 2) Building capital, which is partly owned and used by firms parallel to machinery capital, and partly owned by households to form the main part of the owner-occupied dwellings in which households live. 3) Land, which is exclusively owned by households to form the remaining part of their owner-occupied dwellings.

#### Natural resources from the North Sea

A fourth kind of real assets might have been the oil and gas reserves in the North Sea. Natural resource extraction from the North Sea does play an economic role in DREAM, but it is modelled in a very simple way: What chiefly distinguishes resource extraction from other kinds of production in a broad economical sense is the existence of a pure resource rent: The revenue provided by the value of the resource when all production expenses have been subtracted. This pure resource rent is included directly in DREAM as an extra (time-dependent and diminishing) income stream to the private manufacturing sector, so that the resource stocks themselves do not appear in the model.

#### Financial assets

There exist two kinds of financial assets: Bonds and shares. Bonds can be issued by private households, firms, the government and the rest of the world, though the model assumes these various bonds to be perfect substitutes: There is only one common interest rate. Shares are issued by the two representative domestic private firms and owned by households and pension funds. Neither the government (if it acquires positive net assets, which often happens in current DREAM projections because some sustainability rules imply saving in advance to

---

<sup>1</sup>CGE models which have tried to quantify consequences for the interest rate of world demographic changes are Fehr, Mannheim and Ingenue ...

meet future government obligations caused by demographic changes) nor the rest of the world invest in shares, nor do domestic asset owners buy foreign shares.

As mentioned, firms do not optimize when financing investment decisions, but always have a financial debt which is a fixed fraction (currently 60 per cent) of the replacement value of their capital stock. Also households always possess shares as a fixed fraction of their financial assets. These financial assets may be either positive or negative for each individually-aged household as well as for all private households taken together. Only pension funds adjust their portfolio optimally when conditions change, following an arbitrage rule which states that the returns to shares must equal the risk-adjusted returns to bonds. Note that even though there is no uncertainty in DREAM, shares still yield a higher return than bonds because of an exogenous risk premium. This risk premium has been introduced to replicate the fact that equity on average yields a higher return than loans.

The fact that only pension funds adjust portfolios optimally amounts to assuming that they constitute the marginal investor effectively determining the value of the stock of shares. For this reason also, changes in the taxation of the income of pension funds, but not changes in e.g. the tax on capital income of households, have first-round effects on the value of shares.

### 3.1.7 Pension systems

In DREAM, elderly people receive a number of different pensions benefits. The following income streams are modelled explicitly in the model:

- 1) unfunded government transfers, which comprise old age people's pensions, civil servants' pensions, and non-income-compensating transfers like housing benefits.
- 2) funded labour-market pensions (in DREAM accumulated in an aggregate labour-market pension fund which is simply referred to as "the pension fund")
- 3) funded individual pensions (which are called private pensions)
- 4) funded ATP pensions
- 5) funded SP pensions
- 6) funded LD pensions

Of these, government transfers to elderly people follow the general rules for government transfers which are treated above. Note that civil servants' pensions are paid out as a lump-sum transfer to all generations from the age of 66 years (from 2005) in the model. The aggregate sum of civil servants' pensions paid out in each future year is determined on the basis of a separate exogenous projection delivered by the Ministry of Finance.

SP pensions and individual pensions are treated conceptually alike, even though they are

modelled as two independent systems. In each case, each generation saves an exogenous share of its labour-income, which is accumulated in the fund in question. In the case of private pensions, these are paid out to the generation as once-and-for-all payments during the two years when the members of the generation are 75 and 76 years old, respectively. SP pensions are received as a once-and-for-all payment at the age of 68 years. In effect, each system works as a simple bank account for the generations in question, as there is no intragenerational redistribution involved in these arrangements.

LD pensions work in the same way as the private pensions, except that the LD fund receives no contributions, but only pays out benefits to the generations who once contributed to it (all generations who received labour-market income in 1977-79). The payments are modelled as a once-and-for-all payment to the generations who are (from 2005) 66 years old.

The aggregate labour-market pension fund in DREAM is more complex. It covers three types of pensions: retirement pensions, spouse pensions and disablement pensions. For each type, premiums and pensions are calculated so that the pensions arrangement is actuarially fair. Contributions to the pensions fund is a fixed percentage of wage income which is accumulated and paid out as members of each generation become disabled, reach their retirement age (presently, retirement takes place continually from the age of 65 to the age of 75 years), or die, in which case their spouses (who always belong to the same generation) receive a spouse pension for the rest of their lives. The size of initial pensions (the pension undertaking) is calculated using various principles of precaution capturing the fact that labour-market pension funds use conservative forecast procedures in order to ensure solvency. The principles of precaution include a base interest rate which is lower than the bonds interest rate in DREAM, moderate probabilities of death, disablement and marriage, and constant future contributions, pensions and pension undertakings. Each year, a correction in the form of a bonus on the assets of all surviving individuals is made to account for the fact that actual disablement and mortality rates have differed from the precautionary estimates used. Together with corrections for the deviation in assumed interest rates and the fact that actual contributions differ from forecasted ones, this assures a growth in paid-out pensions per average individual for each generation during its life-time which is higher than the general productivity growth rate of the economy.

ATP pensions are paid as a fixed sum by all employed persons (unemployed persons pay 50 per cent of this amount). The sum is indexed to DREAM's wage rate. Payments are paid out to people as annual benefits from the age of 67 years.

Of the five funded pension arrangements, the oldest is the ATP arrangement, which is now approximately fully mature. The LD fund is being phased out as the last generations who are entitled to a payment become 66 years old, and its wealth will decline to 0 by 2031. The remaining three arrangements will grow considerably during the projection, cf. below.

