A CGE Analysis of the Danish Ageing Problem

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Computable General Equilibrium Modelling

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Chapter 1
INTRODUCTION AND NON-TECHNICAL SUMMARY

This paper presents an analysis of the economic consequences of the Danish ageing problem using the DREAM model. The results concerning the period until 2035 were published as part of the semi-annual report of the Danish Economic Council in December 1998, see Economic Council (1998). The present paper represents a full documentation of both the methodology and the exogenous forecast on which the analysis is based. The emphasis here is on structural and long run issues: Is there a permanent economic burden due to the demographic development or is the problem temporary in the sense that long run fiscal pressures on the working population is reduced to the present level without imposing significant changes to the Danish welfare system? The analysis distinguishes itself from similar analyses, see e.g. Miles (1999) and Broer & Lassila (eds.) (1997), by the incorporation of an initially non-matured funded pension system based on actuarial principles and by a more elaborate demographic structure and complex composition of life cycle income profiles. Furthermore the developed solution method to the model with forward looking agents enables the simulated solution to reproduce the 1995 values of the Danish national acounts as a temporary equilibrium.

The Danish ageing problem is caused by the change in the demographic composition of the Danish population leading to an increase in the number of pensioners by approximately 50 percent from 1998 to around 2035 and a simultaneous reduction in the number of persons in the labor force by approximately 8 percent. 2030-2035 represents the period where the demographic old age burden peaks. In the long run the number of pensioners stabilizes at a level approximately 40 percent higher than the present level. The long run labor force is only slightly reduced compared to the present size. Thus the long run demographic old age burden is significantly higher than the present. The problem initiates around 2010 as the large generations born in the post Second World War period begin to retire. Until 2010 the demographic composition remains fairly constant implying that there exists a period where no structural pressures emanate from the demographic composition of the population. In the political discussion this time span has been termed a ”breathing space” for the economy. The period leaves room for political actions to deal with the change in the anticipated demographic composition.

One very important example of such actions that has been initiated is the gradual shift from a pure pay-as-you-go pension system towards a system based on both pay-as-you-go elements and fully funded pension systems (called labor market pension schemes) that cover the (vast) majority of individuals in the labor market. Historically labor market pension schemes were initiated for groups of academic workers in the 1950'ies. These pension schemes are generally based on a contribution rate of 15 per cent of the annual wage sum. In the 1960'ies and 1970'ies labor market pension schemes were introduced for various groups of mainly publicly employed white collar workers (such as nurses, teachers, laboratory technicians, and nursery teachers). These pension schemes are generally based on a contribution rate of 12 per cent of the annual wage sum. For the large group of manual workers in the private labor market, labor market pension schemes have been introduced in the beginning of the 1990'íes. The contributions to these schemes were initiated at a very low level and are gradually increased over time and are expected to reach a level of 9 per cent of the annual wage sum. In the present analysis this level is assumed to be reached in 2010.

Pension savings receive special tax treatment in Denmark. Contributions are deductible in the current income taxation whereas pension receipts from the pension schemes are taxed according to the income tax system at the time of the payment. Capital income from pension saving receive a tax relief compared to personal capital income. These tax arrangements imply that the tax revenue is postponed permanently. This leads to decreased tax revenue in the phase where the pension funds are ”young”. A ”young” pension fund is defined as a fund where the pension of present pensioners is based on contributions that were initiated at some point in time after their debut on the labor market. Contrary, a ”mature” pension fund is defined as a fund where the pensions of present pensioners are based on contributions from their entire period of labor market participation. When the system is mature the tax revenue from the pension receipts exceeds the loss in tax revenue from deductible pension contributions. Therefore public revenue has a tendency to increase automatically as the labor market schemes approach maturity.

On the other hand public expenditures also have a tendency to increase automatically due to the ageing of the population. Both public transfers such as pensions and public expenditures to e.g. health care and residential homes for elderly people are expected to increase with the number of pensioners.

Finally, the fact that the labor force is reduced has negative effects on the financing of the increased cost of the ageing population leading to higher tax rates.

To sum up, the ageing problem is in the present analysis defined as the net-effect of three shocks: 1) The automatic increase in the tax revenue from increased labor market pension receipts, 2) The increase in the public expenditures due to the change in the demographic composition of the population (for given level of age dependent public transfers and publicly provided products), 3) The reduced number of persons in the labor force.

The analysis is initiated from the 1995 level of the national accounts using the dynamic calibration procedure developed in Knudsen *et al.* (1998a). The analysis should not be regarded as a forecast for the following reasons: 1) Foreign demand, foreign inflation, world market interest rate, and the exchange rate are all assumed to remain at their initial level through out the time horizon. The interpretation of these assumptions is that the analysis focuses on the effects of the domestic ageing problem on the economic performance given that the stimulus to the economy from the rest of the world remains unaffected. Of course most OECD countries face a demographic evolution that is similar to the Danish or even more dramatic. This fact alone indicates that the assumption of neutrality with respect to economic stimulus from the rest of the world may be considered a very optimistic scenario. 2) The analysis abstracts from factor productivity growth. Therefore the reduction in the part of the population that belongs to the labor force implies that the real value of domestic production has a tendency to fall. [1] 3) The foreign price level (the numeraire) is assumed constant in Dkr., i.e. imported inflation is absent. All prices should be considered as being implicitly measured relative to the foreign price level. On the other hand the analysis allows for relative price changes due to domestic economic activity. The nominal value of a variable in a given year in DREAM is transformed into true nominal values by inflating the figure with the expected accumulated price increase on foreign goods.

Furthermore, domestic public spending rules are assumed to remain unaltered through

[1] A version of DREAM allowing for Harrod-neutral technological progress i currently being developed.

the demographic transition. This should not be considered a forecast of future economic policy but rather an analysis of the consequences of not adjusting policy rules. The public pensions are assumed to remain at the current level per person except for the indexation that follows from the current indexation rule.[2] Public services supplied to individuals are assumed to remain at the current real level for an individual of a given gender in a given age-group. As many public services supplied to individuals increase with the age of the person in question, this assumption implies an increase in total spending on public services due to the demographic change. Finally, it is assumed that income taxation is adjusted to keep a balanced public budget from year 2006.

The applied dynamic calibration procedure differs from traditional forecasts using macroeconomic models in several respects. The most important one is the fact that the agents in the DREAM model have forward looking expectations. This implies that the decisions of the agents today are based on their expectation of e.g. the economic policy in the future. In the projection period we assume that agents know (and believe) the policy rules specified above and forecast the consequences for their own intertemporal budget.

1.1 The economic consequences of the ageing problem

The qualitative effects of the demographic change and the introduction of labor market pension schemes on main economic indicators are indicated in tables 1.1 and 1.2 below.

Observe that figures for total public expenditures follow National Accounting principles and include the general labor market related supplementary pension scheme (ATP) and the $LD-$pension fund in the public sector.[3] From 1995 to 2031-35 the net increase in the ATP-pensions accounts for approximately one-third of the total increase in total age-dependent public expenditures. The real value (measured in units of the foreign good) of total public expenditures peaks in 2031-2035 where the expenditures are approximately 33 percent higher than in 1995. In 2046-2050 this

[2] This ignores the fact that increased payments from the funded pension system tend to reduce social pensions as a part of these are reduced with increasing income from other sources. Therefore total expenditures to social pensions are overestimated in the present analysis.

[3] New definitions (not applied here) of the National Accounts include only ATP in the definition of the public sector. For the present analysis this change in the definition will only have marginal effects given the relatively small size of the LD pension fund.

figure has been reduced to 25 percent, due to the decreasing number of pensioners in the period from 2035. This level of real expenditures remain approximately contant throughout the time horizon.

Total public expenditures as percent of GDP increase by 9.0 percentage points from 43.6 percent in 1995 to 52.6 percent in the peak period from 2031-2035. Adjusting for the increase in the ATP-pension (which is a fully funded pension system) the increase reduces to 6.3 percentage points. The reduction in the real public expenditures from 2035 and minor increase the real GDP imply that real public expenditures as percent of GDP only increase by 6.1 percentage points in the long run.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Social security pension	73	74	114	99	101	102
Civil servants pension, ATP, LD	15	22	42	42	42	42
Age-dependent public service	155	162	193	188	186	188
Age-dependent public transfers	45	43	47	46	46	46
Total age-dependent public expenditures	**288**	**302**	**396**	**375**	**375**	**378**
Other expenditures	135	129	129	128	128	128
Total public expenditures	423	431	525	503	502	506
Gross domestic product (GDP)	969	1001	995	996	1002	1016
Total public expenditures as percent of GDP	43.7	43.0	52.7	50.5	50.1	49.8
Change in total public expenditures as percent of GDP from 1995		-0.6	9.1	6.8	6.5	6.1

Note: Average annual rates, deflated by foreign price level

Table 1.1: Public expenditures and Gross Domestic Product

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Tax on labor market pensions	5	5	20	30	36	35
Tax on civil servants pensions, ATP, LD	6	8	17	16	15	14
Tax on private pension schemes	7	5	13	12	13	13
Tax on social security pensions	21	18	37	28	27	26
Other revenues	381	395	438	417	412	418
Total revenue	**419**	**431**	**525**	**503**	**502**	**506**
Effective average wage-income tax rate	42.7	39.0	47.0	43.2	40.9	39.6

Note: Average annual rates, deflated by foreign price level

Table 1.2: Evolution in public revenues

From table 1.2 one observes that there is a significant increase in the tax revenue from labor market pension schemes. But the increase is insufficient to finance the increased public expenditures in the period up to 2035. However, the increase continues beyond 2035 and the revenue is stabilized around 2075, where the revenue form the taxation of labor market pension receipts has increased to 36 billion Dkr. compared to the 20 billion Dkr. in 2031-35 and 5 billion in 1995. Therefore the analysis reveals that in the period of structural change in the demographic composition of the population from 2010 to 2035 the tax revenue from labor market pensions does not increase sufficiently to fully finance the annual increases in public expenditures. However, in the following period the increase in the tax revenue continues whereas public expenditures are moderated. The result is that the long run effective average income tax rate is than the the 1995 level and comparable to the year 2000 level. Therefore the fact that the labor market pension system matures is sufficient to finance the long run increase in the public expenditures that is due to the permanently higher demographic age-burden.

The economic impact of the change in the size and the composition of population may be assigned to a combination of an anticipated supply and an anticipated demand shock to the economy. The anticipated supply shock consists of the gradual reduction in the number of persons in the labor force. This effect is enforced by the increased effective average tax rate on wage-income. Ceteris paribus this reduction in the labor force tends to generate a similar long run reduction in the capital stock and therefore also in domestic production. However, the analysis is performed given the assumption that the foreign demand for the domestic produced good is unaffected throughout the time horizon. Therefore the reduction in domestic production leads to an improvement in the terms of trade that reduces the relative price of capital generating an increase in the capital labor ratio. The supply effect tends to generate a permanent reduction in the total employment of 6 per cent and a reduction in net output of approximately 3 per cent. The total capital stock remains approximately at the initial level through out the analysis.

The demand effects are more complicated. First of all the introduction of labor market pension schemes implies that the savings rate in the economy is increased. This effect is due to the fact that the effective time horizon of the labor market pension fund is longer than the assumed time horizon of the households due to the precautionary behavior of the pension fund. The household determines the optimal consumption path over the lifetime given a time horizon that is equivalent to the expected life time of a woman at the time where she enters the labor market. However, the pension fund has to ensure funds to pay pensions to persons lives longer than the expected life time of a 17 years old woman. This implies that a part of the capitalized contribution to the labor market pension fund is spend on pensions that are paid out after the expiration of the households time horizon. Therefore these pension receipts do not affect the economic planning of the household. The reaction of the household is to increase gross savings.

This initially tends to reduce the level of private consumption. When the demographic age-burden materializes the net-effect of the increase in the public expenditures and the associated tax increase also tends to reduce private consumption. Over time the increased savings reverse this effect, however this do not happen before 2040.

Therefore the conclusion is that the demographic ageing of the population leads to two types of economic problems : First, a negative labor supply effect which reduces the productive capacity of the economy and second a financing problem due to increased public expenditures (for a given level of public service and income transfers per person of a given age). Observe however that in the period until 2010 the supply shock is reversed as the labor force in increase in this period.

The gradual increase in labor market pensions contributes to a reduction of the ageing problem , but the consequences for the working part of the population remain a significant increase in the tax burden from 2005 to at least 2055. However from this point on the larger tax based associated with the increased pension receipts is sufficient to finance the permanent increase in age-dependent public expenditures.

By decomposing the total exogenous demographic shock and the evolution in the funded pension system one obtains the following conclusions: First, if the total labor supply is kept at the 1995 level then the increase in the average tax rate that is associated with the increased expenditures to the rising number of pensioners is reduced to a maximum of around 2 percentage points in the peak period from 2031-35 compared to the level in 1995. Furthermore, the permanent effect on the average income tax rate is a reduction of no less than 6 percentage point compared to the 1995 level. Therefore, given the simulated evolution in the funded pension system, the demographic age burden may be almost eliminated by keeping the labor supply at the current level. In other words, the negative labor supply effect from the 8 per cent reduction in the labor is a major part of the simulated ageing problem.

Second, by assuming that contributions to the labor market pension system is forced to zero in 2005 the long run consequences for the economy without a funded pension system is analyzed. In this scenario the ageing problem becomes a major long run problem even if the problem at the peak of the demographic transition in 2035 is somewhat reduced. The reason for this development is that in the short run the tax base is increased by the removal of the deferred tax from contributions to the labor market pension. This contributes to the financing of the expenditures in the first period (until 2035) of the demographic problem. However, as pension receipts from the funded pension system is gradually phased out due to the lack of contributions, the tax base is significantly reduced compared to the base scenario. This reduction in the tax base implies that the increased cost in the long run has to be financed through higher average income tax rates. The long run increase in the effective average wage income tax rate is no less than 10 percentage point compared to 1995 if no funded pension receipts are present.

The conclusion therefore is that even if the funded pension system is not sufficient to avoid a relatively severe temporary burden of financing increased public expenditures in the transition period until 2035, the funded pension system has a significantly positive effect on the long run properties of the Danish economy.

The rest of this introduction gives an informal presentation of DREAM which is intended to illustrate the specific modelling of the demographic changes and of the pensions fund. The second part of the paper gives a formal presentation of DREAM including the behavior of the pension funds. This is followed by a presentation of the data material, the projection of the exogenous variables and the parameters (elasticities) that are fixed in advance. A brief description of the dynamic calibration procedure and a presentation of the results concerning behavioral parameters that are determined in the procedure lead up to the presentation of the model-generated projection of the behavior of the Danish economy for the next 75 years. Finally the total effects of the demographic shock and the evolution of the funded pension system is decomposed into three parts: 1) The effect of the funded pension system, 2) The effect of the increased expenditures 3) The effect of the reduced labor supply.

1.2 An overview of DREAM

For the present analysis the DREAM model is extended to allow for a specific modelling of the pension sector in the economy. The basic idea of this type of model is that the macroeconomic consequences is a result of the sum of the behavior of the agents in the model. Therefore the behavioral assumptions of the different agents and the types of markets are central parts of the model. The types of agents are households that differ according to their age, private firms, governmental producers, pension funds, the public sector and the foreign sector. We start by giving an overview of the behavior of the households. This is followed by a description of the labor market pension schemes, the behavior of firms and an overview of the labor market, and finally the behavior of the public sector.

1.2.1 Households

The household sector consists of overlapping generations of households with a finite and deterministic time horizon. Households are defined as representative couples with children. The size of the representative household is affected by the exogenous age and gender specific mortality rates of the members, and by the age specific fertility rates of women. The adult members of the households are divided into workers and pensioners, according to an exogenously given retirement age which is 62 years implying that all persons are assumed to participate in the early retirement scheme from this age.[4] The explicit incorporation of both children and a retirement period gives rise to a life-cycle motive for saving (and dis-saving) in the model. At the end of the planning horizon, each generation of households leaves a bequest to their children. Agents may be alive *after* the end of the planning horizon - these persons are assumed to consume their total income in each period. The income consists of the after tax value of the public transfers and pension receipts from labor market pension schemes. Younger generations derive utility from consumption of the domestic and the foreign good, and incur disutility from time spent working. Bequest also has a positive effect on the donor's utility.

Income for the consumers arise from 10 main sources. The first is wages received for the hours worked, and the second is unemployment benefits for the hours the average member of the household is unemployed. By the construction of the model, work-sharing prevails so that everyone in the labor force is equally underemployed. The third source of income is age-dependent income transfers from the public sector, such as child-care transfers, education benefits, and sickness benefits. The fourth source of income is social pensions, which are given to all persons who are 62 years old or older. The pension is identical for all pensioners.[5] The fifth source of income is retirement pension from labor market pensions schemes (a description of the different types of labor market pension is given below). The sixth type of income is disablement pension from the labor market pension system, the seventh is spouse pension from the labor

[4] There is only one reperesentative person of each gender in each generation. The age 62 years is chosen (in stead of 60 years) to account for the fact that not all persons in the labor force retire when the age of 60 year is reached.

[5] This implies that no distinction between early retirement benefits and social pension is made. Social pensions therefore are higher than actual pensions whereas payments to persons on early retirement schemes are lower that the actual level.

market pension system. The eighth type of income arises from bonds and shares held by the consumers. The ninth source of net income is transfers from abroad. The final type of income arises from inheritance left by the parent household.

Observe that pension receipts from the labor market pension system is defined as income even if these partly represent a reduction in accumulated savings in the pension fund. On the other hand the reserves of the pension fund are not included in the stock of financial assets of the households. This convention is in line with most literature in the field. However as noted by Miles (1999) this implies that the amount of saving as young and dissaving by elderly households both are underestimated in data given these conventions.

The combined assumption of perfect capital markets and perfect foresight implies that the household's consumption of the aggregate consumption bundle depends on the sum of the stock of financial assets and the discounted value of the lifetime non-interest income. Furthermore consumption of the aggregate good is decreasing in the current consumer price index relative to an index of future consumer prices. Demand for each of the goods in the aggregate consumption bundle depends of the relative prices of the goods.

1.2.2 Labor market pension schemes and private pension schemes

Two different types of labor market pension schemes exist in Denmark. Both types are fully funded. The first type is the so-called ATP pension scheme[6] (which actually consists of two separate pension schemes) that is compulsory and covers all employed persons and all unemployed persons receiving unemployment benefits. The second type of schemes, which we name labor market pensions, are voluntary in the sense that the part of the wage that is paid into the pension fund is determined as part of the collective bargaining agreements between labor unions and employers confederations. For the individual household the saving in labor market pension schemes is compulsory in the sense that employment in a given bargaining area determines the labor market pension savings. By 1995 virtually all collective bargaining agreements in the private labor market contain agreements concerning contributions to pension funds as part of total labor costs. For white collar workers who are not covered by collective bargaining agreements, employment contracts normally specify a part of the wage costs as pension contributions. Also for this group we consider the labor

[6] ATP is an abbreviation in Danish for labor market related supplementary pension.

market pension schemes as compulsory. Finally some groups of mainly blue collar workers working in non-organized parts of the labor market may not be covered by labor market pension schemes. The same holds for unemployed and persons who do not belong to the labor force.