## 3.2 DREAM's base-line projection

The following section presents the current base-line projection of DREAM. This projection is based on DREAM's population projection from November 2004 and calibrated to 2003. Fiscal sustainability is ensured by a one-time permanent cut in collective government consumption from 2007 at the size of 2.57 per cent of GDP.

### 3.2.1 Population

The projected development in the Danish population during the 21st century is shown in Figure 3.1. Two main developments appear from the figure:

The first one is that the population is projected to diminish by more than 250.000 people or about 5 per cent during the century. From 5.38 million people in 2003 the projected population reaches its peak in 2010 at 5.42 million, after which it declines to 5.06 million in 2080. During the last 20 years of the century, the projection rises slightly again to 5.11 million people. These figures are the collective results of the assumptions made concerning the projections fertility, mortality and migration rates. These imply a rise in the mean living age of ... years, a fertility rate which gradually rises to 1.85 and an annual net immigration of 5-10.000 people during the projection period, cf. Koch et al (2004) for a more thorough description of the projection method.

The second noteworthy feature is the changed composition of ethnical groups within the population. The two immigrant groups make out 6.1 per cent of the population in 2003, but 13.1 per cent in 2100. Their descendants rise relatively even more: The two descendant groups make out 1.9 per cent of the total population initially, but 9.1 per cent at the end of the century. The largest ethnical group, the residual population, consequently falls from 92 to 78 per cent. This changed composition has various consequences for the economic development because the various groups have different economic behaviour in the base-line projection, cf. Schou (2006).

At the same time, the age composition of the population changes. This is reflected in the demographic dependency ration in 3.2. The demographic dependency ratio is the number of people less than 15 or at least 65 years old divided by the number of people between 15 and 64 years. This ratio rises considerably from 100 in 2003 to 169.5 in 2040. After some years of decline, it reaches a new peak around 2075.

However, the demographic dependency ratio does not take into account the fact that also the labour-market participation for people of a given age may change because of changes in the ethnical composition of the population. The labour-market dependency ratio shows the projected number of people in the population outside the labour force relative to the number of people in the labour force. It has the same qualitative double-hump-shaped look as the demographic dependency ratio, but is generally at a lower level, as can be seen from 3.2.

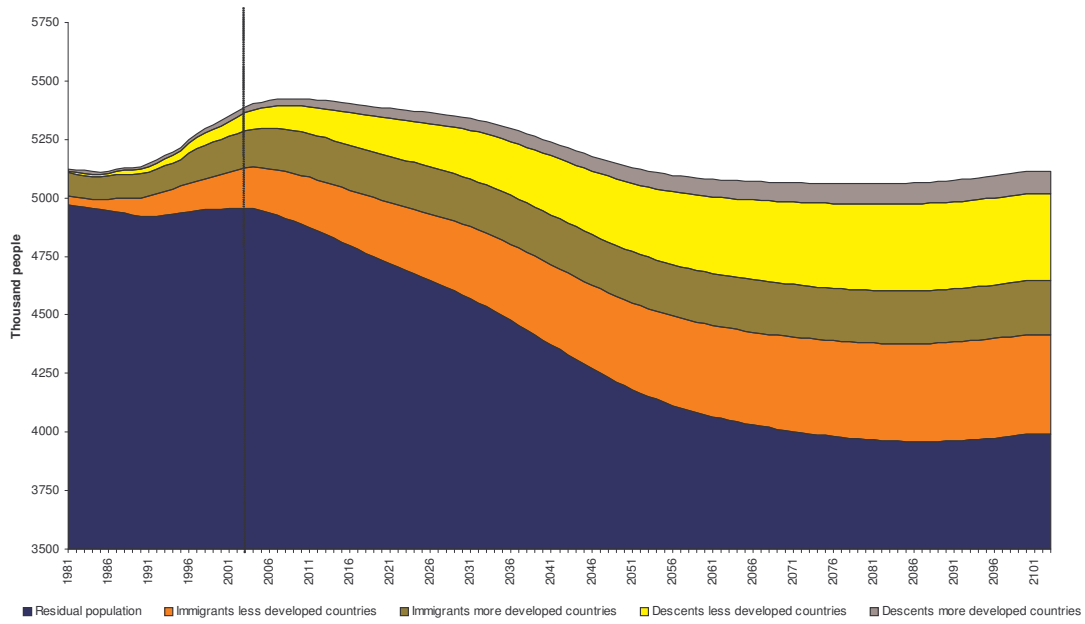


Figure 3.1:

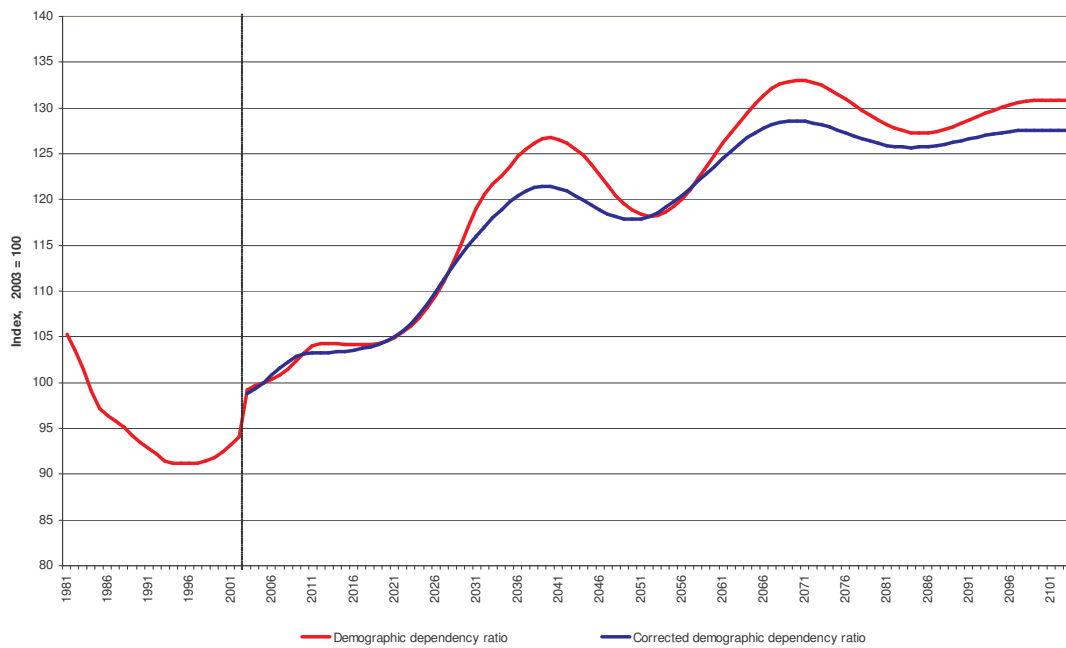


Figure 3.2:

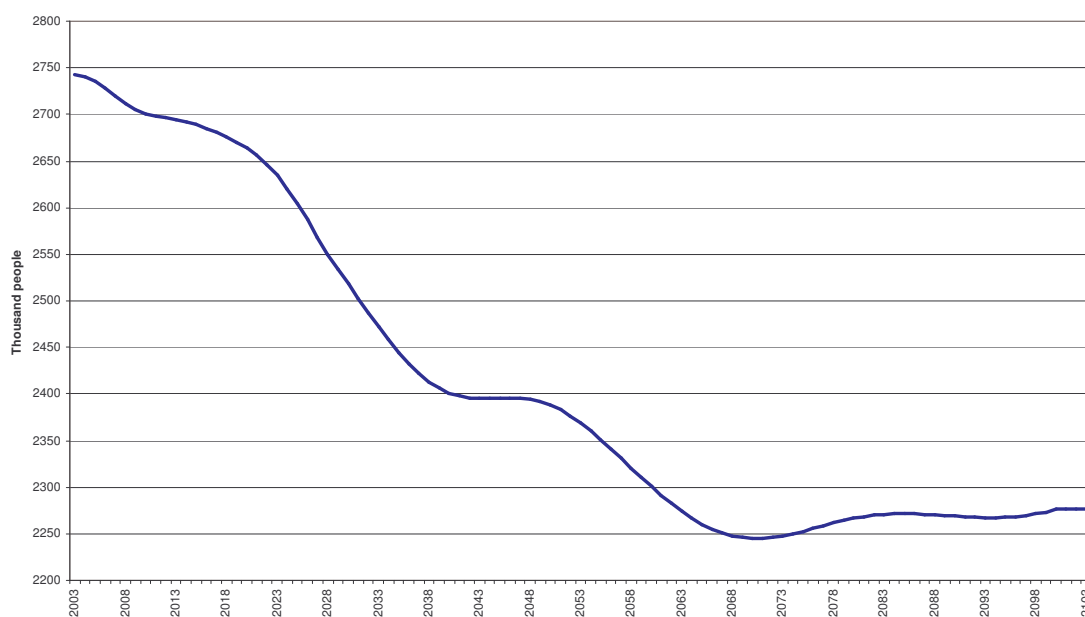


Figure 3.3:

The consequences of the demographic developments for the labour force are seen in 3.3: Because of a changed composition with regards to age and ethnical origin, the labour force falls by about half a million people from 2001 to 2070, after which year it rises slightly. This is equivalent to a fall of 28 per cent.

### 3.2.2 Aggregate macroeconomic development

The macroeconomic development of the base-line projection can be seen in 3.4. To understand the figures, it is instructive to first observe the projected employment development. The employment level in the base year is measured as a wage level<sup>2</sup> and is presented as an index *<denne formulering skal lige finpudes>*. That is, when average working hours or demographically-induced productivity changes, the index is affected, whereas it ignores the labour-augmenting productivity growth (because the basic measure is a time unit). Employment measured in this way falls by more than 13 per cent by 2040 and about 19 per cent during the whole century. This fall is dominated by demographic factors.

Despite the fall in employment, projected production (measured by real GDP) nevertheless increases considerably during the projection period. By 2040, it has grown by about 80 per cent, and at the end of the century by more than 400 per cent. This is due to the assumed constant labour-augmenting productivity growth of 2 per cent annually which is ultimately due to technological progress and increased knowledge in the economy.

<sup>2</sup>So that it takes account of changes in demographically-induced productivity or working hours. In the projection, labour-augmenting productivity growth as well as endogenous changes in the over-all wage level are ignored.