In the national accounts - and in this version of DREAM - the ATP fund is defined as a part of the public sector. Contributions to the ATP pension therefore affect the budgets of the public and private sector in the same way as do taxes. Similarly pensions from the ATP fund have the same budgetary effect as pay-as-you-go pensions. As one of the main goals of the modelling is to capture effects of the special tax treatment of savings in pension funds, rather than to model the total value of the accumulated stock of assets in pension savings arrangements, we have chosen to model pensions from the ATP system as exogenous. For given assumptions concerning wages and employment the pensions have been forecasted by the ATP fund for this analysis. Contributions to the ATP fund are modelled similar to taxes and the contribution rate is calibrated in the base year to yield the observed revenue. As the value of contribution to the ATP system is endogenous whereas the pensions are exogenous in the model, there is no guarantee that the ATP system is fully funded in the present modelling.

Contributions to the labor market pension schemes are modelled as an exogenous fraction of the pre-tax wage rate. The size of the contribution rate is set equal to the average contribution rate of employed persons, who are at least 22 years old, in the data from 1995. In the period from 1995 the average rate is gradually increased as a consequence of the negotiated increase in the rate in the 1997 bargaining agreements for privately employed blue collar workers. For future periods a gradual increase in the contribution rate for blue collar workers in the private labor market is assumed. The increase continues until it reaches 9 percent.

Pensions from the labor market pension schemes is calculated endogenously in DREAM. The calculation implies that these schemes are fully funded and actuarial principles are obeyed. We assume that the labor market pension schemes provide the following pensions: Retirement pension, disablement pension, and spouse pension. In the data the size each of the pension types differ from one pension fund to another. The relative sizes of the types of pension is determined as the average over the funds in the data in 1995.[7]

[7] The pension funds for blue collar workers in the private sector presently only offer retirement

In addition to the two labor market related types of pensions schemes the exists a significant amount of private pension arrangements that are also subject to special tax treatment. Ideally, the size of these private pension arrangement should be chosen by the optimizing behavior of the households as these contributions are not exogenous from the point of view of the household. However, the absence of uncertainty and the existence of perfect capital markets in DREAM implies that the household would speculate in the beneficial tax treatment of pension saving and no equilibrium would exist. Therefore we have chosen to model private pensions arrangements exogenously. This implies that the measured age and gender specific contributions to these arrangements are assumed to be a constant fraction of the wage sum of the group in question. It is assumed that these savings are withdrawn (and taxed) at two points in the life time. The first part is withdrawn as the generation reaches the age of 67 years and the rest is withdrawn at the age of 72.

Finally, there exists two additional types of labor market related pension schemes, which are treated exogenously in DREAM: First, civil servants receive pension from the public sector. These pensions are not funded and the expected future (real) value of these pensions have been forecasted by the Danish Ministry of Finance. This forecast is indexed to the wage rate in DREAM. Second, the part of the population that had labor market income in 1977-79 receive a so-called LD pension.[8] These pensions are assumed to be paid out as a once and for all payment when the generation in question reaches 62 years.

1.2.3 Firms

The fundamental behavioral assumption of the corporate sector is that firms strive to maximize the discounted value of current and future net-of-tax-dividends to the owners of the stock of shares in the firm. It is assumed that the firm finances investments by an exogenous combination of debt and retained profits. This implies that the discounted value of the stream of dividends is positive (and thus the shares have a positive economic value). The positive discounted value of dividends is composed of the earnings on the non-debt financed part of the capital stock and the discounted value of the pure profits of the firm. Pure profits appear due to the fact, that the net

pension. As the contribution rate for these pension funds is increased one should expect that other types of pensions is introduced as well. The present procedure ignores this difficulty.

[8] LD is not receiving contributions and the fund simply pays out pensions and administrate the accumulated stock of assets from the contributions in 1977-79.

production function of the firm is strictly concave, because of convex costs of installation of new capital (and also of deploying existing capital). Furthermore, these costs imply that an individual firm will want to adjust its capital stock gradually towards a stationary capital stock, if the firm is in an environment with constant prices and demand. Investments are driven by the so-called marginal q-theory of investment. Contrary to this, we abstract for the time being from any turn-over costs of labor, which implies that the employment decision does not depend directly on future prices and wages.

The presence of pension funds that is not taxed according to the personal capital income taxation implies that there are two financial investors in DREAM. We assume that the value of shares have been determined through an arbitrage condition, which implies that the professional investors of the pension funds are indifferent between investing in shares and in bonds. As the relative taxation of the two assets is not identical for private investors and institutional investors (pension funds) the same is not true for the private investors. To overcome this problem we assume that private investors simply keep a fixed proportion of their total assets as bonds (and the residual as shares). This implies that the average yield after tax for private investors is affected by the development in the value of shares.

1.2.4 Labor market

The labor market is assumed to be competitive. Labor is a homogenous good. The DREAM group's 10 per cent sample of the total register of persons and workplaces in Denmark has been used to calculate the distribution of wages across gender and generations for 1995. It is assumed this distribution of relative wages remains constant through the analysis. As labor is homogenous the differences in wages totally reflect differences in productivity, such that the productivity corrected wage is identical across gender and generations. Persons in the age groups from 17 years to 62 years are potentially in the labor force. For each age group and gender an exogenous labor market participation rate is assumed. These rates are considered fixed at the 1995 level through out the analysis. Persons, who belong to the work force and supply less labor to the market than an institutionally fixed maximum supply, are entitled to (supplementary) unemployment benefits. Thus the labor market equilibrium implies that agents voluntary reduce their individual labor supply such that underemployment and work sharing prevails. The calibrated level of unemployment in the model is thus by assumption entirely voluntary. However, the construction of the labor market implies that the market equilibrium is affected as if imperfections in the market had prevailed. Therefore one may interpret the labor market as an imperfectly competitive market.

1.2.5 International relations

The economy is integrated in the world economy through trade and capital flows. Materials and foreign consumer goods are imported while the domestic product is exported. Domestic production is an imperfect substitute for imported goods, implying that the terms of trade are endogenous. Financial capital is assumed to be perfectly mobile internationally, and the exchange rate is fixed. These assumptions, the absence of uncertainty and the presence of residence based taxation of interest income, imply that the domestic and the foreign pre-tax interest rate are equal.

The foreign demand for the domestic good can be thought of as demand functions derived from intertemporal optimization of foreigners. Assuming that foreign consumers have utility functions, which are similar to those of domestic consumers, it may be plausible to assume that the foreign demand function for the domestic good is isoelastic. For simplicity we assume that the position of the export demand curve is fixed through time.

1.2.6 Public sector

The government collects taxes, distributes income transfers and produces goods. Following the conventions of the national accounts the governmental sector produces mainly non-marketed goods and services of which an overwhelming part are delivered directly to public consumption. Income transfers are assumed to be both age and gender specific. Data for these transfers in 1992 are obtained from the generational accounting project joint organized by the Ministry of Finance and EPRU (see Jacobsen *et al.*; 1997). Income transfers per person are currently indexed to the wage rate according to Danish law. In the projection period this indexation is assumed to be maintained. This implies that for a fixed real wage the real expenses to income transfers will evolve according to the demographic change in the number of recipients. Similarly public consumption and services to education, health care and social expenses are assumed to be age and gender specific. The distribution of these expenditures follows The Danish Ministry of Economic Affairs (1998). The real value per person of these expenses are kept at the 1995 level. Also for these public expenditures the result is that demographic changes in the number of recipients are transmitted to the level of expenditures. For the remaining part of public expenditures it is assumed that the real value is a constant fraction of GDP.

Consistency requires that the agents do not violate their intertemporal budget constraints. For the government this implies that the discounted value of government debt has to converge to zero for time approaching infinity; violating this means, that private agents are not willing to hold government bonds. Until the year 2005 tax rates are assumed to remain at the current or the announced level. From 2005 we assume that the government runs a balanced budget and uses the personal income tax system as the residual source of funds. The dynamic calibration process implies that implicitly these policy responses are known to the general public and affects current and future behavior.

Chapter 2
LABOR MARKET PENSION FUNDS

Danish labor market pension schemes are in general part of collective bargaining agreements between labor unions and confederations of employers. Therefore, different labor market segments have different labor market pension schemes. Historically, the labor market pension schemes were initiated within the labor market segments of academic employees during the 1950s. These schemes have contribution rates of 15 percent of the gross wage income and support the following types of pensions: retirement, disablement, spouse and child pension. During the 1960s and 1970s, the labor market pension schemes were introduced to public employees with middle-range training, for instance teachers, nurses and nursery school teachers. These schemes typically have contribution rates of 12 percent of wage income and support the same types of pensions as the pension funds of academic workers. The lower contribution rate is to some extent counteracted by the fact that members of these schemes are enrolled earlier in life and therefore contribute for a longer period of time. Therefore, the coverage of retirement pension as percent of wage income is approximately similar for the two categories of workers. During the 1990s, the labor market pension schemes have been extended to cover all parts of the organized labor market by including the large segment of privately employed blue collar workers. The bargaining agreements for these groups of workers include contributions to labor market pensions schemes that presently are in the range from 5 to 6 percent, but are expected to increase gradually and to reach 9 percent by 2010. Presently the pension fund for these workers only support retirement pension schemes, but as the contribution rate reaches 7 percent other pension schemes are expected to be introduced.

Both young pension funds with low contribution rates and more mature pension funds with higher contribution rates are modelled within DREAM as a unique collective labor market pension fund. We make the following assumptions: the contribution rate is equal to the average contribution rate (defined as total contributions relative to the relevant wage sum), all employed persons are members and all new entries to the pension schemes take place at the age of 22 years (which on average may be considered a somewhat low entry age). The resulting pensions should therefore be interpreted as pensions of the average person within a given generation. DREAM's pension fund supports three types of pensions: retirement, disablement and spouse pension. This implies that unlike the pension fund of academic employees, for example, the child pension is ignored. This, however, has only a small positive effect on the forecasted value of retirement pensions.

Labor market pension funds in Denmark are regulated by the authorities. Calculations of pension undertakings follow procedures that are approved by the authorities. The modelling of the pension fund in DREAM is based on these procedures. Thus annual pension undertakings are derived from conservative forecast procedures of the funds. For each member generation this yields a difference equation in the annual pension undertakings. One may view this as equivalent to the "Keynes-Ramsey rule" of households, which is a difference equation in their annual consumption. The initial level of household consumption is derived from the fact that the household, given this initial level, should be able to finance the consumption path over the life cycle that follows from the Keynes-Ramsey rule. Similarly the pension fund determines the initial pension undertaking such that the capitalized value of expected contributions equals the capitalized pension over the lifetime, given the difference equation in annual pension undertakings. In the following, the behavioral equations of the pension fund are derived.

2.1 Members of the pension fund

All persons who are employed and at least 22 years of age are assumed to be members of the pension fund. At any given moment of time there are two types of members in the pension fund: active members and disabled members. In addition to this, the pension fund supports spouse pensions, i.e. pensions to persons that are not necessarily members of the pension fund. The following subsections outlines the evolution over time of a given generation of members. In each period a fraction of the members become disabled. The evolution in the amount of disabled is derived as a function of the number of members of the given generation. Similarly the fact that a fraction of the members die in each period implies that a number of new spouse pensioners appear. The number of spouse pensioners is also given as a function of the number of members of the given generation.

2.1.1 The total number of members

The development in the total number of members is calculated using the number of new members and the age and gender specific mortality rates. New members enter the pension fund at age A^0 assumed to be 22 years.[1] The number of new members of gender g, age A^0 at time t, $M^g_{A^0,t}$, is given exogenously from the demographic forecast of the population. Hereafter the development over time in the number of members of a specific generation is given by

$$M^g_{x,t} = (1 - \mu^g_x)\, M^g_{x-1,t-1}$$

where μ^g_x is the probability for an individual of gender g and age $x - 1$ to die before reaching the age of x. Solving forward from the time of entry, $t - (x - A^0)$ implies that

$$\begin{aligned} M^g_{x,t} &= M^g_{A^0,t-(x-A^0)} \prod_{v=A^0+1}^{x} (1 - \mu^g_v) \\ &= M^g_{A^0,t-(x-A^0)} \frac{l^g_x}{l^g_{A^0}} \end{aligned}$$

where the probability of survival from time of birth to the age of x, l^g_x, is given by

$$l^g_x \equiv \prod_{v=1}^{x} (1 - \mu^g_v)$$

so that $\frac{l^g_x}{l^g_{A^0}}$ is the probability that a member of gender s will still be alive at the age of x, given that the person is A^0 years old. Observe that by assumption these survival probabilities used by the pension fund are assumed to be independent of time, i.e. gender and age specific mortality rates are assumed to remain constant through time.

The number of active members

The number of *active* members of gender g and age x at time t, $M^{ga}_{x,t}$, is given by

$$M^{ga}_{x,t} = \left(1 - \mu^{g,ai}_x\right)(1 - \mu^g_x)\, M^{ga}_{x-1,t-1}$$

[1] Of course this is a simplification. In reality, entrance of new members appears distributed over several years.

where $\mu_x^{g,ai}$ is the probability for a person of gender g and age $x-1$ becoming disabled before turning x years. By definition, members are not disabled when they enter the pension fund

$$M_{A^0,t}^{ga} = M_{A^0,t}^{g}$$

Based on this definition it can be calculated that the number of active members who are of gender s and age x at time t is given by

$$M_{x,t}^{ga} = \frac{l_x^{g,ai}}{l_{A^0}^{gai}} M_{x,t}^{g}$$

where $\frac{l_x^{g,ai}}{l_{A^0}^{g,ai}}$ is the probability that a member of gender g will remain active when the person in question becomes x years of age given that the person is active at the age of A^0 years. $l_x^{g,ai}$ is the probability that a person is not disabled at the age of x years, and is defined as

$$l_x^{g,ai} \equiv \prod_{j=1}^{x} \left(1 - \mu_j^{g,ai}\right)$$

Observe that also the age and gender specific disablement probabilities are assumed to remain constant through time.

The number of disabled members

Let $M_{y,x,t}^{gD}$ be the number of disabled members of gender g and age y at time t who became disabled at the age of x, $y \geq x$.

Consider the number of members of gender g and age x at time t, who have just become disabled at this age. By the introduced terminology this number is called $M_{x,x,t}^{gD}$. Since these members were active in the preceding period, the group must consist of individuals who survived period $t-1$, $(1-\mu_x^g)\, M_{x-1,t-1}^{ga}$, and became disabled

$$M_{x,x,t}^{gD} = \mu_x^{g,ai} \left(1 - \mu_x^g\right) M_{x-1,t-1}^{ga}$$

To follow the evolution of this group, consider the number of individuals of gender g

at time $t+n$, n periods after the disabling event, which occurs at age x

$$\begin{aligned} M^{gD}_{x+n,x,t+n} &= \mu^{g,ai}_x (1-\mu^g_x) \frac{l^g_{x+n}}{l^g_x} M^{ga}_{x-1,t-1} \\ &= \mu^{g,ai}_x (1-\mu^g_x) \frac{l^g_{x+n}}{l^g_x} \frac{l^{g,ai}_{x-1}}{l^{g,ai}_{A^0}} M^g_{x-1,t-1} \\ &= \mu^{g,ai}_x \frac{l^g_{x+n}}{l^g_{x-1}} \frac{l^{g,ai}_{x-1}}{l^{g,ai}_{A^0}} M^g_{x-1,t-1} \\ &= \mu^{g,ai}_x \frac{l^{g,ai}_{x-1}}{l^{g,ai}_{A^0}} M^g_{x+n,t+n} \end{aligned}$$

In general we have the following relation

$$M^{gD}_{y,x,t} = \mu^{g,ai}_x \frac{l^{g,ai}_{x-1}}{l^{g,ai}_{A^0}} M^g_{y,t} \tag{2.1}$$

Finally define the total number of disabled members of gender g, and age y at time t

$$M^{gD}_{y,t} = \sum_{x=22}^{y} M^{gD}_{y,x,t} \tag{2.2}$$

The number of spouse pensioners

The number of spouse pensioners is determined by the mortality among the members. Let $\hat{M}^{gS}_{x,y,t}$ be the number of persons who receive spouse pension from a late member of gender g who at time t would have reached age x but died earlier at age y. We have that

$$\hat{M}^{gS}_{x,x,t} = p^{gS}_x \mu^g_x M^g_{x-1,t-1}$$

where p^{gS}_x is the probability that a person of gender g and age x has a living spouse. Assuming that the member and the spouse have the same age (when both still alive)[2], we have that at time $t+\theta$ the number of persons who receive spouse pension after a member who died at time t is given by

$$\hat{M}^{gS}_{x+\theta,x,t+\theta} = \frac{\hat{l}^g_{x+\theta}}{\hat{l}^g_x} p^{gS}_x \mu^g_x M^g_{x-1,t-1}$$

[2] This simplifying assumption of household structure is used in the present analysis, see the section on household behavior.

where $\frac{\hat{l}^g_{x+\theta}}{\hat{l}^g_x}$ is the probability that a person of gender g, who is widowed at the age of x, will survive to become $x+\theta$ years of age (or more). Thus we define $\hat{l}^g_x$ as

$$\hat{l}^g_x = \begin{cases} l^{female}_x & \text{for } g = male \\ l^{male}_x & \text{for } g = female \end{cases}$$

To be able to write the number of persons of gender g and age $x+\theta$ receiving spouse pension at time $t+\theta$ after a member who died at time t as a function of the number of members of gender g and age $x+\theta$ at time $t+\theta$ the following calculations are made

$$\begin{aligned} \hat{M}^{gS}_{x+\theta,x,t+\theta} &= \frac{\hat{l}^g_{x+\theta}}{\hat{l}^g_x} p^{gS}_x \mu^g_x M^g_{x-1,t-1} \\ &= \frac{\hat{l}^g_{x+\theta}}{\hat{l}^g_x} p^{gS}_x \frac{\mu^g_x}{1-\mu^g_x} M^g_{x,t} \\ &= \frac{\hat{l}^g_{x+\theta}}{\hat{l}^g_x} \frac{l^g_x}{l^g_{x+\theta}} p^{gS}_x \frac{\mu^g_x}{1-\mu^g_x} M^g_{x+\theta,t+\theta} \end{aligned}$$

In general we have that

$$\hat{M}^{gS}_{y,x,t} = \frac{\hat{l}^g_y}{\hat{l}^g_x} \frac{l^g_x}{l^g_y} p^{gS}_x \frac{\mu^g_{,x}}{1-\mu^g_x} M^g_{y,t} \tag{2.3}$$

Finally define the total number of spouses of late members of gender g, and age y at time t

$$\hat{M}^{gS}_{y,t} = \sum_{x=22}^{y} \hat{M}^{gS}_{y,x,t} \tag{2.4}$$

2.2 Pension undertakings and insurance premiums

Pension undertakings are the basis for the calculation of the various pension types that are offered by the pension fund. All information relevant for the calculation of a specific pension in case of an event that releases the pension is summarized in the pension undertaking. The pension undertakings are calculated every year for each member until the member reaches the retirement age, A^P. The first year's receipts of retirement and disablement pensions are equal to the pension undertaking at the time of the event. The first year's spouse pension is equal to a pre-specified fraction of the pension undertaking for the late member at the time of death.