Table: Macroeconomic development														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level, billion DKK	Index in fixed prices, 2003=100												
Private consumption	552.6	100.0	102.2	104.9	106.9	109.5	112.2	114.6	116.9	141.2	197.8	281.9	412.8	617.2
Real GDP	1210.6	100.0	101.9	104.0	106.3	108.6	110.2	112.0	113.8	133.7	180.8	255.4	368.8	551.2
Unemployment*	6.2	6.2	6.4	5.7	5.0	5.0	4.9	4.9	4.8	5.0	5.0	5.2	5.2	5.2
Employment	800.0	100.0	100.0	100.5	100.9	100.4	100.0	99.5	99.0	95.2	86.7	82.4	80.5	81.0
Private non-construction sector	497.1	100.0	99.0	100.1	100.5	102.7	101.9	101.3	100.7	95.9	84.0	78.5	74.9	75.2
Construction sector	58.8	100.0	104.2	106.8	111.4	123.6	122.8	121.3	119.3	111.2	99.8	96.5	97.8	98.3
Public sector	244.1	100.0	100.9	99.8	99.2	90.2	90.8	90.7	90.6	89.8	89.0	86.8	87.7	88.6
Capital stock														
Private non-construction sector	248.2	100.0	102.6	103.7	106.1	108.6	111.0	113.1	115.1	134.9	178.0	249.1	352.9	526.3
Construction sector	10.2	100.0	101.1	101.6	107.7	115.1	119.4	121.9	123.4	142.1	192.8	279.3	421.0	628.5
Public sector	25.8	100.0	98.8	98.0	97.4	96.9	90.3	92.0	93.8	113.4	167.2	242.8	363.5	545.9
Net foreign assets**	-178.0	-12.7	-7.5	-2.8	1.8	7.0	12.2	17.5	22.9	72.7	145.9	196.3	213.5	213.7

\* Level in per cent

\*\* Index is assets in per cent of GDP

Figure 3.4:

Table: Macroeconomic development, Growth-corrected														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level, billion DKK	Index in fixed prices, 2003=100, Growth-corrected												
Private consumption	552.6	100	100.2	100.8	100.7	101.2	101.6	101.7	101.8	100.8	95.1	91.2	89.8	90.4
Real GDP	1 210.6	100	99.9	100.0	100.2	100.3	99.8	99.5	99.1	95.5	86.9	82.6	80.3	80.7
Unemployment*	6.2	6.2	6.4	5.7	5.0	5.0	4.9	4.9	4.8	5.0	5.0	5.2	5.2	5.2
Employment	800.0	100	100.0	100.5	100.9	100.4	100.0	99.5	99.0	95.2	86.7	82.4	80.5	81.0
Private non-construction sector	497.1	100	99.0	100.1	100.5	102.7	101.9	101.3	100.7	95.9	84.0	78.5	74.9	75.2
Construction sector	58.8	100	104.2	106.8	111.4	123.6	122.8	121.3	119.3	111.2	99.8	96.5	97.8	98.3
Public sector	244.1	100	100.9	99.8	99.2	90.2	90.8	90.7	90.6	89.8	89.0	86.8	87.7	88.6
Capital stock														
Private non-construction sector	248.2	100	100.6	99.7	100.0	100.4	100.5	100.4	100.2	96.3	85.6	80.6	76.8	77.1
Construction sector	10.2	100	99.1	97.7	101.5	106.3	108.2	108.3	107.5	101.5	92.7	90.3	91.6	92.1
Public sector	25.8	100	96.8	94.2	91.8	89.5	81.8	81.7	81.0	80.4	78.5	79.1	80.0	
Net foreign assets**	-178.0	-12.7	-7.5	-2.8	1.8	7.0	12.2	17.5	22.9	72.7	145.9	196.3	213.5	213.7

\* Level in per cent

\*\* Index is assets in per cent of GDP

Figure 3.5: Macroeconomic developments

Private consumption grows even more than domestic production; it almost doubles during the next 40 years and grows by more than 500 per cent throughout the century. Also capital stocks grow by a rather large number. Whereas the reported consumption levels etc. of table y are the most relevant to judge possible consequences for economic welfare, the permanently positive trend growth causes these figures to grow towards infinity in the long run, making it difficult to distinguish consequences of other shocks. Because of this, it is often convenient to show the same figures adjusted for labour-augmenting productivity growth. This makes it possible to judge the isolated effects of the remaining projected exogenous changes, among which the demographic changes are the most important ones. The result is seen in 3.5. In growth-corrected terms there is a fall in real GDP which closely follows the fall in employment. Private consumption falls by close to 5 per cent in 2040 and 10 per cent at the end of the century. The fall in private consumption is consequently smaller than the fall in GDP. The reason is that household savings are accumulating rather rapidly when the projection is initiated, mainly because of the funded pensions arrangements, and during the projection period people enjoy increasingly the returns from this accumulation of wealth. One result of this is that net imports can rise to a permanently higher level, enabling consumption levels to rise relative to production in the long run. This can be confirmed by looking at GDP composition in greater detail.



Table: GDP identity in projection														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level in billion kr.				Fixed price index, 2003 = 100, Growth adjusted									
GDP	1401.8	100.0	100.7	100.5	100.8	100.4	100.3	100.0	99.7	97.0	91.1	87.9	87.0	87.5
	Pct. of GDP, market prices													
Private consumption	673.1	48.0	47.8	48.2	48.0	48.3	48.4	48.5	48.7	49.8	50.7	50.6	50.9	50.9
Government consumption	370.4	26.4	26.6	26.1	25.8	23.1	23.2	23.4	23.5	24.4	26.8	27.7	28.7	28.9
Investments	273.5	19.5	19.5	20.1	20.8	22.5	22.4	22.1	21.9	21.2	20.6	20.7	21.6	21.5
Private inv. in production	188.5	13.4	11.8	13.7	13.9	13.8	13.5	13.3	13.3	13.0	12.6	12.2	12.6	12.5
Private inv. in housing	65.3	4.7	6.3	5.0	5.4	9.4	6.8	6.7	6.5	6.0	5.7	6.2	6.5	6.5
Inventory investments	0.9	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Government investments	18.8	1.3	1.5	1.5	1.5	-0.7	2.1	2.1	2.1	2.2	2.3	2.3	2.5	2.5
Net export	84.7	6.0	6.0	5.6	5.4	6.0	6.0	6.0	6.0	4.6	1.8	0.9	-1.2	-1.3
Export	634.9	45.3	44.8	44.9	44.8	45.5	45.4	45.4	45.5	45.0	43.7	43.5	42.1	41.9
Import	-550.2	-39.3	-38.7	-39.3	-39.4	-39.5	-39.4	-39.4	-39.5	-40.4	-41.9	-42.5	-43.3	-43.2
Sum	1401.8	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Figure 3.6:

### 3.2.3 Composition of GDP

The changing composition of GDP during the projection reflects the economic adjustments following the demographic changes, cf. 3.6. Private consumption as share of GDP rises by almost 4 percentage points during the 21st century. This reflects the falling aggregate savings rate of the households.

Government consumption falls by 0.6 per cent of GDP from 2003 to 2006, following the official forecast of the Ministry of Finance. In 2007, there is a sharp drop in government consumption of 2.7 percentage points, almost all of which is a result of the fiscal sustainability rule which makes a large permanent adjustment in collective government consumption from this year. Thereafter, the demographic development results in a considerable rise of government consumption of 5.8 per cent of GDP during the whole century.

Also investments rise somewhat during the projection period. Whereas investments relative to GDP falls in the manufacturing sector, this is more than offset by a rise in housing and in government investments. Government investments follow the forecast of the Ministry of Finance until 2006. From 2007 they adjust to maintain a constant capital/output ratio for the government producer. This is the reason for the sharp fall in 2007 when government production drops because of the fiscal sustainability adjustment, and the government capital stock diminishes correspondingly (some of the capital stock is privatized). In all future years, however, government investments rise to a higher level than before, reflecting the need for more services for elderly people.

With both private and government consumption and investments rising during the century, net exports must fall considerably. From positive net exports making out 6 per cent of GDP, the figure falls from around 2010 and becomes negative in the latter half of the century. This is the counterpart of the falling savings rate of both the private and government sectors.

### 3.2.4 Prices

Table 3.7 shows how some central indices of domestic prices develop relative to the numeraire of the model: the price of foreign goods (which is itself assumed to grow by an inflation level of 1.75 per cent annually). In the steady state, domestic goods prices grow with the same rate as the foreign price, and the nominal (hourly) wage grows with the product of productivity growth and foreign inflation. During the first years of the projection, nominal wages fluctuate somewhat: After a rise in the first projection year, it falls by more than one per cent. From 2007, it slowly rises, illustrating the fact that Danish wage inflation is a little larger than foreign wage growth. This process continues during most of the century. By the end of the century, the domestic wage level has grown by close to 10 per cent relative to foreign wages. This is due to the decline in the Danish labour force combined with the assumption that there are no demographic changes in the rest of the world. The Armington assumption and the fact that the construction and government sectors basically are home-market sectors ensure that domestic prices can differ from the foreign price level.

Table: Prices in projection														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level in billion kr.		Fixed price index, 2003 = 100											
Nominal wage index	1.000	100.0	101.0	99.9	99.9	98.6	99.2	99.6	100.0	101.8	105.9	107.9	109.5	109.4
Consumer price index	1.218	100.0	100.2	100.1	100.1	99.9	99.5	99.3	99.2	99.8	101.2	101.6	102.7	102.7
Government consumption price index	1.005	100.0	100.7	100.1	100.1	99.1	99.6	99.8	100.1	101.5	104.7	106.3	107.6	107.4
Government transfer regulation index	1.000	100.0	100.6	99.2	98.9	97.3	97.6	97.6	97.8	98.6	102.9	104.4	105.5	105.6
Real wage index	0.821	100.0	100.8	99.9	99.9	98.7	99.8	100.3	100.7	102.0	104.6	106.2	106.6	106.6
Housing price index	0.049	100.0	101.1	101.3	101.8	102.3	99.7	99.1	98.5	98.6	98.9	97.6	100.5	100.5

Figure 3.7: Prices

The rise in nominal wages is reflected in the price index for government consumption which is relatively labour-intensive and consequently rises by 7.5 per cent above foreign inflation during the century. Note here that production in the government sector is assumed to have the same productivity growth as the private production sectors unlike the assumption of the National Accounts that the productivity growth rate of government production is zero.

The consumer price index can be thought of as a weighted average of the domestic wage rate and the foreign price level. It consequently follows the nominal wage index partly. Consequently, also the real wage index (which is the nominal wage index divided by the CPI) falls during the first years and then rises by close to 7 per cent during the century.

Finally, the regulation index for government transfers mirrors the nominal wage level, but with adjustments made for changes in annual working time agreements and changes in labour-market pensions