For a given generation, the pension undertakings and the actual pensions are calculated so that capitalized contributions and capitalized costs are equalized from the point of view of the pension fund. Consider therefore a specific generation. The generation enters the pension fund at time t at the age of A^0 years, is active on the labor market until the retirement age, A^P, and is assumed extinct at the age of 120 years. After the extinction of the generation, the development in contributions and pensions should satisfy the following relation (the index of the gender is suppressed in the rest of this chapter)

$$\sum_{x=A^0}^{A^P-1} M^a_{x,t+(x-A^0)} \left(I_{x,t+(x-A^0)} - q^D_{x,t+(x-A^0)} - q^S_{x,t+(x-A^0)} \right) \prod_{s=t}^{t+\left(x-A^0\right)} \frac{1}{1+\tilde{r}_s}$$
$$= \sum_{x=A^P}^{120} M^a_{x,t+(x-A^0)} \left(f^{RP}_{x,t+(x-A^0)} + q^S_{x,t+(x-A^0)} \right) \prod_{s=t}^{t+\left(x-A^0\right)} \frac{1}{1+\tilde{r}_s} \tag{2.5}$$

where

$M^a_{x,t+(x-A^0)}$ is number of active members,
$I_{x,t+(x-A^0)}$ is contributions,
$q^D_{x,t+(x-A^0)}$ is disablement pension premiums,
$q^S_{x,t+(x-A^0)}$ is spouse pension premiums,
$f^{RP}_{x,t+(x-A^0)}$ is retirement pensions,
$\tilde{r}_{t+(x-A^0)}$ is after tax interest rate

The left hand side of equation (2.5) is the discounted sum of net contributions from the generation in question. Annual net contribution is given as the gross contributions minus the premiums to disablement and spouse pensions. The right hand side of (2.5) is the discounted sum of retirement pension and the premium to spouse pension in the years where the generation in question is retired. The premiums to the spouse pensions appears as payment from the pension fund to the member after the retirement, because retired persons do not contribute to the spouse pension. Since retirement pensioners are still covered by the spouse pension insurance, the premium is implicitly paid by the pension fund and may therefore be considered as part of a "gross" retirement pension from the fund. Therefore equation (2.5) implies that for the generation as a whole, the discounted value of the contributions to the retirement pension is equal to the discounted value of the retirement pension receipts. For the whole pension fund to be in balance, the same kind of equality between contributions and payments has to apply in case of disablement and spouse pension. This is ensured by the calculation of the premiums (see the subsection below) and from an assumption that the pension fund's expectations with respect to the age and gender specific disablement and mortality probabilities are correct.

To calculate the undertakings from (2.5) we need to calculate the premiums and to introduce some basic concepts. This is done in the next two sections.

2.2.1 Principles of precaution

The pension fund calculates the pensions given the following principles of precaution.[3]

- **Low interest rate.** In its forecasts, the pension fund uses a *base interest rate,* $\bar{r}$, that is lower than the (expected) after tax interest rate $\tilde{r}_t$. This is done to reduce the risk of promising too high future returns. Each year a correction of the pension undertaking is made to take account for the fact that the achieved interest rate after tax in the specific year deviate from the base interest rate.
- **Constant future contributions**. The pension fund assumes the future contributions to remain at the present level. Each year a correction of the pension undertaking is made to take account for the fact that the achieved contributions in the specific year deviate from the contributions of the previous year.
- **Constant future pension undertakings**. In the forecasts the pension fund assumes constant pension undertakings over time. Forecasts are corrected each year according to the rules stated above
- The actual pensions are initially given by the undertakings in force at time of retirement. After this, the pensions increases over time to correct for the assumptions of low interest rate.

2.2.2 Actuarial constants

The calculations of the pension fund is based on the so-called "calculation basis" ("beregningsgrundlag" in Danish). This basis contains the base interest rate, $\bar{r}$, and

[3] In addition to the precautionary principles listed below actual pension funds also apply precautionary principles with respect to the forecast of age and gender specific death and disablement probabilities. As mentioned the present analysis assumes that these propabilities are forecasted correctly.

age and gender specific probabilities of mortality, disablement, marriage and divorce. From this the following actuarial constants are defined

$$D_x \equiv l_x \left(\frac{1}{1+\bar{r}}\right)^x$$
$$\bar{N}_x \equiv \sum_{b=x}^{120} D_b$$

These variable are used to define two important fractions

1. $\frac{D_{x+\theta}}{D_x} = \frac{l_{x+\theta}}{l_x}\left(\frac{1}{1+\bar{r}}\right)^\theta$ which is the discounted, expected costs (determined by the mortality rates and the base interest rate used) faced by the pension fund when promising a member of age x, 1 monetary unit in θ years (conditional on the person being alive).

2. $\frac{\bar{N}_x}{D_x} = \sum_{b=x}^{120} \frac{D_b}{D_x}$ is the *total* discounted, expected costs faced by the pension fund when promising a member of age x, 1 monetary unit every year from now to the age of 120 (conditional on the person being alive).

Similar constants including probabilities of disablement are

$$l_x^a \equiv l_x^{ai}\, l_x$$
$$D_x^a \equiv D_x\, l_x^a$$
$$\bar{N}_x^a \equiv \sum_{b=x}^{120} D_b^a$$

Observe that $\frac{l_{x+\theta}^{ai}}{l_x^{ai}}$ is the probability that an active member of age x is not disabled within the next θ years, given that mortality does not occur in this period. $\frac{l_{x+\theta}^{a}}{l_x^{a}}$ is the probability that a member of age x is still active and alive θ years later.

Again we use the variable to define the fraction $\frac{D_{x+\theta}^a}{D_x^a}$, which is the discounted, expected costs faced by the pension fund when promising a member of age x 1 monetary unit in θ years, on condition that the member is both alive and active.

Assuming that the demographic development of the members follows the calculation basis of the pension fund, we have that

$$\frac{M_{x+\theta,t+\theta}}{M_{x,t}} = \frac{l_{x+\theta}}{l_x}$$
$$\frac{M_{x+\theta,t+\theta}^a}{M_{x,t}^a} = \frac{l_{x+\theta}^a}{l_x^a}$$

for all t. Dividing with $M^a_{A^0,t}$ on both sides of (2.5) yields

$$\sum_{x=A^0}^{A^P-1}\left(I_{x,t+(x-A^0)}-q^D_{x,t+(x-A^0)}-q^S_{x,t+(x-A^0)}\right)\frac{M^a_{x,t+(x-A^0)}}{M^a_{A^0,t}}\prod_{s=t}^{t+\left(x-A^0\right)}\frac{1}{1+\tilde{r}_s}$$
$$=\sum_{x=A^P}^{120}\left(f^{RP}_{x,t+(x-A^0)}+q^S_{x,t+(x-A^0)}\right)\frac{M^a_{x,t+(x-A^0)}}{M^a_{A^0,t}}\prod_{s=t}^{t+\left(x-A^0\right)}\frac{1}{1+\tilde{r}_s}$$

Using the definition of D^a_x yields

$$\sum_{x=A^0}^{A^P-1}\left(I_{x,t+(x-A^0)}-q^D_{x,t+(x-A^0)}-q^S_{x,t+(x-A^0)}\right)\frac{D^a_x}{D^a_{A^0}}\prod_{s=t}^{t+\left(x-A^0\right)}\frac{1+\bar{r}}{1+\tilde{r}_s}$$
$$=\sum_{x=A^P}^{120}\left(f^{RP}_{x,t+(x-A^0)}+q^S_{x,t+(x-A^0)}\right)\frac{D^a_x}{D^a_{A^0}}\prod_{s=t}^{t+\left(x-A^0\right)}\frac{1+\bar{r}}{1+\tilde{r}_s} \tag{2.6}$$

This is the basic equation in what follows.

2.2.3 Premiums

Concerning the spouse pension the pension fund promises to pay a fraction ζ of the pension undertaking to the spouse of the member in case of the member's death. This may be a lifelong pension or perhaps more typically a pension that is payed in a limited number of years (e.g. 10 years). Since both types of spouse premiums are present in Danish labor market pension schemes, we approximate the combination of the two by assuming that the pension to the spouse is lifelong, but decreases with a constant rate g_S every year. *If* a member of age $x+1$ dies and *if* there is a living spouse, the expected discounted costs faced by the pension fund are given as

$$S^S_{x+1,t+1}=\zeta\sum_{a=x+1}^{120}\left(\frac{1}{1+g^S}\right)^{a-x-1}f^{UP}_{x,t}\frac{\hat{D}_a}{\hat{D}_{x+1}}=\zeta f^{UP}_{x,t}\sum_{a=x+1}^{120}\left(\frac{1}{1+g^S}\right)^{a-x-1}\frac{\hat{D}_a}{\hat{D}_{x+1}}$$

as it is assumed that the spouse has the same age as the late member. Define $f^{UP}_{x,t}$ as the pension undertaking at x years of age, and let the hat-sign denote that probabilities of the opposite gender is used. To see that $S^S_{x+1,t+1}$ in fact is the discounted, expected cost of the spouse pension to the spouse of a member who dies at the age of x years recall that $\frac{\hat{D}_a}{\hat{D}_{x+1}}$ is the discounted, expected costs faced by the pension fund when promising a member of age x, 1 monetary unit in $(a-x-1)$ years.

The spouse premium is calculated so that it is "fair" in the sense that the premium is equal to the expected, discounted costs of the pension fund. The premium $q_{x,t}^S$ can therefore be calculated as

$$q_{x,t}^S = \mu_{x+1} p_{x+1}^M \frac{1}{1+\bar{r}} S_{x+1,t+1}^S = \zeta f_{x,t}^{UP} \mu_{x+1} p_{x+1}^M \frac{1}{1+\bar{r}} \sum_{s=x+1}^{120} \left(\frac{1}{1+g^S}\right)^{s-x-1} \frac{\hat{D}_s}{\hat{D}_{x+1}}$$

where μ_{x+1} is the probability for a member of age x not to survive to become $x+1$ and p_{x+1}^M is the probability of an $x+1$-aged member having a living spouse. $S_{x+1,t+1}^S$ is discounted one period due to the assumption that the spouse pension is initiated in the period after the member's death.

Defining

$$\beta_x^S \equiv \mu_{x+1} p_{x+1}^G \frac{1}{1+\bar{r}} \sum_{s=x+1}^{120} \left(\frac{1}{1+g^S}\right)^{s-x-1} \frac{\hat{D}_s}{\hat{D}_{x+1}}$$

the spouse premiums may be written as

$$q_{x,t}^S = \zeta f_{x,t}^{UP} \beta_x^S \tag{2.7}$$

In case of disablement, the pension fund promises to pay the pension undertaking the rest of the life of the disabled. If a member becomes disabled at age $x+1$ at time $t+1$ the expected cost is given as

$$\begin{aligned} S_{x+1,t+1}^D &= \sum_{a=x+1}^{120} \left(1+\zeta\beta_a^S\right) f_{x,t}^{UP} \frac{D_a}{D_{x+1}} \\ &= f_{x,t}^{UP} \sum_{a=x+1}^{120} \left(1+\zeta\beta_a^S\right) \frac{D_a}{D_{x+1}} \end{aligned}$$

where the part given by $\zeta f_{x,t}^{UP} \sum_{a=x+1}^{120} \beta_a^S \frac{D_a}{D_{x+1}}$ is the discounted, expected premiums to the spouse pension which is paid by the pension fund, since disablement pensioners do not contribute.

Similar to the premium of the spouse pension, the premium of the disablement pension is calculated to be "fair" in the sense that the premium equals the expected, discounted costs of the pension fund. The premium $q_{x,t}^D$ is therefore given by

$$\begin{aligned} q_{x,t}^D &= \left(1-\mu_{x+1}\right) \mu_{x+1}^{ai} \frac{1}{1+\bar{r}} S_{x+1,t+1}^D \\ &= \left(1-\mu_{x+1}\right) \mu_{x+1}^{ai} \frac{1}{1+\bar{r}} f_{x,t}^{UP} \sum_{a=x+1}^{120} \left(1+\zeta\beta_a^S\right) \frac{D_a}{D_{x+1}} \end{aligned}$$

where $\left(1-\mu_{x+1}\right)\mu_{x+1}^{ai}$ is the probability that an active member of x years survives to become $x+1$ years, but has become disabled.

Defining

$$\beta_x^D \equiv \left(1-\mu_{x+1}\right)\mu_{x+1}^{ai}\frac{1}{1+\bar{r}}\sum_{a=x+1}^{120}\left(1+\zeta\beta_a^S\right)\frac{D_a}{D_{x+1}}$$

we may write the premium of the disablement pension as

$$q_{x,t}^D = \beta_x^D f_{x,t}^{UP} \tag{2.8}$$

From (2.7) and (2.8) we have that premiums to both the spouse and the disablement pension are functions of the pension undertaking for the generation in question.

2.2.4 Pension undertakings

The pension undertakings are calculated using (2.6). For generations that are currently alive, (2.6) is partly based on forecasted values. These values are forecasted using the forecast procedure given in the principle of precautions. To be specific consider first a member of age y years at time t. By assumption y is less than the retirement age. Variables that are dated before time t are known to the pension fund. Variables that are dated after time t are forecasted. The forecasting procedure implies that for $x > y$

$$\begin{aligned}
I_{x,t+(x-y)} &= I_{y,t} \\
f_{x,t+(x-y)}^{RP} &= f_{y,t}^{UP} \\
\tilde{r}_{t+(x-y)} &= \bar{r} \\
q_{x,t+(x-y)}^{D} &= \beta_x^D f_{y,t}^{UP} \\
q_{x,t+(x-y)}^{S} &= \zeta\beta_x^S f_{y,t}^{UP}
\end{aligned}$$

For $x \le y : \tilde{r}_{t+(x-y)} = \tilde{r}$.

Given these forecasts and the expressions for the premiums of spouse and disablement pension in (2.7) and (2.8) we may rewrite (2.6) for the generation that is y years at time t as

$$\begin{aligned} & \sum_{x=A^0}^{y} \left(I_{x,t+(x-y)} - \left(\beta_x^D + \zeta \beta_x^S \right) f_{x,t+(x-y)}^{UP} \right) \frac{D_x^a}{D_{A^0}^a} \left(\frac{1+\bar{r}}{1+\tilde{r}} \right)^{x-A^0} \\ & + \sum_{x=y+1}^{A^P-1} \left(I_{y,t} - \left(\beta_x^D + \zeta \beta_x^S \right) f_{y,t}^{UP} \right) \frac{D_x^a}{D_{A^0}^a} \left(\frac{1+\bar{r}}{1+\tilde{r}} \right)^{y-A^0} \\ = & \sum_{x=A^P}^{120} \left(f_{y,t}^{UP} + \zeta \beta_x^S f_{y,t}^{UP} \right) \frac{D_x^a}{D_{A^0}^a} \left(\frac{1+\bar{r}}{1+\tilde{r}} \right)^{y-A^0} \end{aligned} \tag{2.9}$$

where the first line is the sum of the discounted, actual net contributions from this generation up till period t. The second line is the discounted value of the forecasted net contributions for the generation and finally the third line is discounted value of the forecasted gross pension receipts to the generation.

Similarly, at time $t+1$ where the generation in question is $y+1$ years, we have that the forecasted version of (2.6) becomes

$$\begin{aligned} & \sum_{x=A^0}^{y+1} \left(I_{x,t+(x-y)} - \left(\beta_x^D + \zeta \beta_x^S \right) f_{x,t+(x-y)}^{UP} \right) \frac{D_x^a}{D_{A^0}^a} \left(\frac{1+\bar{r}}{1+\tilde{r}} \right)^{x-A^0} \\ & + \sum_{x=y+2}^{A^P-1} \left(I_{y+1,t+1} - \left(\beta_x^D + \zeta \beta_x^S \right) f_{y+1,t+1}^{UP} \right) \frac{D_x^a}{D_{A^0}^a} \left(\frac{1+\bar{r}}{1+\tilde{r}} \right)^{y+1-A^0} \\ = & \sum_{x=A^P}^{120} \left(f_{y+1,t+1}^{UP} + \zeta \beta_x^S f_{y+1,t+1}^{UP} \right) \frac{D_x^a}{D_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}} \right)^{y+1-A^0} \end{aligned} \tag{2.10}$$

The difference between (2.9) and (2.10) of course is that in the latter relation variables dated $t+1$ are observed whereas they are forecasted in the former relation. Subtracting (2.9) from (2.10) one obtains

Proposition 1 The difference equation governing the evolution of pension undertakings over time is given as

$$f_{y+1,t+1}^{UP} = (1+g) f_{y,t}^{UP} + \xi_{y+1}^I \left(I_{y+1,t+1} - (1+g) I_{y,t} \right) \tag{2.11}$$

where

$$\begin{aligned} g & \equiv \frac{\tilde{r} - \bar{r}}{1+\bar{r}} \\ \xi_{y+1}^I & \equiv \frac{\sum\limits_{x=y+1}^{A^P-1} \frac{D_x^a}{D_{A^0}^a}}{\sum\limits_{x=A^P}^{120} \left(1 + \zeta \beta_x^S \right) \frac{D_x^a}{D_{A^0}} + \sum\limits_{x=y+1}^{A^P-1} \left(\beta_x^D + \zeta \beta_x^S \right) \frac{D_x^a}{D_{A^0}^a}} \end{aligned}$$

According to (2.11) the evolution over time in pension undertakings for a given generation of members can be explained in two parts. The first part is an adjustment with respect to last year's pension undertaking. This adjustment is explained by the fact that the base interest rate is smaller than the actual after tax interest rate. The second part is a contribution correction part, that adjusts for the assumption of constant contributions.

The undertakings can now be calculated if we know the initial values, i.e. $f^{UP}_{A^0,t}$. From (2.11) we have that

$$f^{UP}_{A^0,t} = \xi^I_{A^0} I_{A^0,t}$$

2.3 Pensions

Since all types of pensions initially are related to the pension undertakings the present section derives the evolution over time of the various types of pension for a certain generation of pensioners given their initial pension level. For disablement and spouse pension the actual level of the pension does not only depend on the generation of the pensioners but also on the seniority as pensioner of the given type. Therefore we derive the evolution over time of the average pension for a given generation of members for these two types of pension.

2.3.1 Retirement pensions

The initial retirement pensions is given by the pension undertaking

$$f^{RP}_{A^P,t} = f^{UP}_{A^P,t}$$

To calculate the further development of the retirement pensions we use the same method as in the previous section. Consider a generation aged y at time t, where the age y is higher than the retirement age ($y > A^P$). For this generation we may write (2.6) as

$$\begin{aligned}
&\sum_{x=A^0}^{A^P-1} \left(I_{x,t+(x-y)} - q^D_{x,t+(x-y)} - q^S_{x,t+(x-y)}\right) \frac{D^a_x}{D^a_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{x-A^0} \\
= &\sum_{x=A^P}^{y} \left(f^{RP}_{x,t+(x-y)} + \zeta\beta^S_x f^{RP}_{x,t+(x-y)}\right) \frac{D^a_x}{D^a_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{x-A^0} \\
&+ \sum_{x=y+1}^{120} \left(f^{RP}_{y,t} + \zeta\beta^S_x f^{RP}_{y,t}\right) \frac{D^a_x}{D^a_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{y-A^0}
\end{aligned}$$

The first line is the discounted value of actual net contributions. The second line is the discounted value of actual gross pension receipts, whereas the third line is the discounted value of forecasted gross pension receipts to the generation. The similar equation taken at time $t+1$ where the generation is $y+1$ years becomes

$$\begin{aligned} & \sum_{x=A^0}^{A^P-1} \left(I_{x,t+(x-y)} - q^D_{x,t+(x-y)} - q^S_{x,t+(x-y)}\right) \frac{D^a_x}{D^a_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{x-A^0} \\ = & \sum_{x=A^P}^{y+1} \left(f^{RP}_{x,t+(x-y)} + \zeta\beta^S_x f^{RP}_{x,t+(x-y)}\right) \frac{D^a_x}{D^a_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{x-A^0} \\ & + \sum_{x=y+2}^{120} \left(f^{RP}_{y+1,t+1} + \zeta\beta^S_x f^{RP}_{y+1,t+1}\right) \frac{D^a_x}{D^a_{A^0}} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{y+1-A^0} \end{aligned}$$

Subtracting these two equations one obtains

Proposition 2 The difference equation governing the evolution over time of retirement pension of a specific generation is given as

$$f^{RP}_{y+1,t+1} = (1+g)\, f^{RP}_{y,t} \tag{2.12}$$

Observe that actual pensions of a given generation of pensioners are only corrected according to the difference between the actual yield and the base interest rate as no contributions are made once the member has become a pensioner.

2.3.2 Disablement pensions

Let $f^{DP}_{y,p,t}$ be the disablement pension at time t, for a member of age y that became disabled when he/she was p years old ($p \leq y$).

At the time of disablement, p, the discounted, expected costs for the pension fund, $\overline{a}^D_{p,t}$, can be calculated as (given the principles of precaution)

$$\overline{a}^D_{p,t} = \sum_{x=p}^{120} f^{DP}_{p,p,t} \left(1+\zeta\beta^S_x\right) \frac{D_x}{D_p} = f^{DP}_{p,p,t} \sum_{x=p}^{120} \left(1+\zeta\beta^S_x\right) \frac{D_x}{D_p}$$

The total expected costs to members that became disabled at age p and time t is therefore this amount times the number of persons who become disablement pensioners in period p. This is given by

$$\overline{a}^{D}_{p,t}\mu^{ai}_{p}\left(1-\mu_{p}\right)M^{a}_{p-1,t-1}=f^{DP}_{p,p,t}\mu^{ai}_{p}\left(1-\mu_{p}\right)M^{a}_{p-1,t-1}\sum_{x=p}^{120}\left(1+\zeta\beta^{S}_{x}\right)\frac{D_{x}}{D_{p}} \tag{2.13}$$

In the previous period the members paid the premium

$$q^{D}_{p-1,t-1}=\beta^{D}_{p-1}f^{UP}_{p-1,t-1}=\left(1-\mu_{p}\right)\mu^{ai}_{p}\frac{1}{1+\bar{r}}f^{UP}_{p-1,t-1}\sum_{s=p}^{120}\left(1+\zeta\beta^{S}_{s}\right)\frac{D_{s}}{D_{p}}$$

so that contributions and interest payment to the pension fund amounts to

$$\left(1+\tilde{r}\right)q^{D}_{p-1,t-1}M^{a}_{p-1,t-1}=\frac{1+\tilde{r}}{1+\bar{r}}\mu^{ai}_{p}\left(1-\mu_{p}\right)f^{UP}_{p-1,t-1}M^{a}_{p-1,t-1}\sum_{s=p}^{120}\left(1+\zeta\beta^{S}_{s}\right)\frac{D_{s}}{D_{p}} \tag{2.14}$$

Equalizing costs from relation (2.13)and contributions from relation (2.14) yields

Proposition 3 The initial period's disablement pension is given as

$$f^{DP}_{p,p,t}=\left(1+g\right)f^{UP}_{p-1,t-1} \tag{2.15}$$

Hence the first period's disablement pensions is equal to the pension undertaking of previous period corrected for the difference between the actual yield and the base interest rate.

To determine the evolution over time of the disablement pension for these pensioners observe that after the extinction of all persons in the generation, the discounted value of actual pensions to a member is equal to capitalized value of the expected costs, $\overline{a}^{D}_{p,t}$, per disabled for the generation as a whole.[4] Therefore the pensions must satisfy

$$\sum_{x=p}^{120}f^{DP}_{x,p,t+(x-p)}\left(1+\zeta\beta^{S}_{x}\right)\frac{D_{x}}{D_{p}}\prod_{s=t}^{t-(x-p)}\frac{1+\bar{r}}{1+\tilde{r}}=\overline{a}^{D}_{p,t} \tag{2.16}$$

[4] The value, $\overline{a}^{D}_{p,t}$, may be thought of as the stock of wealth that the pension fund withdraw at the time of disablement to finance the lifelong pension payments to the disabled.

For a member of age y at time t that became disabled at age p we may rewrite (2.16) as follows

$$\begin{aligned} & \sum_{x=p}^{y} f^{DP}_{x,p,t+(x-y)} \left(1+\zeta\beta^S_x\right) \frac{D_x}{D_p} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{x-p} \\ & + \sum_{x=y+1}^{120} f^{DP}_{y,p,t} \left(1+\zeta\beta^S_x\right) \frac{D_x}{D_p} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{y-p} \\ = \; & \overline{a}^D_{p,t-(y-p)} \end{aligned} \tag{2.17}$$

where the first line is the discounted value of actual pension receipts from the age p to the age y. The second line is the discounted value of expected future pension receipts and finally the third line is total stock of wealth which is withdrawn at the time of disablement to finance the disablement pension receipts.

Similarly at time $t+1$, where the member in question is $y+1$ years old, we may rewrite (2.16) as

$$\begin{aligned} & \sum_{x=p}^{y+1} f^{DP}_{x,p,t+(x-y)} \left(1+\zeta\beta^S_x\right) \frac{D_x}{D_p} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{x-p} \\ & + \sum_{x=y+2}^{120} f^{DP}_{y+1,p,t+1} \left(1+\zeta\beta^S_x\right) \frac{D_x}{D_p} \left(\frac{1+\bar{r}}{1+\tilde{r}}\right)^{y+1-p} \\ = \; & \overline{a}^D_{p,t+1-(y+1-p)} = \overline{a}^D_{p,t-(y-p)} \end{aligned} \tag{2.18}$$

Subtracting (2.17) from (2.18) yields

Proposition 4 The difference equation governing the evolution over time of the disablement pension of a specific generation that became disabled at a given point in time is given as

$$f^{DP}_{y+1,p,t+1} = (1+g)\, f^{DP}_{y,p,t} \tag{2.19}$$

For a given disablement pensioner, payments are increased at the rate $1+g$, i.e. next period's disablement pension equals the pension of the current period corrected for the difference between the actual yield and the base interest rate.

We may now use the fact that we know the seniority-distribution of disabled to aggregate the disablement pensions. Define the average disablement pension $f^{DP}_{y,t}$ of a disabled person of age y at time t as: the sum over seniority-types of number of disabled of a given generation and with a given seniority times the pension of the seniority-type in question divided by the total number of disabled in a given generation.

$$f_{y,t}^{DP} \equiv \frac{\sum_{p=A^0+1}^{y} M_{y,p,t}^{D}\, f_{y,p,t}^{DP}}{\sum_{p=A^0+1}^{y} M_{y,p,t}^{D}} \tag{2.20}$$

Observe that a given generation of disabled persons, who (by assumption) have the same contributions until they become disabled, have different disablement pensions as the pensions are affected by seniority as disablement pensioner.

Recalling that we may write the number of disabled members of a given generation as a function of the total number of members in the generation, see relation (2.1), we substitute this into (2.20) to yield (as the index of gender is suppressed)

$$\begin{aligned} f_{y,t}^{DP} \sum_{p=A^0+1}^{y} \mu_p^{ai} \frac{l_{p-1}^{ai}}{l_{A^0}^{ai}} M_{y,t} &= \sum_{p=A^0+1}^{y} \mu_p^{ai} \frac{l_{p-1}^{ai}}{l_{A^0}^{ai}} M_{y,t}\, f_{y,p,t}^{DP} \quad \Leftrightarrow \\ f_{y,t}^{DP} \sum_{p=A^0+1}^{y} \mu_p^{ai} \frac{l_{p-1}^{ai}}{l_{A^0}^{ai}} &= \sum_{p=A^0+1}^{y} \mu_p^{ai} \frac{l_{p-1}^{ai}}{l_{A^0}^{ai}}\, f_{y,p,t}^{DP} \end{aligned}$$

Define

$$\lambda_p \equiv \mu_p^{ai} \frac{l_{p-1}^{ai}}{l_{A^0}^{ai}}$$

Therefore

$$f_{y,t}^{DP} \sum_{p=A^0+1}^{y} \lambda_p = \sum_{p=A^0+1}^{y} \lambda_p\, f_{y,p,t}^{DP}$$

By definition the average disablement pension is zero at the time that the generation enters the economy, $f_{A^0,t}^{DP} = 0$. For disabled individuals who are $A^0 + 1$ years of age, we have that they have become disablement pensioners in the current year. Therefore the average disablement pension is equal to the individual pension for this age group.

$$f_{A^0+1,t}^{DP} = f_{A^0+1,A^0+1,t}^{DP} = (1+g)\, f_{A^0,t-1}^{UP}$$

where the last equation follows from relation (2.15).

To find an expression for the average disablement pension for persons who are a least

A^0+2 years old we rewrite the expression above for $y \geq A^0+2$. This yields

$$\begin{aligned} f_{y,t}^{DP} \sum_{p=A^0+1}^{y} \lambda_p &= \sum_{p=A^0+1}^{y} \lambda_p \, f_{y,p,t}^{DP} \\ &= \lambda_y \left(1+g\right) f_{y-1,t-1}^{UP} + \sum_{p=A^0+1}^{y-1} \lambda_p \, f_{y,p,t}^{DP} \\ &= \lambda_y \left(1+g\right) f_{y-1,t-1}^{UP} + \left(1+g\right) \sum_{p=A^0+1}^{y-1} \lambda_p \, f_{y-1,p,t-1}^{DP} \\ &= \lambda_y \left(1+g\right) f_{y-1,t-1}^{UP} + \left(1+g\right) f_{y-1,t-1}^{DP} \sum_{p=A^0+1}^{y-1} \lambda_p \end{aligned}$$

so that

$$\lambda_y f_{y,t}^{DP} + f_{y,t}^{DP} \sum_{p=A^0+1}^{y-1} \lambda_p = \lambda_y \left(1+g\right) f_{y-1,t-1}^{UP} + \left(1+g\right) f_{y-1,t-1}^{DP} \sum_{p=A^0+1}^{y-1} \lambda_p \quad \Leftrightarrow$$

$$\left(f_{y,t}^{DP} - \left(1+g\right) f_{y-1,t-1}^{DP}\right) \sum_{p=A^0+1}^{y-1} \lambda_p = \lambda_y \left(\left(1+g\right) f_{y-1,t-1}^{UP} - f_{y,t}^{DP}\right)$$

Defining

$$\hat{\xi}_y^D \equiv \frac{\lambda_y}{\sum_{p=A^0+1}^{y-1} \lambda_p}$$

and rearranging implies

Proposition 5 The difference equation governing the evolution over time in the average disablement pension of a given generation is given by

$$f_{y,t}^{DP} = \left(1+g\right) f_{y-1,t-1}^{DP} + \hat{\xi}_y^D \left(\left(1+g\right) f_{y-1,t-1}^{UP} - f_{y,t}^{DP}\right)$$

This difference equation governs the evolution in the average disablement pension for a given generation over time. Observe that the evolution in the average disablement pension of a given generation can be explained in two parts. The first part is an adjustment with respect to last year's average disablement pension. This adjustment is explained by the fact that the base interest rate is smaller than the actual after tax interest rate. The second part reflects the fact that the disablement pensions of persons who has become disablement pensioner in the current year differ from the pension of persons who became disablement pensioners earlier in their life.

2.3.3 Spouse pensions

To derive the evolution of the spouse pension to spouses of a given generation of late members we follow the exact same procedure as in the previous subsection concerning disablement pension.

Let $\hat{f}^{SP}_{y,p,t}$ be the spouse pension at time t, for a person of age y whose spouse died when he/she was p years old ($p \leq y$).

At the time of death p the discounted, expected costs for the pension fund, $\overline{a}^{S}_{p,t}$, can be calculated as (given the principles of precaution)

$$\overline{a}^{S}_{p,t} \equiv \sum_{x=p}^{120} \hat{f}^{SP}_{p,p,t} \left(\frac{1}{1+g^S}\right)^{x-p} \frac{\hat{D}_x}{\hat{D}_p}$$

The total expected costs to members that died at age p and time t is therefore

$$\overline{a}^{S}_{p,t}\mu_p p^G_p M_{p-1,t-1} = \mu_p p^G_p M_{p-1,t-1} \hat{f}^{SP}_{p,p,t} \sum_{x=p}^{120} \left(\frac{1}{1+g^S}\right)^{x-p} \frac{\hat{D}_x}{\hat{D}_p} \tag{2.21}$$

where $\mu_p p^G_p M_{p-1,t-1}$ is the number of living spouse to members who died at the age of p year at time t.

In the previous period the members paid the premium

$$q^S_{p-1,t-1} = \beta^S_{p-1} f^{UP}_{p-1,t-1} = \zeta f^{UP}_{p-1,t-1}\mu_p p^G_p \frac{1}{1+\bar{r}} \sum_{s=p}^{120} \left(\frac{1}{1+g^S}\right)^{x-p} \frac{\hat{D}_s}{\hat{D}_p}$$

such that the pension fund has gained (including interest payments)

$$(1+\tilde{r})\, q^S_{p-1,t-1} M_{p-1,t-1} = \frac{1+\tilde{r}}{1+\bar{r}} \mu_p p^G_p M_{p-1,t-1} \zeta f^{UP}_{p-1,t-1} \sum_{s=p}^{120} \left(\frac{1}{1+g^S}\right)^{x-p} \frac{\hat{D}_s}{\hat{D}_p} \tag{2.22}$$

Equalizing costs from relation (2.21) and gains from relation (2.22) yields

Proposition 6 The initial period's spouse pension is given as

$$\hat{f}^{SP}_{p,p,t} = (1+g)\, \zeta f^{UP}_{p-1,t-1}$$

The proposition states that the first year's spouse pension is equal to the indexed pension undertaking from the previous period multiplied by the fraction ζ.

To determine the evolution of the spouse pension for the pensioners in question we follow the procedure of the disablement pension and consider the following: At the time of death the pension fund withdraws a stock of wealth which is equal to the discounted, expected costs for the pension fund, $\overline{a}_{p,t}^{S}$ at the time of death. The discounted value of actual pensions must equal this amount. Therefore we have

$$\sum_{x=p}^{120} \hat{f}_{x,p,t+(x-p)}^{SP} \frac{\hat{D}_x}{\hat{D}_p} \prod_{s=t}^{t-(x-p)} \frac{1+\overline{r}}{1+r_s} = \overline{a}_{p,t}^{S}$$

For a member that would have been y years old at time t that died at age p, we have the following "budgetary" condition:

$$\begin{aligned} & \sum_{x=p}^{y} \hat{f}_{x,p,t+(x-y)}^{SP} \frac{\hat{D}_x}{\hat{D}_p} \left(\frac{1+\overline{r}}{1+r}\right)^{x-p} \\ & + \sum_{x=y+1}^{120} \hat{f}_{y,p,t}^{SP} \left(\frac{1}{1+g^S}\right)^{x-y-1} \frac{\hat{D}_x}{\hat{D}_p} \left(\frac{1+\overline{r}}{1+r}\right)^{y-p} \\ = \; & \overline{a}_{p,t-(y-p)}^{S} \end{aligned}$$

where the first line is the discounted value of actual spouse pension receipts. The second line is the discounted value of the expected spouse pension and the third is the stock of wealth that was withdrawn to finance the spouse pension.

Similarly, at time $t+1$ where the member in question would have been $y+1$ years old had he/she not died at age p:

$$\begin{aligned} & \sum_{x=p}^{y+1} \hat{f}_{x,p,t+(x-y)}^{SP} \frac{\hat{D}_x}{\hat{D}_p} \left(\frac{1+\overline{r}}{1+r}\right)^{x-p} \\ & + \sum_{x=y+2}^{A^P-1} \hat{f}_{y+1,p,t+1}^{SP} \left(\frac{1}{1+g^S}\right)^{x-y-2} \frac{\hat{D}_x}{\hat{D}_p} \left(\frac{1+\overline{r}}{1+r}\right)^{y+1-p} \\ = \; & \overline{a}_{p,t+1-(y+1-p)}^{S} = \overline{a}_{p,t-(y-p)}^{S} \end{aligned}$$

Substraction implies

Proposition 7 The difference equation governing the evolution over time of the spouse pension of a specific generation that became spouse at a given point in time is given as

$$\hat{f}_{y+1,p,t+1}^{SP} = \frac{1+g}{1+g^S} \hat{f}_{y,p,t}^{SP} \tag{2.23}$$

For a given spouse pensioner the spouse pension evolves over time according to the difference equation above. Observe that the general to tendency to increase the pension due to the difference between the interest rate after tax and the base interest rate is modified in the case of the spouse pension due to our assumption that the spouse pension is exponentially decreasing in the number of year the pensioner in questions has received the pension.

As with disablement pension we can use the seniority-distribution of the spouse pensioner of a given generation to aggregate the spouse pensions. Define the average spouse pension:

$$\hat{f}^{SP}_{y,t} \equiv \frac{\sum_{p=A^0+1}^{y} \hat{M}^S_{y,p,t} \hat{f}^{SP}_{y,p,t}}{\sum_{p=A^0+1}^{y} \hat{M}^S_{y,p,t}}$$

Using the same method as in case of disablement pension we can show that

Proposition 8 The difference equation governing the evolution over time in the average spouse pension of a given generation is given by

$$\hat{f}^{SP}_{y,t} = \frac{1+g}{1+g^S} \hat{f}^{SP}_{y-1,t-1} + \xi^S_y \left((1+g)\, \zeta f^{UP}_{y-1,t-1} - \hat{f}^{SP}_{y,t} \right) \tag{2.24}$$

where

$$\xi^S_y \equiv \frac{\lambda^S_y}{\sum_{p=A^0+1}^{y-1} \lambda^S_p}$$

$$\lambda^S_y \equiv \frac{l_y}{\hat{l}_y} p^G_y \frac{\mu_y}{1-\mu_y}$$

The average spouse pension of a given generation evolves over time according to (2.24) . This evolution is similar to the evolution in the average disablement pension of a given generation with the exception that the assumption that the spouse pension is exponentially decreasing with the seniority of the spouse pensioner tends to reduce the annual growth in the pension. Again we observe the correction of the average pension of the generation in question due to the fact that the new spouse pensioners this generation receive a pension that differs from the average pension of the generation.

2.4 Summary of the behavior of the pension fund

The preceding sections have outlined the determination of the various types of pension of a given generation of members. In short the following rules were obtained

1. As new members enter the pension fund a pension undertaking is calculated. The initial pension undertaking is based on a forecast of future contributions, where it is assumed that current contributions are constant until the retirement age.

2. Pension undertakings are revised annually. The revision has two parts: First the undertakings are increased with a rate which is equal to the difference between the actual interest rate after tax and the base interest rate. Second undertakings are revised to correct for a change in the annual contribution.

3. In case of disablement the initial disablement pension is calculated as the pension undertaking of the previous year increased with a growth rate equal to the difference between the actual interest rate after tax and the base interest rate.