Table: Public expenditure and revenue														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level, billion DKK						per cent of GDP							
Expenditure	710.11	50.66	51.10	50.04	49.32	44.00	47.57	47.81	48.12	50.43	54.34	55.07	56.23	56.34
Public transfers	243.02	17.34	17.45	16.89	16.49	16.39	16.52	16.63	16.81	18.15	19.44	19.31	19.27	19.23
- to immigrants, LDC	10.97	0.78	0.80	0.80	0.81	0.82	0.85	0.87	0.90	1.23	1.93	2.34	2.55	2.65
- to immigrants, MDC	7.56	0.54	0.54	0.52	0.52	0.53	0.55	0.56	0.58	0.68	0.82	0.92	0.96	0.97
- to decendants	2.50	0.18	0.19	0.19	0.20	0.21	0.21	0.22	0.24	0.37	0.70	1.13	1.52	1.61
Age-dep. pub. consumption	255.36	18.22	18.36	17.97	17.75	17.68	17.80	17.92	18.04	18.94	21.40	22.24	23.27	23.41
- to immigrants, LDC	5.89	0.42	0.43	0.43	0.44	0.44	0.46	0.47	0.48	0.64	1.06	1.43	1.64	1.77
- to immigrants, MDC	5.62	0.40	0.41	0.41	0.42	0.42	0.43	0.44	0.44	0.49	0.62	0.74	0.81	0.83
- to decendants	6.88	0.49	0.52	0.53	0.55	0.58	0.60	0.63	0.66	0.89	1.21	1.55	1.99	2.18
Non-age-dep. pub. consumption	115.09	8.21	8.26	8.11	8.02	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45
Other Expenditure	96.65	6.89	7.03	7.08	7.06	4.49	7.80	7.82	7.82	7.90	8.05	8.07	8.24	8.25
Revenue	736.77	52.56	52.22	52.40	51.77	51.80	51.56	51.55	51.59	52.70	54.93	55.12	55.34	55.22
Primary budget surplus	26.66	1.90	1.12	2.35	2.45	7.80	3.99	3.73	3.47	2.26	0.60	0.06	-0.89	-1.11
Net interest expenses	27.73	1.98	0.78	0.74	0.63	0.53	0.18	0.00	-0.18	-1.59	-3.34	-4.95	-5.59	-5.62
Net public debt	240.12	17.13	16.04	13.87	11.51	3.86	-0.09	-3.82	-7.34	-37.36	-74.20	-109.24	-122.33	-122.93
GDP in 2003-prices	1401.81	1401.81	1439.94	1466.33	1499.91	1523.62	1552.80	1579.23	1604.69	1903.27	2657.48	3809.78	5603.45	8373.08

Figure 3.8: Government expenditure and revenue

### 3.2.5 Government finances

The projected development in government finances is seen in table 3.8. In the base year, there is a primary budget surplus of 1.9 per cent of GDP: Total expenditure makes out 50.7 per cent of GDP, whereas revenue is 52.6 per cent. Over the whole century, both revenue and expenditure are expected to increase as per cent of GDP, but expenditure considerably more so than revenues, so that the primary budget surplus turns into a deficit in 2061. In 2100, the deficit is 1.11 per cent of GDP. The budget surplus does not fall monotonously, however. The particular policy reaction rule ensuring fiscal sustainability which is used during the projection creates a budget surplus of more than 3 per cent for a number of years from 2007.

Table 3.9 takes a closer look at the components of the government budget. More than half of total expenditure is used for government consumption. Here, expenditure for individual government consumption as a percentage of GDP is determined by demographics, by the price index for government services relative to other prices, and by the labour-augmenting productivity growth rate relative to the growth rate of real GDP. Whereas all three factors contribute to the increase as per cent of GDP, the demographic component is the most important one. Expenditures for individual government consumption rises from 18.2 per cent of GDP to 21.4 per cent in 2040 and 23.4 per cent in 2100.

Collective government consumption is by assumption a constant percentage of GDP, except that in the base-line scenario fiscal sustainability is ensured by a one-time permanent adjustment in this expenditure. The fall of 2.6 per cent of GDP in 2007 measures hence the size of the fiscal sustainability problem of the Danish economy in this projection.

After government consumption, income transfers to households are the largest expenditure component. They rise by almost four percent of GDP during the century from 17.3 per cent in 2001 to 19.2 per cent in 2100. Again, demographics account for this development as increasingly many people outside the labour force enter various transfer schemes. The expenditure is also affected by the regulation of transfers which closely, but not fully reflects the rise in wages, cf. chapter on government finances. Additionally, transfer regulations are

Table: Public expenditures														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level, billion DKK						per cent of GDP							
Expenditure	710.11	50.66	51.10	50.04	49.32	44.00	47.57	47.81	48.12	50.43	54.34	55.07	56.23	56.34
Government consumption	370.45	26.43	26.62	26.07	25.77	23.13	23.25	23.36	23.49	24.38	26.85	27.69	28.71	28.86
Income transfer	243.02	17.34	17.45	16.89	16.49	16.39	16.52	16.63	16.81	18.15	19.44	19.31	19.27	19.23
Government investments	22.66	1.62	1.76	1.79	1.78	-0.83	2.49	2.51	2.51	2.60	2.80	2.83	3.03	3.04
Product specific subsidies	19.72	1.41	1.39	1.40	1.40	1.42	1.42	1.42	1.42	1.42	1.40	1.40	1.37	1.37
Transfers to foreign countries	13.61	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97
Other production subsidies residual	16.14	1.15	1.15	1.15	1.16	1.17	1.17	1.16	1.16	1.16	1.13	1.12	1.11	1.10
VAT contribution to EU	4.26	0.30	0.30	0.30	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Subsidies to dwelling interest insurance	5.67	0.40	0.40	0.40	0.41	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
GNI contribution to EU	7.33	0.52	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.54	0.55	0.55	0.56	0.56
Transfers to households	5.75	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
Capital transfers to private production sectors	3.68	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
Other expenditures	-2.16	-0.15	-0.15	-0.15	-0.15	-0.16	-0.16	-0.16	-0.16	-0.16	-0.17	-0.18	-0.18	-0.18
GDP in 2003-prices	1401.81	1401.81	1439.94	1466.33	1499.91	1523.62	1552.80	1579.23	1604.69	1903.27	2657.48	3809.78	5603.45	8373.08

Figure 3.9:

affected by labour-market pensions schemes and changes in annual working time as well as various taxes.