4. Disablement pension for a specific disabled is revised annually. The pension is increased with a rate which is equal to the difference between the actual interest rate after tax and the base interest rate.

5. The initial retirement pension is equal to the pension undertaking.

6. The retirement pension is revised annually. The pension is increased with a rate which is equal to the difference between the actual interest rate after tax and the base interest rate.

7. The initial spouse pension is equal to a fraction of the previous year's pension undertaking increased with a growth rate equal to the difference between the actual interest rate after tax and the base interest rate.

8. Spouse pension for a specific spouse is revised annually. The pension is increased with a rate which is equal to the difference between the actual interest rate after tax and the base interest rate.

This behavior implies that for each generation as a whole the discounted value of the contributions over the life time is equal to the discounted value of all pension types paid to the specific generation.

If the economy is in an otherwise stationary environment and if one considers a situation where only new generations have contributions, then the behavior of the pension will imply that the stock of wealth held by the fund will increase towards a stationary level during (100 - 22) 78 years. The fact that the contribution rate is gradually increased in the Danish labor market pension schemes implies that the conversion takes a longer time, therefore a steady level of the stock of wealth in the pension funds should not be expected until after year 2100.

2.5 Accumulation in labor market pension funds

Contributions in the pensions fund sector is defined as an age independent fraction of the wage sum for the specific age group in question

$$y_t^{LP} = \sum_{g\in\{F,M\}} \sum_{b=22}^{61} \vartheta_t^{LP} \left(1 - t_t^{\ell} - t_t^{ATP2}\right) w_{g,b,t} \ell_{b,t} N_{b,t}^{gW}$$

where y_t^{LP} is the total contributions after labor market contribution tax to the labor market pension system. ϑ_t^{LP} is the exogenous fraction of the wage sum that each person pays into the pension fund. Observe that even though the persons may enter the labor market at the age of 17 years we assume that no contributions to the labor market pensions are made until the age of 22. This assumption based on the empirical observation that only a very limited part of the labor force has contribution before the age of 22 years. t_t^{ℓ} is gross income tax rate on labor income (labor market contribution tax). The accumulation of wealth, A_t^{LP}, in the pension sector is defined as

$$A_t^{LP} = (1 + \tilde{r}_t)\, A_{t-1}^{LP} + y_t^{LP} - F_t^{RP} - F_t^{DP} - F_t^{SP} \tag{2.25}$$

where

$$\tilde{r}_t \equiv \left(1 - t_t^{rz}\right) r_t$$

and where F_t^{PR} is the sum of all pensions to retired persons form the pension funds, F_t^{PD} is the sum of all pensions to disabled persons, F_t^{PS} is the sum of all pensions to spouses. The total sum of pensions from the labor market pension system is the sum of the individual pensions to persons of different age and gender.

$$F_t^{RP} = \sum_{b=67}^{120} M_{b,t}^{Fa} f_{b,t}^{FRP} + \sum_{b=67}^{120} M_{b,t}^{Ma} f_{b,t}^{MRP}$$

$$F_t^{DP} = \sum_{b=23}^{120} M_{b,t}^{FD} f_{b,t}^{FDP} + \sum_{b=23}^{120} M_{b,t}^{MD} f_{b,t}^{MDP}$$

$$F_t^{SP} = \sum_{b=23}^{120} M_{b,t}^{FS} f_{b,t}^{FSP} + \sum_{b=23}^{120} M_{b,t}^{MS} f_{b,t}^{MSP}$$

where $f_{b,t}^{FRP}$, $f_{b,t}^{MRP}$ is the individual retirement pension to females respectively men of age b at time t. $M_{b,t}^{Fa}$, $M_{b,t}^{Ma}$ is the number of females respectively men who are defined as active members and therefore receive retirement pension and is of age b at time t. Similarly for the two other categories of pension.

If the pension fund is initiated gradually by introducing new member at the age of 22 years and if the mortality rates (and disablement rates) used by the fund are correct then the rules of precaution imply that the stock of accumulated wealth by the pension fund converges to a stationary state if the average wage remains constant. In other words, the rules ensures that the fund neither over-accumulates nor becomes insolvent. However, as pension schemes have existed for substantial period, the pension fund in DREAM has to be initiated as an existing fund with some initial distribution of members with different seniority and some initial level of aggregate stock of wealth. Therefore the saddle point property is not necessarily ensured since this would imply that the initial data are 100 percent consistent. As our data do not cover all companies this requirement is not met. To ensure that the fund in DREAM neither over-accumulates nor becomes insolvent a terminal condition is imposed. This implies that initial pension undertakings and pension are adjusted to satisfy the terminal condition.

Furthermore, since mortality rates are not constant in the official forecast of Statistics Denmark the behavior of the pension fund when applying the rules of precaution is not perfectly correct. Also this over-accumulation is corrected by the terminal condition. In the real world this adjustment would be more smooth due to the existence of reserves, which is not a feature in the present version of DREAM.

Chapter 3
THE ARBITRAGE CONDITION AND THE BEHAVIOR OF FIRMS

The fundamental behavioral assumption of the firms is that each firm seeks to maximize the value of the outstanding stock of shares. The value of the shares is determined from an arbitrage condition that ensures that the after tax yield on shares is equal to the interest rate after tax.

It is assumed that the value of the outstanding stock of shares is determined by the behavior of the institutional investors (i.e. the pension fund) by assuming that the tax rates, which apply to the pension savings, are used to determine the after tax value in the arbitrage condition of the financial markets. In this way the behavior of the firm is indirectly affected by the taxation of the institutional investor.[1]

Concerning the financing decision of the firms we apply the so-called "new view" of dividend taxation, see Sinn (1987), according to which a corporation should retain all earnings which can profitably be invested within the firm and pay out only the remainder as dividends. Dividends are therefore determined residually and corporations abstain from financing increases in activity by issuing new shares. In addition we assume that the corporations maintains a fixed ratio of debt to the value of the capital stock.[2]

Therefore, suppose that there exists a fixed number of shares in the economy, n. We normalize this number to 1. Define the value of the outstanding stock of shares as

$$V_t = nv_t \equiv v_t$$

where v_t is the value of a share at time t. Similarly, define total stream of dividends

[1] Previous versions of DREAM (version 1.2) do not include a pension fund and therefore the relevant tax rates are assumed to be the personal tax rates.

[2] This latter ad hoc assumption is applied to prevent the firm from choosing either a debt ration of zero or 100 percent depending on the tax system.

as

$$D_t = nd_t \equiv d_t$$

where d_t is the amount of dividend per share at time t

Institutional investors of the pension fund sector are assumed to be indifferent between investing in shares or in bonds. The arbitrage condition read

$$r_t\left(1-t_t^{rz}\right)v_{t-1} = \left(1-t_t^{dz}\right)d_t + \left(1-t_t^{gz}\right)\left(v_t - v_{t-1}\right) \tag{3.1}$$

where t_t^{rz} is the rate of taxation of interest income in pension funds, t_t^{dz} is the tax rate of dividends, t_t^{gz} is the rate of taxation of capital gains from shares. In fact the present[3] Danish tax system for pension funds implies that all types of yields from shares are taxed identically.

Aggregation and exploiting the fact that the number of shares is fixed yields

$$r_t\left(1-t_t^{rz}\right)V_{t-1} = \left(1-t_t^{dz}\right)D_t + \left(1-t_t^{gz}\right)\left(V_t - V_{t-1}\right)$$

Solving this equation forward in time and ruling out explosive paths yields an expression for the total value of stock in the economy.

$$V_s = \sum_{t=s+1}^{\infty} \frac{1-t_t^{dz}}{1-t_t^{gz}} D_t \prod_{v=s+1}^{t} \frac{1}{1+r_v\left(\frac{1-t_v^{rz}}{1-t_v^{gz}}\right)} \tag{3.2}$$

The representative firm in the private production sector maximizes the tax-adjusted cum-dividend value of the firm subject to the production technology and the constraint on the financial decision that the debt of the firm is a fixed fraction of the value of the stock of capital.

Figure 3.1 illustrates the assumed technology of the producers. It is assumed that the technology of the representative private firm and the governmental producer can be represented by the same functional form. The upper part of figure 3.1 outlines the assumed production function which is specified as two-factor CES (sub) production functions nested as indicated at the figure. At the top level materials is combined with value added to produce gross output. Materials are obtained by combining governmentally produced materials and privately produced materials. Finally, privately produced materials can be either of domestic or foreign origin.

[3] In this projection we assume that the taxation of pension funds is given by the rules adopted by the parliament in May 1998. (In Danish "pinsepakken")

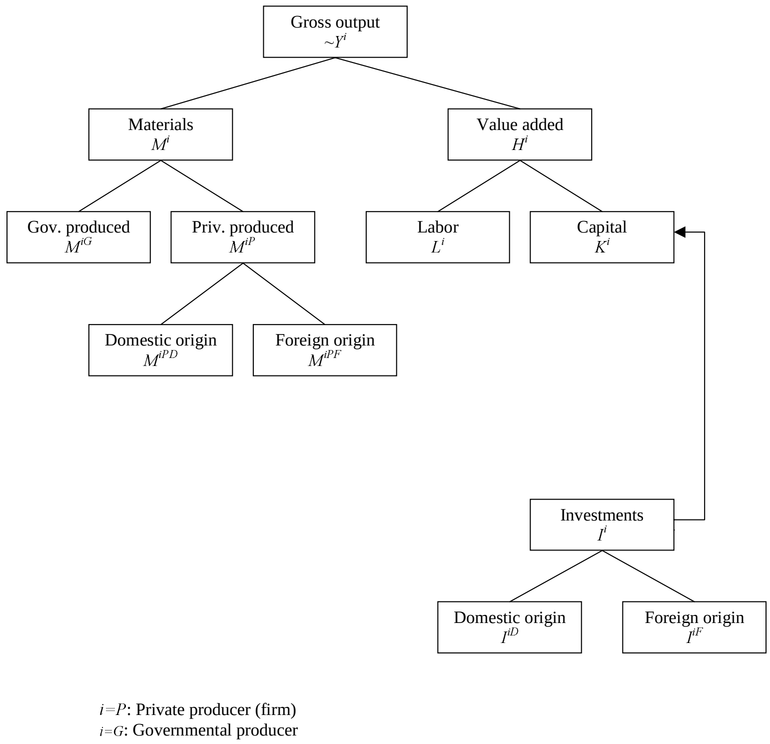

Figure 3.1: The technology of the producers

The solution to the maximization problem of the representative private firm leads to the following first order conditions:

$$\frac{\partial F^P}{\partial L^P}\left(M_t^P, K_{t-1}^P, L_t^P\right) = \left(1 + t_t^a\right)\frac{W_t}{p_t^P} \tag{3.3}$$

where F^P is the gross production function of the private sector. L^P is demand for the labor index in the private sector, M^P is demand for the index of materials, K_{t-1}^P is the capital stock, t_t^a is the pay roll tax (in Danish "arbejdsgivernes arbejdsmarkedsbidrag"), W_t is the wage index, p_t^P is the output price of the private sector.

This is the standard labor demand condition, stating that labor is employed until the marginal product is equal to the producer wage inclusive payroll tax.

$$p_t^P \frac{\partial F^P}{\partial M^P}\left(M_t^P, K_{t-1}^P, L_t^P\right) = p_t^{PM} \tag{3.4}$$

where p_t^{PM} is the price of the index of materials.

Also this condition is a standard first stating that materials are used until the value of the marginal product is equal to the price (index) of materials.

Define Φ^P as the cost of installation of capital, which is a function of the capital stock and the level of investment, I_t^P. The function is specified as follows

$$\Phi^P(I_t^P, K_{t-1}^P) = \phi^P \left(\frac{|I_t^P|}{K_{t-1}^P} \right)^{\iota} |I_t^P|, \ \iota > 0 \tag{3.5}$$

The first order condition with respect to investments becomes

$$\frac{1-t_t^{dz}}{1-t_t^{gz}} \left(1 - g + (1-t_t^c) \frac{p_t}{p_t^{PI}} \frac{\partial \Phi^P}{\partial I^P}(I_t^P, K_{t-1}^P) \right) = \frac{\lambda_{1t}}{p_t^{PI}} + \lambda_{2t} \tag{3.6}$$

where g is the debt ratio of the firm, t_t^c is the corporate tax rate, Φ^P is the cost of installation of capital, which is a function of the capital stock and the level of investment, I_t^P. λ_{1t} is the shadow price of the capital stock, p_t^{PI} is the price index of investments so that $\frac{\lambda_{1t}}{p_t^{PI}}$ is Tobin's marginal q. λ_{2t} is the shadow price of the book value of the capital stock i.e. the value of future depreciation allowances.

The interpretation of this equation is also that in optimum the marginal cost of investment is equal to the marginal benefits from investment. The marginal cost is given on the left-hand side. The costs are the sum of the direct costs of investment to the owners of the firm, and the indirect costs associated with the loss of production, due to the installation of the additional capital equipment. Both types of costs are corrected by the tax adjustment factor, which applies to the stream of dividends. Observe that increasing the debt ratio of the firm reduces the direct cost to the owners, as they have to finance a smaller fraction of the investment. The benefits of an investment are twofold. First, it adds to the capital stock of the firm. The shadow price, λ_{1t}, is the marginal value of an additional unit of capital to the owners of the firm. Second, it adds to the depreciation allowance of the firm. The value of the depreciation allowance increases with the direct cost of the additional capital unit. The shadow price, λ_{2t}, measures the marginal value of a unit depreciation allowance to the owners of firm. The total benefit is the sum of these two effects. The determination of the expressions for the two shadow prices are given in the two final first order conditions. These are the conditions for the stock variables of the problem.

$$\begin{aligned} &\frac{1-t_t^{dz}}{1-t_t^{gz}} \left\{ (1-t_t^c)\, p_t^P \left[\frac{\partial F^P}{\partial K^P} \left(M_t^P, K_{t-1}^P, L_t^P \right) - \frac{\partial \Phi^P}{\partial K^P}(I_t^P, K_{t-1}^P) - r_t g \frac{p_{t-1}^{PI}}{p_t^P} \right] \right. \\ &\left. + g p_t^{PI} \left(1-\delta^P\right) - g p_{t-1}^{PI} \right\} = r_t \frac{1-t_t^{rz}}{1-t_t^{gz}} \lambda_{1(t-1)} + \delta^P \lambda_{1t} - \left(\lambda_{1t} - \lambda_{1(t-1)} \right) \end{aligned} \tag{3.7}$$

$$\frac{1-t_t^{dz}}{1-t_t^{gz}} t_t^c \hat{\delta}_t = r_t \frac{1-t_t^r}{1-t_t^g} \lambda_{2(t-1)} + \hat{\delta}_t \lambda_{2t} - \left(\lambda_{2t} - \lambda_{2(t-1)} \right) \tag{3.8}$$

where $\hat{\delta}_t$ is the annual depreciation rate for tax purposes..

The first order conditions (3.7) and (3.8) may be interpreted directly as user cost expressions (or arbitrage conditions). Equation (3.7) states, that in optimum the marginal increase in the dividend from a marginal capital unit (the left hand side of the equation) is equal to the user cost of holding capital (the right hand side). The user cost of holding capital (the right hand side) consists of the sum of the tax adjusted foregone interest payments and the (true) costs of physical depreciation plus a possible capital loss due to the change in the shadow price of capital to the owners. Equation (3.8) has a similar interpretation.

Assuming that the cost of installation of capital, $\Phi^P(I_t^P, \frac{K_{t-1}^P}{1+n})$, is homogenous in degree one[4], we may follow Hayashi (1982), proposition 1 and write the value of the firm as

$$V_t = \lambda_{1t} K_t + \lambda_{2t} \hat{K}_t \tag{3.9}$$

Observe that since the value of shares depends on the tax rates of the institutional investors, see relation (3.2) , then these tax rates affects the relevant shadow prices of capital and the book value of capital. Therefore the tax rates faced by the institutional investors affect the relevant cost of capital for the firms, and hence both short and long run supply behavior in the model.

3.0.1 Governmental producers

Following the national accounts conventions, the governmental sector in DREAM produces mainly non-marketed goods and services of which an overwhelming part are delivered directly to public consumption. Publicly owned firms which mainly produces for a market are classified as part of the private sector. However, approximately 10 per cent of the production, which in DREAM is classified as public is delivered to the private sector.

The governmental production and factor demands are modelled in much the same way as the production and factor demands of the private sector firms. One important exception is that we do not assume any intertemporal optimization of the public sec-

[4] In the computer program of DREAM the cost of installation function is specified as

$$\Phi^P(I_t^P, \frac{K_{t-1}^P}{1+n}) = (1+n)\,\phi^P \frac{\left(I_t^P\right)^2}{K_{t-1}^P}$$

tor. Public (gross) investments are instead determined by the requirement that the capital output ratio in the public sector is constant. Together with the capital accumulation identity this determines the development of the public capital stock. Thus, the value of the marginal product of capital may deviate between the private and the public sector because the public capital stock is not determined by (intertemporal) optimization. This is a crude way to model the behavior of policy-makers, which just assures that the size of the public production plant increases with increased demand for publicly produced goods. For simplicity it is assumed that there are no installation costs of investments in the public sector.

Given the politically determined investments and capital stock, the public demands for the remaining inputs labor and materials of different origins are determined along the lines of the private sector firms, i.e. it is assumed that the public sector minimizes costs subject to a nested CES-technology and given the capital stock. Thus even though the public sector does not substitute between capital and labor plus materials in an economic optimal way it is still assumed that it substitutes between labor and various materials in an optimal way - given the capital stock.

Public demand for publicly produced goods are divided into two different categories: Goods which are delivered to individuals as public services (this includes education, health care, and residential homes for elderly) and goods that cannot be attributed to specific categories of individuals. Public demand for the first category is assumed to determined by the demographic evolution as the real value of these services is assumed to remain fixed for each age and gender group of individuals. The real value of the second category of public demand for the publicly produced goods is assumed to be a fixed proportion of real GDP.

Chapter 4
BEHAVIOR OF THE HOUSEHOLD

In DREAM the population is divided into generations of households. The households consist of men, women and children. In the maximization problem of the household children are measured in adult equivalents to account for the fact that children consume less than adults. We assume that there is one representative household in each generation. The size of this household is equal to the number of persons in the generation of the household in question. In DREAM the age of the household is identical to the age of the women in the household. Men are assigned to the household according to a "couple matrix" that for each generation of women measures the age distribution of the men, who are married to the specific generation of women. Children are assigned to households by assigning each generation of newborns to a distribution of mothers with different ages. The distribution of mothers to newborns are calculated for each year according to the assumption in the official population forecast from Statistics Denmark.

In the present analysis the construction of the household is simplified by assuming that the couple matrix is a diagonal matrix. This implies that men and women of a specific household have the same age. As the mean value of the age-difference in couples is approximately 3 years, this implies that the men in the household becomes younger than what is actually observed. This again implies that the household as such faces a too small mortality rate, since the mortality rate is increasing with the age. This affects household behavior, but the effect is minor.[1]

Projection of age and gender specific mortality rates follows the official population

[1] The reason why this assumption has been made is the following: If the couplematrix is not diagonal, then (e.g) 17 years old men will in general not be matched to women who are 17 year or older. In the model these 17 years old men therefore remain as part of their parent household and enter the "new" young households gradually.

In the modellering of the labor market pension schemes it is assumed that all persons enter the pension fund at a given age. Therefore the gradual inflow of young men become inconsistent. The assumption of a diagonal couple matrix solves this inconsistency.

forecast of Statistics Denmark. The underlying assumption behind this forecast is a rather significant gradual reduction in all age and gender specific mortality rates until 2040. The population forecast is prolonged in the present paper. We assume that mortality rates remain at the level reached in 2040 for the rest of the projection period.[2]

The official forecast of Statistics Denmark is adjusted with respect to immigrants. In the official forecast net immigration is positive for all age groups. As immigrants in DREAM form households of their own this implies that a very large number of different immigrant households (that differ according to age and time of immigration) will appear. To avoid this technical problem in the model net immigration has been converted into 17 year's equivalents, which means that it is assumed that the entire net-immigration consists of persons who are 17 years old. As this assumption implies that each immigrant is longer in the economy than the actual net-immigrant the number of immigrants has be calculated so that the proportion of net-immigrants to the total adult population in the economy is the same as in the official forecast.

The household is assumed to be utilitarian in the sense that it maximizes the sum of utilities of the members of the household. Therefore the household maximizes the following utility function

$$U_{b-1,t-1} \equiv \left[\sum_{i=b}^{77} \left(Q_{i,t-b+i}\right)^{\frac{S-1}{S}} v_{b-1,i} \cdot N_{i,t-b+i}^{EF} \right]^{\frac{S}{S-1}} \quad (4.1)$$

$$v_{b-1,i} \equiv \xi_i \left(\frac{1}{1+\theta} \right)^{i-b+1} \text{ where } \xi_i = 1 \text{ for } i = b, ..., 76, \text{ and } \xi_{77} = \xi \quad (4.2)$$

The utility, (4.1), is defined as the sum of annual total household utilities (equal to instantaneous utilities per adult-equivalent, $(Q_{b,t})^{\frac{S-1}{S}}$, times the number of adult-equivalents in the household, $N_{b,t}^{EF}$) in the remaining lifetime (from age b to age 76 years), discounted to the beginning of year t. In addition to this sum of annual utilities, the household obtains positive utility from leaving a bequest to the children. This additional utility is measured as an extra period of (pure) consumption (year 77)

[2] The fact that mortality rates decrease until 2040 is in fact inconsistent with the behavior of the pension fund, where it is assumed that the mortality rates remain constant through out. As the pension fund uses mortality rates that applies in 2020 this implies that *ceteris paribus* the pension fund calculates too high pension undertakings in the long run and therefore pays too high annual pensions. This effect is considered small and is "corrected" in the initializing of the pension undertakings of existing members of the pension fund.

differentiated only from normal consumption by a preference parameter ξ. $Q_{77,t-b+77}$ is the real value (as measured by the price index of consumption) of the bequest per adult(-equivalent) that the generation leaves to future generations at the time when it reaches 77 years of age and the household ceases to exist, ξ is the weight that the donor generation associates with the bequest.

The instantaneous utility function is given as

$$Q_{b,t} = C_{b,t} - Z_{b,t} \tag{4.3}$$

where $C_{b,t}$ is the demand for the aggregated consumption index per adult equivalent of generation b at time t. $C_{b,t}$ is an index over domestic private goods, foreign private goods and the domestic publicly produced good.

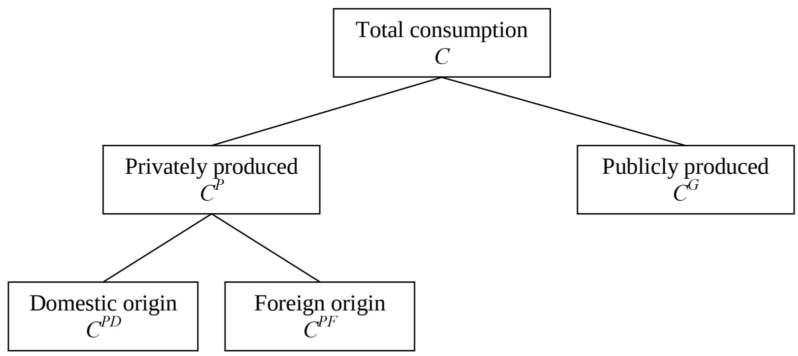

Figure 4.1: The intratemporal utility function of the household

Figure 4.1 illustrates the assumptions about the intratemporal sub utility function of households. At the top level private consumption goods which are supplied from the private sector substitutes private consumption goods supplied from the public sector. At the bottom level privately produced consumption goods of domestic and foreign origin substitute each other. There are no imports of the publicly produced good. This "utility tree" is specified as nested CES-functions.

The variable $Z_{b,t}$ in (4.3) is the disutility of work per adult equivalent of generation b. It is assumed that the disutility of work of the individual in question varies with the productivity of the individual. This assumption and the additivity of the disutility together imply that the number of hours supplied becomes independent of age and gender.

The representative household maximizes this utility function by choosing the consumption of each of the goods in the economy in each period. This defines a value of the consumption index in each period. Similarly the representative household chooses the labor supply of each of the adult member in the household.[3]

Observe, that the financial decisions of the households are not part of the optimization problem of the household. The household has the following financial decisions: Total savings has to divided between savings in the labor market pension system, the private pension savings and savings that is not tied until a specific age is reached. For the latter type of savings the household further has to decide how to divide the savings between the two assets in the economy: Shares and bonds.

Regarding the decision as to how much saving should take place in the labor market pension system we assume that this is exogenous to the household. This is so since Danish labor market pension contributions are determined as part of the employment contract, where pension contributions are determined by collective bargaining between the unions and employers associations. For high income employees in the private sector, who generally are not covered by collective bargaining agreements, the pension contributions also tend to be part of the employment contract. We assume that also in this case the pension contribution may be considered exogenously given.

For the rest of the savings decisions the choice ought ideally to be the outcome of a maximization. Infinite arbitrage would however imply that no equilibrium would exist, since the perfect capital market implies that it is optimal for the household to borrow in the private capital market and to invest in the market for pension savings given the tax subsidy to private pension savings. Therefore it has been assumed that a fixed part of the wage sum for each age and gender group is saved in private pension arrangements. This savings ratio is calculated from the data for 1995 and the private pension savings rate is assumed to be fixed through time.

Finally, the taxation of income from shares relative to the taxation of bonds for private saving differ from the relative taxation of the two assets for pension funds. This implies that the private savings should optimally take place in the asset where the private savings has a comparative tax advantage (given the calibration of the model this asset is bonds). To avoid this type of corner solution we assume that a fixed part of the value of assets takes the form of shares.

[3] A complete derivation of the demand for each of the goods in the aggregate consmption index is found in Knudsen *et al.* (1998a).

4.1 The accumulation of private savings

We initially consider the exogenous composition of assets in the value of the accumulated wealth that is not part of any pension saving scheme. Recall that we assumed that the value of this stock of wealth is split into bonds and shares in a fixed proportion. Formally we may write this assumption as

$$n_{b,t}^{v} v_t = \omega b_{b,t}^{h}, \quad \omega > 0$$

where $n_{b,t}^{v}$ is the number of shares held by the household that is b years old at time t and v_t is the value of one share. $b_{b,t}^{h}$ is the stock of bonds held by the household. The total stock of wealth (that is not part of a pension scheme) of the household of age b, $a_{b,t}$ is therefore defined as

$$a_{b,t}^{h} = n_{b,t}^{v} v_t + b_{b,t}^{h} \quad \Rightarrow \quad a_{b,t}^{h} = \frac{(1+\omega)}{\omega} n_{b,t}^{v} v_t$$

The average yield after tax on the asset position of the household is given by

$$\hat{r}_t a_{b-1,t-1}^{h} = \left(1 - t_t^{g}\right)\left(v_t - v_{t-1}\right) n_{b-1,t-1}^{v} + n_{b-1,t-1}^{v}\left(1 - t_t^{d}\right) d_t + \left(1 - t_t^{r}\right) r_t b_{b-1,t-1}^{h}$$

$\Leftrightarrow$

$$\hat{r}_t = \left(\frac{\omega}{1+\omega}\right)\frac{1}{v_{t-1}}\left[\left(1 - t_t^{g}\right)\left(v_t - v_{t-1}\right) + \left(1 - t_t^{d}\right) d_t\right] + \left(\frac{1}{1+\omega}\right)\left(1 - t_t^{r}\right) r_t$$

where t_t^{g}, t_t^{d}, t_t^{r} are tax rates of the personal capital income taxation system. t_t^{g} is the tax rate on capital gains, t_t^{d} is the tax rate on dividends, t_t^{r} is the tax rate on interest income.

Observe that the average yield is a weighed average of the yields of the two types of assets. The weights are defined as the proportions of the two types of assets in the total stock of wealth.

Using this expression for the after tax yield on the composite asset in the model yields the correct accumulation rule for the private household.

$$a_{b,t} = \left(1 + \hat{r}_t\right)\frac{N_{b-1,t-1}^{EF}}{N_{b,t}^{EF}} a_{b-1,t-1} + y_{b,t} - P_t C_{b,t} \tag{4.4}$$

where $N_{b,t}^{EF}$ is the number of adult equivalents in the household that is b years old at time t. $y_{b,t}$ is the non-interest income of the household.

Observe that due to the assumption that each generation uses the same fixed proportion the assets, the weighted average yield is identical across generations.

4.2 Accumulation in private pension arrangements

Contributions to private pensions arrangements are defined as an age and gender specific fraction of the pre-tax labor income. Capital income from the accumulated savings in private pension schemes are taxed according to the same rules as applies to the accumulation in labor market pension schemes. The withdrawals from the private pension arrangements are assume to take place at 2 instances: When the person in question turns respectively 67 years, and 72 years.

The accumulation equation for the stock of saving of generation that is age b at time t is therefore given as

$$\begin{aligned} A_{b,t}^{PP} &= \left(1+\left(1-t_t^{rz}\right)r_t\right)A_{b-1,t-1}^{PP}+\vartheta_{F,b,t}^{PP}\left(1-t_t^{\ell}\right)w_{F,b,t}\ell_t N_{b,t}^{FW} \\ &\quad +\vartheta_{M,b,t}^{PP}\left(1-t_t^{\ell}\right)w_{M,b,t}\ell_t N_{b,t}^{MW}-f_{b,t}^{PP}\left(N_{b,t}^{F}+N_{b,t}^{M}\right) \end{aligned}$$

where $A_{b,t}^{PP}$ is the accumulated stock of savings in private pension schemes for the generation that is b years old at time t. $\vartheta_{j,b,t}^{PP}$ is the gender, age and time specific fraction of income, which is saved as private pension schemes, $\left(1-t_t^{\ell}\right)w_{j,b,t}\ell_{b,t}N_{b,t}^{jW}$ is the total income net of labor market contribution (which is a pay roll tax) from employment of persons of gender j, age b at time t. $f_{b,t}^{PP}$ is the withdrawal per person age b at time t. $N_{b,t}^{F}$, $N_{b,t}^{M}$ is the number of female and male adults respectively . The withdrawal per person is given by

$$f_{b,t}^{PP}=\begin{cases} \sigma_1 \dfrac{A_{66,t-1}^{PP}\left(1+\left(1-t_t^{rz}\right)r_t\right)}{N_{67,t}^{F}+N_{67,t}^{M}} & \text{for} \quad b=67 \\ \dfrac{A_{71,t-1}^{PP}\left(1+\left(1-t_t^{rz}\right)r_t\right)}{N_{72,t}^{F}+N_{72,t}^{M}} & \text{for} \quad b=72 \\ 0 & \text{for} \quad b\neq\{67,\ 72\} \end{cases} \tag{4.5}$$

where $0<\sigma_1<1$,

The parameter σ_1 is calibrated so that the observed reduction in the stock of accumulated savings from the age of 67 year to 72 year is reproduced.

Total accumulation in private pension arrangements is given by the following equation

$$y_t^{PP} = \sum_{j\in\{F,M\}} \sum_{b=17}^{66} \vartheta_{j,b,t}^{PP} \left(1 - t_t^{\ell} - t_t^{ATP2}\right) w_{j,b,t} \ell_{b,t} N_{b,t}^{jW}$$

where y_t^{PP} is the total contributions to the private pension schemes net of labor market contribution tax

Total accumulation of wealth in private pension schemes, A_t^{PP}, is given as

$$A_t^{PP} = (1 + (1 - t_t^{rz})\, r_t)\, A_{t-1}^{PP} + y_t^{PP} - F_t^{PP}$$

where the total payment from the private pension schemes is defined as the withdrawal of the total stock of assets for persons who turn 67 years

$$F_t^{PP} = \left[f_{67,t}^{PP} \left(N_{67,t}^{F} + N_{67,t}^{M}\right) + f_{72,t}^{PP} \left(N_{72,t}^{F} + N_{72,t}^{M}\right)\right] (1 + (1 - t_t^{rz})\, r_t)$$

Since we in this special analysis apply a very simple household structure where the men and the women in the household have the same age, we do not need to have separate savings accounts for men and women, since the savings of the different gender apply to the same household. Furthermore, we stick to the assumption that the household maintains undivided possession of the estate until the household reaches the age of 77. Therefore we abstract from the fact that a part of the generation dies and apply the simple rule that the total stock of accumulated savings of a specific generation is withdrawn as the generation reaches the age of 67 years.

4.3 The non-interest income of households

Total non-interest income in the economy is distributed between the representative households according to the following principles: First, we define three income categories: Income from time consuming activities, income from pensions from the pension fund sector and other types of non-interest income. The latter consists of public transfers that are distributed according criteria other than income compensation. Examples of this type of income are housing subsidy, transfers to families with children, medicine subsidies. Presently, these transfers also include early retirement pension and education benefits although these activities are time consuming. These public transfers are distributed across the different households according to the distribution across gender and generations of the total expenses of the specific type of transfer. These distribution of expenses are obtained from the generational accounting system of the Ministry of Economic Affairs. There are two kinds of age specific public transfers: The tax free transfers, $TR^{g}_{i,t}$, $g = F, M$, and the taxable transfers $TRT^{g}_{i,t}$, $g = F, M$ of which the tax $t^{TRT}_{t} \cdot TRT^{g}_{i,t}$ is paid, where t^{TRT}_{t} is the average tax rate relevant to public transfers. Also each individual gets lump sum transfers, τ^{W}_{t} and τ_{t}, from abroad and the domestic government respectively.

4.3.1 Income from time consuming activities

Income from time consuming activities are categorized in the following groups:

1. Wage income
2. Unemployment benefits
3. Pensions (including early retirements benefits)

To simplify we have defined persons who are 62 years or older as retired from the labor market. For persons who are between 17 and 61 years, an age and gender specific labor market participation rate, $\varkappa^{iW}_{b,t}$, is defined, so that the number of persons in the labor force of a specific age and gender is given as

$$N^{gW}_{b,t} = \begin{cases} \varkappa^{gW}_{b,t} N^{g}_{b,t}, & 17 \leq b \leq 61 \\ 0, & b \geq 62 \end{cases} , \quad g \in \{F, M\} \tag{4.6}$$

Based on the DREAM group's 10 per cent representative household sample of the total register of persons and workplaces in Denmark an age and gender specific wage distribution for 1995 is constructed. We assume that persons of different age and gender groups are perfect substitutes so that differences in the wage per hour across different individuals reflect differences in the productivity of the individuals in question.

Let L_t denote demand for labor at time t measured in productivity corrected units

$$L_t = \sum_{g \in \{F,M\}} \sum_{b=17}^{61} \varkappa^{gW}_{b,t} N^{g}_{b,t} \rho^{g}_{b} \ell^{g}_{b,t} \tag{4.7}$$

where ρ^{g}_{b} is the relative productivity of an individual of gender g and age b. Observe that by assumption this relative productivity is constant through time. $\ell^{g}_{b,t}$ is the (average) number of hours that an individual of gender g and age b who belongs to the work force spends working. As the definition of L_t implies that all individuals are perfect substitutes the productivity corrected wage for two different individuals must be identical. Define the productivity corrected wage

$$W_t = \frac{w_{g,b,t}}{\rho_b^g}, \quad \forall g \in \{F, M\}, b \in \{17, ..., 61\} \tag{4.8}$$

Furthermore, it is assumed that work sharing prevails in the economy. Therefore the income of an individual who belongs to the work force is divided into income from time spend as employed and benefit from time spend as unemployed. Let $\bar{\ell}$ be the total annual number of working hours per individual and let $\ell_{b,t}^g$ be the number of hours employed for an individual of gender g and age b at time t, then $\left(\bar{\ell} - \ell_{b,t}^g\right)$ is the number of hours in period t that the individual in question spends unemployed. Using these definitions the non-interest income for a working individual composed of the sum of salaries net of taxes and contributions to different pension schemes and unemployment benefit net of taxes and contributions to pension schemes. This amounts to

$$\begin{aligned} &\left(1 - t_t^w\right)\left[\left(w_{g,b,t}\left(1 - \vartheta_t^{LP} - \vartheta_{g,b,t}^{PP}\right) - t_t^{ATP1}\right)\left(1 - t_t^l - t_t^{ATP2}\right)\right]\ell_{b,t}^g + \\ &\left(1 - t_t^b\right)\left(b_t - \kappa_t t_t^{ATP1}\right)\left(\bar{\ell} - \ell_{b,t}^g\right) \end{aligned} \tag{4.9}$$

where t_t^w is the effective average tax rate on labor income net of the labor market contribution tax, t_t^l, ϑ_t^{LP} is the percentage of wage income allocated to a labor market pension fund as contributions, $\vartheta_{g,b,t}^{PP}$ is the percentage of wage income allocated to a private pension scheme for a person of gender g and age b, t_t^{ATP2} is the part of the contributions to the ATP pension scheme which depends upon the total wage income, $w_{g,b,t}\ell_{b,t}^g$ is the wage income for persons of gender g, and age b at time t, $t^{ATP1}\ell_{b,t}^g$ is the part of the contribution to the ATP pension scheme that depends upon the number of hours worked. t_t^b is the effective average tax rate on unemployment benefits, b_t is the level of unemployment benefits. The term, $\kappa_t t_t^{ATP1}\left(\bar{\ell} - \ell_{b,t}^g\right)$, is the contribution to the ATP pension scheme from unemployed persons. κ_t is equal to 2 except for the first two periods.

Individuals who are at least 60 years old and have contributed to the unemployment insurance system for at least 20 years are entitled to early retirement benefits according to Danish rules.[4] To simplify it is assumed that all agents retire from the labor

[4] As part of the passing of the government budget for 1999 the Parliament introduced extensive changes in the early retirement benefit system. These are not incorporated in the present analysis.

market at the age of 62 years. At the age from 62 year to 67 years the individuals receive early retirement pension from the public sector. For simplicity it is assumed that early retirement and public pension are identical, f_t^G. The value of the public pension per head in 1995 is found as total costs of both the early retirement scheme and the social pension scheme, $\overline{FP}_{1995}$ divided by the sum of the number of persons who are 62 years or older.

$$f_{1995}^G = \frac{\overline{FP}_{1995}}{\sum_{b=62}^{101} N_{b,1995}} \tag{4.10}$$

The pensions are indexed to the wage rate after labor market contribution tax and contributions to the labor market pension schemes. Both public pension and the early retirement scheme is taxed by the effective average tax rate on public pensions, t_t^G

4.3.2 Other types of non-interest income

In addition to wage income, unemployment benefits and public pensions the individuals receive income from various types of pension schemes.