Government investments relative to GDP rise relatively much during the projection, from initially 1.6 per cent to ultimately 3 per cent. These investments are governed by the rule that the capital/production ratio should always be constant in the government sector, and as production of government services rise as a percentage of GDP, so does also government capital and consequently investments. Naturally, the size of gross investments are influenced by the calibrated depreciation rate of the productive government capital stock.

The remaining components of government expenditure are all relatively small and do not change very much during the projection - either because they are by assumption constant percentages of GDP like transfers to foreign countries and capital transfers to private production sectors, or because they naturally follow GDP rather closely.

Also revenues rise considerably as a percentage of GDP during the projection, although less so than expenditures, cf. 3.10. Altogether, revenues are projected to rise by about 2.6 per cent of GDP. By far the largest revenue component is the source tax which falls initially following tax reductions, but rises relative to GDP in the long run as (taxable) government transfers and pensions benefits increase. Also VAT revenues rise in the long run as consumption rises relative to GDP. Product-specific indirect taxes fall relative to GDP, however. Most of the decline takes place during the years until 2010 when the tax freeze undermines revenues, but even after 2010 the percentage falls slightly because of a fall in indirect taxes paid by the private production sectors, whose GDP share decreases.

The labour-market contribution tax is a proportional tax on wages and follows GDP very closely during the entire projection. The corporate tax, on the other hand, falls by about 0.4 per cent of GDP during the century. The decline is due to the fact that the share of the production of the private firms to total GDP diminishes during the century (whereas in 2007, as this share momentarily increases because of the cut in government production caused by the fiscal reaction rule, corporate tax revenues increase correspondingly as well).

Table: Public revenues														
	2003	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	Level, billion DKK	per cent of GDP												
Revenue	736.77	52.56	52.22	52.40	51.77	51.80	51.56	51.55	51.59	52.70	54.93	55.12	55.34	55.22
Source tax	286.88	20.46	20.19	19.92	19.71	19.38	19.13	19.12	19.18	19.81	21.09	21.50	21.56	21.51
Value added tax	135.09	9.64	9.83	9.75	9.80	9.95	10.13	10.14	10.13	10.21	10.32	10.38	10.52	10.53
Product specific taxes	67.42	4.81	4.73	4.68	4.61	4.64	4.54	4.48	4.41	4.45	4.42	4.40	4.38	4.38
Labor market contribution tax	62.13	4.43	4.44	4.43	4.43	4.37	4.39	4.39	4.40	4.43	4.46	4.47	4.48	4.48
Corporate tax	39.93	2.85	2.77	2.74	2.73	2.93	2.86	2.83	2.82	2.64	2.57	2.53	2.48	2.46
Capital income	27.18	1.94	1.88	1.83	1.78	1.74	1.59	1.60	1.60	1.65	1.78	1.82	1.87	1.88
Property taxes	16.51	1.18	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.14	1.08	1.07	1.05	1.05
Registration taxes on vehicles	13.55	0.97	0.93	0.97	0.97	0.95	0.95	0.95	0.95	0.96	0.96	0.94	0.96	0.95
Imputed contribution to civil servant pensions	12.32	0.88	0.87	0.87	0.87	0.88	0.88	0.88	0.88	0.91	0.96	1.00	1.01	1.00
Taxation of pension funds	13.50	0.96	0.96	1.51	1.10	1.14	1.19	1.23	1.27	1.60	1.93	1.97	1.86	1.84
Contribution to unemployment insurance	10.12	0.72	0.72	0.71	0.70	0.69	0.69	0.69	0.69	0.69	0.71	0.71	0.71	0.71
Other revenues	52.13	3.72	3.75	3.82	3.89	3.97	4.05	4.09	4.10	4.22	4.66	4.32	4.46	4.42
GDP in 2003-prices	1401.81	1401.81	1439.94	1466.33	1499.91	1523.62	1552.80	1579.23	1604.69	1903.27	2657.48	3809.78	5603.45	8373.08

Figure 3.10:

### 3.2.6 National wealth

The composition of national wealth changes considerably during the projection, cf. 3.11. One of the noticeable features is that the free financial wealth of households declines considerably during the projection period and indeed becomes negative for several decades. Towards the end of the century, net household financial wealth again becomes positive as a consequence of the generational changes.

Financial wealth in the labour-market and private pensions arrangement naturally grow considerably as these two arrangements mature. They attain their highest total wealth some time around 2040 (?), at which time they have accumulated assets of 2540 billion DKK in growth-corrected terms. This corresponds to more than double their sum of assets in 2003, which amounts to 1142 billion DKK. After 2040, these pension assets diminish a little following the smaller populations in the labour-market-active ages.

The LD fund, which initially possesses bonds of 54 billion DKK, decumulates its assets progressively during the projection and closes down completely in 2031, at which time all its assets have been paid out.

The relatively recent SP fund (founded in 1998?) accumulates assets until around 2060, at which time it administrates assets of close to 170 billion DKK. This makes it around 75 % the size of the ATP fund, which is relatively mature already in the initial year and diminishes from the years from 2020 to 2100 following the decrease in the labour force.

Equilibrium in the financial markets imply that the total financial wealth of households and various pension funds must equal the (equity and debt) value of domestic firms plus domestic government debt plus net foreign assets.

As the government is assumed to follow a savings strategy to preserve fiscal sustainability during the base-line projection, the government debt of 240 billion DKK in 2003 quickly turns into positive net government assets (in 2008), after which the government keeps accumulating assets throughout the whole projection. By 2100, government net assets have reached the formidable sum of 1500 billion DKK.