First, persons who received wage income in the period from 1977-79 has an account at the LD pension fund. The we assume that amount accumulated is withdrawn as the individual in question become 62 years old. The annual receipts from the LD pension fund is forecasted by the fund and converted into an amount per individual in DREAM. We define the receipts per person as

$$f_{b,t}^{\overline{LD}} = \begin{cases} \frac{\overline{LD}_t}{N_{62,t}}, & b = 62 \\ 0, & b \neq 62 \end{cases} \tag{4.11}$$

where $\overline{LD}_t$ is the total annual receipts from the LD pension fund. $f_{b,t}^{\overline{LD}}$ is the average LD pension per individuals who are 62 years. Receipts from the LD pension fund are taxed at the rate t_t^{PP} which is equal to 40 per cent. The economic effect of pensions from LD pension fund is identical to the effect of public pensions to persons who are 62 years old.

Second, pension receipts from the $ATP-$ pension fund initiates when the individual in question turns 67. As mentioned two types of pension schemes from the $ATP-$pension fund exists: The first scheme is a life long retirement pension based on the individuals employment related contributions. The second scheme is a retirement pension to members in the age-group from 67 to 76. Receipts from this scheme is based on the average wage sum dependent contributions in the period where the member in question has paid this type of contributions. Total receipts from each of the two types of pension schemes from the ATP pension fund are forecasted by the fund. These forecasts are used directly in DREAM. As no information on the age distribution of the pension recipients is available, it is assumed that the total annual receipts are distributed equally between all pensioners. Therefore, the average ATP pension per pensioner from the employment dependent contribution , $f_{b,t}^{\overline{ATP_1}}$, is defined as

$$f_{b,t}^{\overline{ATP_1}} = \begin{cases} 0, & b < 67 \\ \dfrac{\overline{ATP1}_t}{\sum_{b=67}^{\infty} N_{b,t}}, & b \geq 67 \end{cases} \tag{4.12}$$

where $\overline{ATP1}_t$ is the total receipts of ATP pensions based on employment dependent contributions at time t. Similarly, the average ATP pension per pensioner from the wage sum dependent contribution, $f_{b,t}^{\overline{ATP_2}}$, is defined as

$$f_{b,t}^{\overline{ATP_2}} = \begin{cases} 0, & b < 67 \\ \dfrac{\overline{ATP2}_t}{\sum_{b=67}^{76} N_{b,t}}, & 67 \leq b \leq 76 \\ 0 & b > 76 \end{cases}$$

where $\overline{ATP2}_t$ is the total receipts of ATP pensions based on employment dependent contributions at time t.

Third, as the individual in question turns 67 years the person in question withdraws the first part of the total private pension savings. The rest is withdrawn as the individual in question turns 72. The average withdrawal per person in each age-group is defined in (4.5). Private pension savings are defined as forced savings that are not part of the "free" stock of financial wealth of the household. Therefore the withdrawals are defined as a part of the human capital of the individual in question and thus included in the non-interest income of the household. The withdrawals are taxed at the rate t_t^{PP} which is equal to 40 per cent.

Fourth, the part of the pensioners who are at least 67 years and who have been civil servant are entitled to civil servants' pension from the public sector. The Danish ministry of Finance has forecasted the expenditures to civil servants' pension in measured in fixed prices. In DREAM this forecast is indexed to the wage rate. As we have no knowledge of the age distributions of present and future civil servant pensioners it is assumed that the average civil servants' pension is identical for all age-groups of pensioners. Therefore the average civil servants pension, $f_t^{\overline{CSP}}$ is defined as

$$f_{b,t}^{\overline{CSP}} = \begin{cases} 0, & b < 67 \\ \dfrac{\overline{CSP}_t}{\sum_{b=67}^{\infty} N_{b,t}}, & b \geq 67 \end{cases} \tag{4.13}$$

where $\overline{CSP}_t$ is the total expenses to civil servants' pensions, and $\sum_{b=67}^{\infty} N_{b,t}$ is the total number of persons, who are at least 67 years. Civil servants' pensions is assumed to be taxed by the effective average tax rate for retirement pensions, t_t^{RP}.

Fifth, all types of pension from the labor market pension fund are defined as non-interest income as savings in the pension fund are not considered part of the individual's personal savings.

Retirement pension receipts from the labor market pension fund are received from the age of 67 years. By definition the retirement pension, $f_{b,t}^{FRP}$, is the average retirement pension of a so-called active female who is b years old at time t. The number of active females of a given generation is defined as the total number of females in the generation in question minus the number of disabled females in the generation in question. The average retirement pension for an individual of gender g who is age b at time t, $\hat{f}_{b,t}^{gRP}$, is therefore given as

$$\hat{f}_{b,t}^{gRP} = \begin{cases} 0, & b < 67 \\ \dfrac{M_{b,t}^{ga}}{N_{b,t}^{g}} f_{b,t}^{JRP}, & b \geq 67 \end{cases}, \quad g \in \{F, M\} \tag{4.14}$$

where $M_{b,t}^{ga}$ is the number of active members of gender g, who are age b at time t.

Similarly, the disablement pension, $f_{b,t}^{FDP}$, is measured as the disablement pension per disabled female who is b years old at time t. The average disablement pension per individual of gender J, who are age b at time t is therefore given as

$$\widehat{\hat{f}}_{b,t}^{gDP} = \begin{cases} 0, & b < 27 \\ \dfrac{M_{b,t}^{gD}}{N_{b,t}^{g}} \hat{f}_{b,t}^{gDP}, & b \geq 27 \end{cases}, \quad g \in \{F, M\} \tag{4.15}$$

Observe that the youngest individual, who receives disablement pension is defined to be 27 years old.

Spouse pensions are defined similarly to disablement pensions. The only complication is that spouse pensions receipts are not distributed to insured individuals themselves but to individuals of the opposite gender. Therefore the average spouse pension of a female of age b at time t, $\widehat{\hat{f}}_{b,t}^{FSP}$, is given as

$$\widehat{\hat{f}}_{b,t}^{FSP} = \begin{cases} 0, & b < 27 \\ \frac{\hat{M}_{b,t}^{MS}}{N_{b,t}^{F}} \hat{f}_{b,t}^{MSP}, & b \geq 27 \end{cases} \tag{4.16}$$

Recall that $\hat{M}_{b,t}^{MS} \hat{f}_{b,t}^{MSP}$ is the total value of spouse pensions from late male (hence the M in the variable names) members of the pension fund who would have been b years old at time t. As these men are married to females who has the same age this value is also the total value of spouse pension receipts to females of age b at time t. Similarly the average spouse pension of a male of age b at time t, $\widehat{\hat{f}}_{b,t}^{MSP}$, is given as

$$\widehat{\hat{f}}_{b,t}^{MSP} = \begin{cases} 0, & b < 27 \\ \frac{\hat{M}_{b,t}^{FS}}{N_{b,t}^{F}} \hat{f}_{b,t}^{FSP}, & b \geq 27 \end{cases} \tag{4.17}$$

The final type of non-interest income is the inheritance per individual of gender g and age i at time t, $B_{i,t}^{\overline{gin}}$,. Note, that the specification of the distribution of births and the assumption that households survives until the woman reaches the age of 77 years, imply that the maximum age of heirs will be $(77 - 17 =)$ 60 years, because the youngest mothers are 18 years old. Similarly, the minimum age of a heir is $(77 - 49 =)$ 28 years, as the maximum age of mothers giving birth is 49 years. This implies that $B_{i,t}^{\overline{gin}}$ is potentially positive only for $i \in [28, 60]$.

4.3.3 Total non-interest income per adult

We may now write the total non-yield income per adult of a household aged b at time t, $y_{b,t}$, as below,

$$
\begin{aligned}
y_{b,t} &= \frac{N_{b,t}^{FW}}{N_{b,t}} \left[(1-t_t^w) \left[\left(w_{F,b,t} \left(1 - \vartheta_t^{LP} - \vartheta_{F,b,t}^{PP}\right) - t_t^{ATP1} \right) \left(1 - t_t^l - t_t^{ATP2}\right) \right] \ell_{b,t}^F \right. \\
&\left. + \left(1 - t_t^b\right) \left(b_t - \kappa_t t_t^{ATP1}\right) \left(\overline{\ell} - \ell_{b,t}^F\right) \right] \\
&+ \frac{N_{b,t}^{MW}}{N_{b,t}} \left[(1-t_t^w) \left[\left(w_{M,b,t} \left(1 - \vartheta_t^{LP} - \vartheta_{M,b,t}^{PP}\right) - t_t^{ATP1} \right) \left(1 - t_t^l - t_t^{ATP2}\right) \right] \ell_{b,t}^M \right. \\
&\left. + \left(1 - t_t^b\right) \left(\left(b_t - \kappa_t t_t^{ATP1}\right) \left(\overline{\ell} - \ell_{b,t}^M\right) \right) \right] \\
&+ \frac{N_{b,t}^F}{N_{b,t}} \left[TR_{b,t}^F + \left(1 - t_t^{TRT}\right) TRT_{b,t}^F \right] + \frac{N_{b,t}^M}{N_{b,t}} \left[TR_{b,t}^M + \left(1 - t_t^{TRT}\right) TRT_{b,t}^M \right] \\
&+ \left[\left(1 - t_t^G\right) f_{b,t}^G + \left(1 - t_t^{PP}\right) \left(f_{b,t}^{\overline{LD}} + f_{b,t}^{\overline{PP}} \right) + \left(1 - t_t^{RP}\right) \left(f_{b,t}^{\overline{CSP}} + f_{b,t}^{\overline{ATP_1}} + f_{b,t}^{\overline{ATP_2}} \right) \right] \\
&+ \frac{N_{b,t}^F}{N_{b,t}} \left[\left(1 - t_t^{RP}\right) \hat{f}_{b,t}^{FRP} + \left(1 - t_t^{DP}\right) \hat{\hat{f}}_{b,t}^{\widehat{FDP}} + \left(1 - t_t^{SP}\right) \hat{\hat{f}}_{b,t}^{\widehat{FSP}} \right] \\
&+ \frac{N_{b,t}^M}{N_{b,t}} \left[\left(1 - t_t^{RP}\right) \hat{f}_{b,t}^{MRP} + \left(1 - t_t^{DP}\right) \hat{\hat{f}}_{b,t}^{\widehat{MDP}} + \left(1 - t_t^{SP}\right) \hat{\hat{f}}_{b,t}^{\widehat{MSP}} \right] \\
&+ \frac{N_{b,t}^F}{N_{b,t}} B_{b,t}^{F\overline{in}} + \frac{N_{b,t}^M}{N_{b,t}} B_{b,t}^{M\overline{in}} + \tau^{p,h} + \tau_t^W + \tau_t
\end{aligned} \tag{4.18}
$$

The equation (4.18) has the following contents: The first line measures the average wage income after tax and contributions to pension schemes from females in the generation, where the fraction $\frac{N_{b,t}^{FW}}{N_{b,t}}$ is the share of females in the generation of households. The second line measures the (average) income after tax from the unemployment benefit system, the third and the fourth line measures the same two income sources for men in the generation. Line five is the total income after tax from age dependent transfers to the household. Line six consist of income from the social pay-as-you-go pension schemes, the after tax value of withdrawals from private pension schemes and the LD pension fund, and finally the after tax value of the average income from civil servants' pensions and average pension receipts from the ATP pension fund. Line seven and eight are the average after tax income from various types of pension receipts the labor market pension fund for women and men respectively. Finally the last line is the total inheritage per adult of the generation of household. To this we add the two lump sum transfers, τ_t^W (transfers from abroad) and τ_t (transfers from the public sector), both of which are defined per adult.

4.3.4 Indexation rules

The social security pensions, the unemployment benefits and the age specific transfers are forecasted endogenously in DREAM. According to Danish rules transfers per individual are indexed to the level of wages net of the labor market contribution tax and contribution to labor market pensions with a lag of approximately two years. In DREAM the lag is ignored and the pre-tax social security pension is given as

$$f_t^G = \varphi^G W_t \bar{\ell} \left(1 - \vartheta_t^{LP} - t_t^{ATP1}\right)\left(1 - t_t^{\ell} - t_t^{ATP2}\right) \tag{4.19}$$

where t_t^{ℓ} is the labor market contribution tax rate. W_t is the (productivity corrected) wage defined in (4.8)

The "replacement" parameter φ^G is determined so that the total spending to public pensions in the initial period is equal to the value of total spending on pensions in 1995

$$\varphi^G = \frac{f_{1995}^G}{W_{1995}\bar{\ell}\left(1 - \vartheta_t^{LP} - t_t^{ATP1}\right)\left(1 - t_t^{\ell} - t_t^{ATP2}\right)}$$

Similarly nominal unemployment benefits (measured per hour) are indexed according to the rule

$$b_t = \varphi^b W_t \left(1 - \vartheta_t^{LP} - t_t^{ATP1}\right)\left(1 - t_t^{\ell} - t_t^{ATP2}\right) \tag{4.20}$$

where φ^b is defined as

$$\varphi^b = \frac{b_{1995}}{W_{1995}\left(1 - \vartheta_t^{LP} - t_t^{ATP1}\right)\left(1 - t_t^{\ell} - t_t^{ATP2}\right)}$$

where

$$b_{1995} = \frac{\overline{UB}_{1995}}{\sum_{g\in\{F,M\}}\sum_{b=17}^{61}\left(\bar{\ell} - \ell_{b,t}^g\right)\varkappa_{b,t}^{gW} N_{b,t}^g}$$

$\overline{UB}_{1995}$ is the total spending on unemployment benefits in 1995, and the numerator is the total amount of hours spend unemployed in 1995.

Finally age specific transfers are indexed according to the same rule. Hence tax free age specific transfers are given as

$$TR_{b,t}^g = \varphi_b^g W_t \left(1 - \vartheta_t^{LP} - t_t^{ATP1}\right)\left(1 - t_t^{\ell} - t_t^{ATP2}\right) \quad g = F, M \tag{4.21}$$

and the taxable age specific transfers are indexed analogously

$$TRT^{g}_{b,t} = \varphi^{gT}_{b} W_t \left(1 - \vartheta^{LP}_{t} - t^{ATP1}_{t}\right)\left(1 - t^{\ell}_{t} - t^{ATP2}_{t}\right) \quad g = F,\, M \tag{4.22}$$

where the age and gender specific parameters φ^{g}_{b}, φ^{gT}_{b} are defined using a similar procedure as in case of e.g. φ^{G}. The revenues used for calculating φ^{g}_{b}, φ^{gT}_{b} are obtained from the generational accounting system of the Danish Ministry of Economic Affairs.

4.4 The result of the household's optimization

The combined assumption of perfect capital markets and perfect foresight implies that the consumption of a household in DREAM is a function of the sum of the stock of financial assets, $a_{b-1,t-1}$ (net of assets held in pension funds and private pension schemes) and the level of human capital, $H_{b-1,t-1}$. As we assume that the intertemporal elasticity of substitution is less than one, the consumption function may be written as a standard CES function in current prices and an index of future prices, $\eta_{b,t}$, where b is the age of the household at time t. Future prices are age specific as the household faces a finite time horizon and therefore the vector of relevant future prices depend upon the remaining life time of the household.[5]

$$C_{b,t} = \xi^{S}_{b} \left(\frac{1+\theta}{\left(1 + r_t\left(1 - t^{r}_{t}\right)\right) N^{EF}_{b-1,t-1}}\right)^{-S} \left(\frac{P_t}{\eta_{b-1,t-1}}\right)^{-S} \frac{a_{b-1,t-1} + H_{b-1,t-1}}{\eta_{b-1,t-1}} + Z_{b,t}, \quad 17 \leq b \leq 77$$

where $\eta_{b-1,t-1}$ is the index of future consumption prices for generation b and $a_{b-1,t-1}$ is the level of non-human wealth, while $H_{b-1,t-1}$ is the level of human capital. $Z_{b,t}$ is the disutility of work.

Given the specified utility function, the labor supply decision that is identical for all generations and both genders, and is given by

$$\ell_t = \left(\frac{\left(1 - t^{w}_{t}\right) \frac{w_{g,b,t}\left(1 - t^{\ell}_{t}\right)}{\rho^{g}_{b}} - \left(1 - t^{b}_{t}\right) b_t}{\gamma_1 P_t}\right)^{\gamma} \tag{4.23}$$

where γ is the labor supply elasticity, and γ_1 is a level parameter for the disutility of labor supply.

[5] For a derivation of the demand for the aggregate consumption index and the labor supply function, see Knudsen *et al.*(1998a).

Observe that due to the assumption of additivity in the utility function the labor supply is a simple function of the difference between the wage rate net of tax and unemployment benefits (per hour) after tax. [6]

Observe also that this very simple labor supply function has the property that if there is a fixed replacement level after tax (i.e. after tax unemployment benefits are increased by the same (absolute) amount as the real after tax wages), then complete real wage flexibility with respect to the labor tax rate prevails. On the other hand if unemployment benefits are indexed to e.g. the wage level (as in the present case), then real wage resistance is the outcome and a decrease in the tax rate of employed persons will increase employment. These effects are the typical outcome of models with imperfect competition in the labor market, irrespectively of whether models of bargaining, search or efficiency wages are employed. (See e.g. Pissarides (1998) for a survey of these models). Therefore one should not expect the qualitative effects to differ from these kinds of models.

4.5 Very old persons

The persons who survive the time horizon of the household, i.e. reaches the age of 77 years, are called very old persons. These persons receive various types of pensions and in each period they consume the after tax value of the pensions.

The total number of very old persons is defined as

$$N_t^O = \sum_{b=77}^{\tilde{B}} N_{b,t}^F + \sum_{a=77}^{\tilde{A}} N_{a,t}^M$$

where N_t^O is the total number of very old persons. $\tilde{B}$ is the maximum age of a female, such that $\sum_{b=77}^{\tilde{B}} N_{b,t}^F$ is the sum of females who are older than 77 years at period t. Similarly $\tilde{A}$ is the maximum age of a male, such that $\sum_{a=77}^{\tilde{A}} N_{a,t}^M$ is the sum of males who are at least 77 years old at period t. In the following we fix both $\tilde{B}$ and $\tilde{A}$ to 101 years.

[6] In fact this is not completely correct. The result would appear if the after tax yield on pension saving were identical to the after tax interest rate faced by the agent. In this case the agent can ignore the contributions to the pension schemes, since she can borrow a similar amount against the saving. However, the fact that pension savings obtain a higher yield after tax implies that the agent can borrow an amount that is a bit larger than the savings against the savings. Therefore there should theoretically be a small positive effect on the labour supply.