Table: Assets in projection													
	2003	2004	2005	2006	2007	2008	2009	2010	2020	2040	2060	2080	2100
	<i>in billion kr., Growth adjusted</i>												
+ Financial wealth of households	666.9	641.5	613.7	582.2	480.2	422.8	367.8	318.6	-22.5	-161.1	-76.4	98.9	118.9
+ Financial wealth, pension funds	1141.7	1255.4	1319.0	1386.1	1449.2	1510.6	1571.9	1630.7	2126.7	2540.0	2529.3	2368.1	2353.4
+ Financial wealth, LD Fund	54.2	52.8	51.3	49.8	48.2	46.5	44.7	42.7	22.6	0.0	0.0	0.0	0.0
+ Financial wealth, SP Fund	41.3	41.4	41.3	41.2	40.9	47.6	54.1	60.5	116.2	161.3	167.7	156.9	158.3
+ Financial wealth, ATP Fund	263.3	265.9	268.2	270.1	271.6	272.9	273.8	274.5	261.0	220.2	211.0	201.9	202.8
- Debt of government sector	240.1	226.5	195.4	162.7	54.4	-1.2	-53.6	-102.5	-507.8	-947.7	-1346.1	-1492.1	-1507.8
- Equity and debt of firms	2105.2	2136.7	2137.7	2141.9	2137.7	2129.9	2120.3	2110.2	2024.0	1844.3	1758.7	1714.2	1720.2
= Net foreign assets	-178.0	-106.1	-39.6	24.8	98.1	171.8	245.6	319.4	987.7	1863.8	2418.9	2603.6	2621.0
Value of household dwellings stock	2324.4	2339.9	2329.1	2323.5	2372.2	2383.1	2391.6	2396.2	2361.6	2239.2	2192.5	2146.7	2157.7
- of which land makes out (in per cent)	22.2	22.5	22.6	22.6	22.1	22.0	21.8	21.6	20.5	19.1	18.6	18.8	18.8
- of which buildings make out (in per cent)	77.8	77.5	77.4	77.4	77.9	78.0	78.2	78.4	79.5	80.9	81.4	81.2	81.2

Figure 3.11:

The sum of equity and debt of firms shrinks during the projection after a minor initial rise. After 100 years, it has fallen to about 82 per cent of its initial value, roughly corresponding to the decline in domestic production.

Initially, net foreign assets are negative, but due to both private and government positive net savings and the decline in domestic physical capital, Denmark accumulates very large foreign assets. In 2100, net foreign assets make out 2621 billion DKK in the projection or more than twice the size of GDP.

The last asset calculated separately in DREAM is the stock of dwellings of households, the value of which is relatively stable during the projection period, but does decline by about 7 per cent.

Total national wealth can be computed as the sum of net financial assets of households, all pensions arrangements and the government sector plus the value of household dwellings. This will also be equal to domestic physical capital (equity and debt of firms plus the value of residential capital) plus net foreign assets. Defined in this way, national wealth grows from less than 4.300 billion DKK in 2003 to 6.500 billion DKK in 2100, even in growth- and inflation-corrected terms. This is mainly due to the assumed development in government debt, but also the total value of household and pensions assets grow by about 500 billion DKK during the projection. The huge government savings are consequently not offset, but supplemented by increased private savings in this scenario.

Intergenerational matters?

Generational accounting

As an overlapping generations model, DREAM is well-suited to analyse intergenerational issues. A central tool is generational accounting which distributes all government expenditures and revenues over the different age groups (and possibly also over genders and origin groups)<sup>3</sup>.

<sup>3</sup>All expenditures and revenues of the primary budget are distributed according to the model. Some entries are distributed lump-sum because of lack of data or because no other distribution principle is deemed sensible (for instance in the case of expenditures for collective government consumption). Cf. Gruno (2005) for a

For 2003, the resulting net contribution from each age group to the government is seen in Figure ...

(Figure of age distribution in base year about here)

It shows the intuitive result that people are on average net recipients vis-a-vis the government from their birth until they are 23 years old, and again from the time when they are 63 years old until death, whereas they are net contributors from 24-62 years old when their tax payments, mainly from labour income, exceeds their transfers and the value of other government expenditures allocated to them. Note that the generational accounting comprises all age groups including children, even though only adult generations (from 17 years of age) are considered to make independent economic decisions in the DREAM model. Children below 17 years are assumed not to receive any market income or pay taxes, but they do receive some government consumption expenditures and transfers.

Figure ... shows a picture of the momentary (or cross-sectional) distribution across all generations alive in 2003. Another question concerns the position of a cohort vis-a-vis the government over its entire life. Figure ... shows the present value of net government contributions over the entire life for all generations born in 2003 and later.

(Figure of discounted life-time net contributions about here)

The figure shows that all generations born from 2003 and onwards - i.e. all generations who live their entire life inside the time horizon of the model - are net recipients vis-a-vis the government in present value terms. Two reasons enable such a result even for an economy where the government respects its intertemporal budget constraint: Firstly, the generations who were born before 2003, but are still alive, also contribute to the government budget. Secondly, discounting implies that contributions paid (or benefits received) early in life weigh more heavily than contributions paid or benefits received later. As individuals are always net beneficiaries during childhood and youth in the Danish welfare system (because of transfers to children and day-care and educational expenses), the benefits received may outweigh the contributions paid later in present value terms even under a balanced budget. This is the opposite effect of a PAYG pensions system.

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description of the method employed in distributing the individual budget entries over the entire population.