Total income of this group is given as

$$
\begin{aligned}
N_t^O y_t^O \; = \; & \sum_{b=77}^{101} N_{b,t}^F \left[TR_{b,t}^F + \left(1 - t_t^{TRT}\right) TRT_{b,t}^F \right] + N_{b,t}^M \left[TR_{b,t}^M + \left(1 - t_t^{TRT}\right) TRT_{b,t}^M \right] \\
& + \sum_{b=77}^{101} N_{b,t} \left[\left(1 - t_t^G\right) f_{b,t}^G + \left(1 - t_t^{PP}\right) \left(f_{b,t}^{\overline{LD}} + f_{b,t}^{PP} \right) + \left(1 - t_t^{RP}\right) \left(f_{b,t}^{\overline{CSP}} + f_{b,t}^{\overline{ATP_1}} \right) \right] \\
& + \sum_{b=77}^{101} N_{b,t}^F \left[\left(1 - t_t^{RP}\right) \hat{f}_{b,t}^{FRP} + \left(1 - t_t^{DP}\right) \widehat{\hat{f}_{b,t}^{FDP}} + \left(1 - t_t^{SP}\right) \widehat{\hat{f}_{b,t}^{FSP}} \right] \\
& + \sum_{b=77}^{101} N_{b,t}^M \left[\left(1 - t_t^{RP}\right) \hat{f}_{b,t}^{MRP} + \left(1 - t_t^{DP}\right) \widehat{\hat{f}_{b,t}^{MDP}} + \left(1 - t_t^{SP}\right) \widehat{\hat{f}_{b,t}^{M\overline{SP}}} \right]
\end{aligned}
$$

where the first line is the total of all age dependent transfers net of tax to females and males of this age-group. The second line is the sum of social pensions after tax, withdrawals after tax from the LD pension fund and the private pension schemes and finally the average pension receipts after tax from civil servants' pension and the ATP pension based on employment dependent contributions. Line three and four are the average after tax income from various types of pension receipts from the labor market pension fund for women and men respectively.

Observe that part of the pensions from the labor market pension system is paid to this group. This affects the savings behavior of the younger households. The reason is the following: When the households make decisions about the optimal consumption path over the life cycle, this optimal plan is made for the for the part of life time consumption that is prior to the age of 77 years. Since the labor market pension system allocates savings to the period after the age of 77 years this part of the labor market pension savings cannot substitute private savings within the optimization horizon of the household. Therefore the household considers this part of the labor market savings as forced savings (or taxes). Therefore the households total savings are affected by the labor market pension schemes by a kind of an income effect in addition to the effects that follow from the special tax treatment of pension savings and the overlapping generation structure of the model.

It is assumed that very old persons do not save and therefore spend the total income in each period.[7] On the other hand it is assumed that the very old persons optimize

[7] Note that part of the "income" of the very old persons is receipts from the pension fund. In a standard definition of income this is not entirely income but includes a reduction in the stock of assets that the pension fund has allocated to the financing of pension to these generations of members. Therefore very old persons have a negative aggregate savings rate in the standard definition.

intra-temporally given a nested CES sub-utility function, which is identical to the sub-utility function of the household. In each period the demand per very old person for the publicly produced good, $C_{o,t}^{G}$ is therefore given by

$$C_{o,t}^{G} = (\mu_{CG})^{\sigma_C} \left(\frac{\left(1+t_t^{VAT}\right)\left(p_t^G + t_t^{CG}\right)}{P_t} \right)^{-\sigma_C} \frac{y_{o,t}}{P_t}$$

where σ_C is the elasticity of substitution between the publicly produced good and the index of privately produced goods. P_t is the consumer price index, p_t^G is the producer price of the publicly produced good, t_t^{CG} is a quantity excise tax on the publicly produced good, t_t^{VAT} is the VAT rate. μ_{CG} is a weight parameter.

Similarly the demand per very old person of the two privately produced goods are given by the following standard demand expressions of nested CES utility functions

$$C_{o,t}^{PD} = (\mu_{CPD})^{\sigma_{CP}} \left(\frac{p_t}{p_t^{CP}} \right)^{-\sigma_{CP}} (\mu_{CP})^{\sigma_C} \left(\frac{\left(1+t_t^{VAT}\right)\left(p_t^{CP} + t_t^{CP}\right)}{P_t} \right)^{-\sigma_C} \frac{y_{o,t}}{P_t}$$

$$C_{o,t}^{PF} = (\mu_{CPF})^{\sigma_{CP}} \left(\frac{1}{p_t^{CP}} \right)^{-\sigma_{CP}} (\mu_{CP})^{\sigma_C} \left(\frac{\left(1+t_t^{VAT}\right)\left(p_t^{CP} + t_t^{CP}\right)}{P_t} \right)^{-\sigma_C} \frac{y_{o,t}}{P_t}$$

where $C_{o,t}^{PD}$, $C_{o,t}^{PF}$ is the demand for the domestic respectively foreign privately produced good. σ_{CP} is the elasticity of substitution between the domestic and foreign privately produced good. p_t the producer price of the domestic privately produced good, p_t^{CP} is the price index of privately produced goods, t_t^{CP} is a quantity excise tax on the privately produced goods. μ_{CPF}, μ_{CPD} are weight parameters.

Chapter 5
THE MACRO ECONOMIC VARIABLES

The description in the two previous subsections constitutes the behavior of the supply and the demand side in the model. The present section aggregates these effects. We start by the equation for the total accumulation in the economy. This is followed by an aggregation over households and finally a presentation of the foreign and the public sector.

5.1 Total accumulation

Total stock of wealth in the economy, A_t^H is given by sum of financial assets in private pension arrangements, A_t^{PP}, labor market pension funds, A_t^{LP}, and private households, A_t^H

$$A_t = A_t^{PP} + A_t^{LP} + A_t^H = A_t^P + A^H$$

where

$$A_t^H = \frac{(1+\omega)}{\omega} V_t \sum_{b=18}^{77} n_{b,t}^v N_{b,t}^{EF}$$

As the price of shares is determined by the arbitrage condition of the pension funds, it follows that the funds are indifferent between holding bonds and equity. Therefore pension funds (including private pensions arrangements) always hold the stock of shares not held by individuals

$$n_t^z = 1 - \sum_{b=18}^{77} n_{b,t}^v N_{b,t}^{EF}$$

where n_t^z is the number (or equivalently the faction) of shares held by the pension funds.

$$n_t^z V_t = \left(1 - \sum_{b=18}^{77} n_{b,t}^v N_{b,t}^{EF}\right) V_t = V_t - \frac{\omega}{1+\omega} A_t^H$$

Observe that V_t is a net value of firms, where it is assumed that all shares are held by households or pension funds (or foreign citizens). However, a part of the total stock of shares is held by other incorporated firms. The value of these firms therefore in part reflect the ownership of shares in other companies. Adding the value of the shares in the economy therefore becomes larger than V_t. As there is no empirical measure of the value of shares held by firms exists, it is hard to measure V_t empirically. As a consequence the (initial) value is determined endogenously in the calibration procedure.

5.2 Aggregation across generations

The total aggregate demand for goods and supply of labor is found by adding together the demand and supply functions of the representative households of all generations alive, at a given moment in time. This is done for all periods of time t. For each generation of households, the level of consumption per adult-equivalent is weighted by the number of adult-equivalents in the generation of households in question. The consumption of the very old persons (who have survived the planning horizon of the household) must be added to obtain total consumption. The same procedure is applied with respect to the labor supply, except for the fact that in this case the labor supply per adult of a generation of households is weighted by the number of adults as children are assumed not to supply labor, and that the labor supply of the very old persons is zero.

The level of aggregate demand and supply of the different goods are defined below as

$$C_t^P = \sum_{b=17}^{76} N_{b,t}^{EF} C_{b,t}^P + N_t^O C_{o,t}^P \quad \text{for all } t \tag{5.1}$$

where C_t^P is the total domestic consumption demand for the privately produced good

$$C_t^G = \sum_{b=17}^{76} N_{b,t}^{EF} C_{b,t}^G + N_t^O C_{o,t}^G \quad \text{for all } t \tag{5.2}$$

where C_t^G is the total domestic consumption demand for the governmentally produced good

$$C_t^{PD} = \sum_{b=17}^{76} N_{b,t}^{EF} C_{b,t}^{PD} + N_t^O C_{o,t}^{PD} \quad \text{for all } t \tag{5.3}$$

where C_t^{PD} is the total domestic consumption demand for the privately produced domestic good

$$C_t^{PF} = \sum_{b=17}^{76} N_{b,t}^{EF} C_{b,t}^{PF} + N_t^O C_{o,t}^{PF} \quad \text{for all } t \tag{5.4}$$

where C_t^{PF} is the total domestic consumption demand for the privately produced foreign good

$$L_t^s = \sum_{b=17}^{61} N_{b,t}^{FW} \ell_t + \sum_{a=17}^{61} N_{a,t}^{MW} \ell_t = \ell_t N_t^W \quad \text{for all } t \tag{5.5}$$

where L_t^s is the total labor supply measured in hours and

$$N_t^W = \left(\sum_{b=17}^{61} N_{b,t}^{FW} + \sum_{a=17}^{61} N_{a,t}^{MW} \right) \tag{5.6}$$

is the number of persons in the work force.

$$A_t^H = \sum_{b=17}^{76} N_{b,t}^{EF} a_{b,t} \quad \text{for all } t \tag{5.7}$$

where A_t is the total stock of non-human wealth held by domestic consumers

$$\overline{TR}_t = \sum_{b=17}^{101} N_{b,t}^F \left(TR_{b,t}^F + TRT_{b,t}^F \right) + \sum_{a=17}^{101} N_{a,t}^M \left(TR_{a,t}^M + TRT_{a,t}^M \right) \quad \text{for all } t \tag{5.8}$$

where $\overline{TR}_t$ is the sum of age specific transfers

$$\overline{UB}_t = b_t \left(\sum_{a=17}^{61} N_{a,t}^{MW} \left(\overline{\ell} - \ell_t \right) + \sum_{b=17}^{61} N_{b,t}^{FW} \left(\overline{\ell} - \ell_t \right) \right) = b_t \left(\overline{\ell} - \ell_t \right) N_t^W \quad \text{for all } t \tag{5.9}$$

where $\overline{UB}_t$ is total gross expenditures on unemployment benefits

$$\overline{FP}_t = f_t^P \left(\sum_{a=62}^{76} N_{a,t}^M + \sum_{b=62}^{76} N_{b,t}^F + N_t^O \right) \quad \text{for all } t \tag{5.10}$$

where $\overline{FP}_t$ is the total gross expenditure on social security pensions including early retirement benefits.

$$\overline{TW}_t = \sum_{b=17}^{76} N_{b,t}^{AF} \tau_t^W \quad \text{for all } t \tag{5.11}$$

where $\overline{TW}_t$ is the total transfers to households from abroad

$$\Upsilon_t = \sum_{b=17}^{76} N_{b,t}^{AF} \tau_t \quad \text{for all } t \tag{5.12}$$

where Υ_t is the total value of the lump sum transfer to the consumers.

5.3 The foreign sector

The domestic economy is integrated in the world economy, through trade and capital flows. As to financial capital, we assume that domestic corporate and government bonds are perfect substitutes for foreign bonds in a perfect world bond market. Residence based taxation of interest income, implies that the domestic pre-tax interest rate is equal to the foreign pre-tax interest rate, which is assumed to be fixed (through time) in units of the foreign good.

The accumulation of domestic claims on the rest of the world, is determined by the current account

$$F_t = (1 + r_t) F_{t-1} + \overline{TW}_t + \overline{TWG}_t + \sum_{i=P,G} p_t^i \left(1 + t_t^{iX}\right) X_t^i \tag{5.13}$$

$$-\frac{1}{1+t_t^t}\left(C_t^{PF} + \sum_{i=P,G}\left(M_t^{iPF} + I_t^{iF}\right)\right) \tag{5.14}$$

where F_t is the domestic net stock of foreign assets, $\overline{TWG}_t$ is total transfers to the domestic government from abroad, X_t^i is exports from production sector i, t_t^{iX} is the associated export tax (normally negative, i.e. an export subsidy). Total import consists of imported consumer goods, C_t^{PF}, imported materials to the production sectors, M_t^{iPF}, and imported investment goods to the production sectors, I_t^{iF}. t_t^t is the tariff. For convenience we assume that an identical tariff applies to all types of goods. Since by definition the foreign price inclusive tariff is equal to 1, then foreign price net of tariff must be $\frac{1}{1+t_t^t}$.

Alternatively, the stock of foreign assets may be expressed as the difference between the stock of total private non-human wealth, A_t on the one hand and sum of the value of assets issued by domestic firms and the public debt on the other hand

$$F_t = A_t - (V_t + B_t^c + B_t^g) \quad (5.15)$$

$$= A_t^H + A_t^P - (V_t + B_t^c + B_t^g) \quad (5.16)$$

where V_t is the value of shares in domestic firms, B_t^c is domestic corporate debt, and B_t^g is domestic public bonds.

(5.15) defines the balance between all the households assets. For most variables the model is only solved for $t \geq 1$. However for some variables including the value of the firm, the model is also solved for $t = 0$. It is therefore likely that the value of the firm at the end of period 0, V_0, changes compared with the calibrated value at the same point in time, V_{t_0}. It is assumed that no other item of households' assets change at the end of period 0, i.e. according to (5.15) the sum of households' assets, A_0, must change by the amount $V_0 - V_{t_0}$. Furthermore, it is assumed that the asset holdings of all generations change by an equal relative amount. The relation between the model solution value of generation b's assets at the end of period 0, $a_{b,0}$, and the calibrated (initial) value for the same measure, a_{b,t_0}, can therefore be determined using (5.7) as

$$a_{b,0} = a_{b,t_0}\left(1 + \frac{V_0 - V_{t_0}}{\sum_{b=18}^{77} N_{b,t}^{EF} a_{b,t_0}}\right)$$

Although the economy is small in the sense that it does not affect the world market interest rate, we assume that the world production of the good produced in the domestic country **is affected** by the level of production in the domestic country. Thus in this respect the country is not small in its own output market. This implies that the economy has endogenous terms of trade with the rest of the world.

Only the private sector supplies exports to the world market. The foreign demand for the privately produced domestic good, can be thought of as demand functions derived from intertemporal optimization of foreigners. For simplicity it is assumed that the foreign demand functions for the domestic goods are isoelastic, and that the positions of the demand curves are fixed through time

$$X_t^i = \chi^i \left(\left(1 + t_t^{iX}\right) p_t^i\right)^{-\varepsilon^i} \quad i = P, G \quad (5.17)$$

where $\chi^i > 0$ are constants, $\varepsilon^i > 0$ is the price elasticity in the export demand.

As is well known from classical trade theory, the introduction of endogenous terms of trade implies that there on welfare grounds exists a positive optimal tariff. In the present version of the model any policy that introduces a marginal cut back on domestic production will generate a positive terms of trade effect that *ceteris paribus* tends to increase the utility of the representative consumer in the domestic economy at the expense of consumers abroad. Thus the endogenous terms of trade introduces the possibility of using "beggar thy neighbor policies" to increase domestic welfare.

5.4 The consumption and the budget of the public sector

Besides being a producer, the (main) role of the government is to collect taxes, to finance unemployment benefits, social pensions, other personal transfers such as family allowance transfers, and public consumption which is government purchases of goods and services supplied solely from the governmental production sector. Notice that this public consumption does not explicitly enter the utility function of the agents. One interpretation of this is, that the utility function of the agents has an implicit additively separable argument containing the public consumption, thus the amount of public consumption does not affect the marginal utility of private consumption and vice versa. This specification of government consumption in the utility function (i.e. it is omitted) implies that the model is ill-suited to analyze welfare effects of changes in the size of government consumption *per se*[1].

Government expenditures to transfers to the household are divided into the following categories: Unemployment benefits, $\overline{UB}_t$, the sum of early retirement and social pensions, $\overline{FP}_t$, and $ATP-$, and LD pensions, $\overline{ATP}_t$, $\overline{LD}_t$ (recall that these two labor market pension funds are defined as part of the public sector. The rest of the transfers are defined as age- and gender specific transfers. The distribution of the various transfers on age and gender group follows the generational accounting set up by EPRU and the Ministry of Finance. The age and gender specific variable are calibrated so that the total expenditure in 1995 equals the recorded expenditures to the transfers in question. In the projection period it is assumed that these age and gender specific transfers are indexed to the wage rate net of labor market contribution tax and contributions to labor market pension schemes. Therefore total expenditures

[1] This would yield a rather peculiar policy finding, since utility is inversely correlated with the size of government purchases - therefore diminishing government expenditures will unambiguously increase welfare. This problem can be solved by altering the utility function to take government expenditures into account. However, this method is not used here, since it would give cause to an even larger problem when calibrating the model; how to determine the weight to place on the expenditures of the public sector?

to income transfers is affected both by the demographic evolution and the domestic wage level.

As this projection is concerned with the effects of demographic evolution we have chosen to consider public services as age and gender specific as well. The idea is that this could be taken to imply that the level of public service per person is kept constant in the projection period. The part of the public service that is considered age specific is the following: Education, Cultural affairs, Hospitals, and Health care services. The procedure that distribute the total expenditures to expenditures per person in a specific age-group is similar to one described in the case of income transfers above. The distribution on age-groups follows Ministry of Economic Affairs (1997).

Finally the part of the value of public expenditures that are not distributed to age groups are assumed to constitute a fixed fraction of GDP.

The government faces an intertemporal budget constraint (the so-called No Ponzi Game Condition), which implies that the discounted value of government debt has to converge to zero for time approaching infinity. This means, that in the long run the increase in the public debt must be less than the interest payments on the debt.

The consolidated budget constraint of the government is derived by using the savings identity and the No Ponzi Game condition. This yields the result that the present discounted value of the stream of tax revenues must be greater than or equal to the sum of the public debt and the present discounted value of the stream of government expenditures

$$\sum_{s=t}^{\infty} \frac{\overline{tax}_s}{\prod_{v=t+1}^{s}(1+r_t)} = \sum_{s=t}^{\infty} \frac{\hat{G}_s}{\prod_{v=t+1}^{s}(1+r_t)} + B_t^g \tag{5.18}$$

where $\overline{tax}_s$ is the total tax revenue at time s, $\hat{G}_s$ is the total public expenditures net of interest payments at time s, B_t^g is the stock of outstanding public debt at time t.

Violating this means, that private agents are not willing to hold government bonds. Consistency thus requires, that a policy reaction function is assumed to fulfill the intertemporal budget constraint.

A simple policy reaction rule, is to assume that the government runs a balance budget in each period and uses the income tax rate as the endogenous instrument. The government budget constraint then becomes

$$
\begin{aligned}
0 &= -r_t B^g_{t-1} - \overline{GR}_t - \hat{g}\hat{Y}_t - p^{GI}_t I^G_t - \overline{TR}_t - \overline{UB}_t - \overline{FP}_t - \overline{ATP1}_t - \overline{ATP2}_t - \overline{LD}_t \\
&\quad -\overline{FCSP}_t + \overline{TWG}_t + \overline{tax}_t + p^G_t \overline{MPK}^G_t K^G_{t-1} - \Upsilon_t
\end{aligned}
\tag{5.19}
$$

where $\overline{GR}_t$ is age distributed public service expenditures, $\hat{g}\hat{Y}_t$ is the non-age-distributed part of public consumption. $0 < \hat{g} < 1$ is a constant fraction and

$$
\hat{Y}_t = \sum_{i=P,G} p^i_t Y^i_t - p^{iM}_t M^i_t \tag{5.20}
$$

is nominal GDP. $p^G_t \overline{MPK}^G_t K^G_{t-1}$ is the total reward to the governmental capital stock. Υ_t is a constant lump sum tax that is used to correct for differences between actual revenue and the endogenously generated revenue in the initial period. $\overline{tax}_t$ is the tax revenue from distortionary taxes in the economy. $\overline{tax}_t$ is the variable that closes the budget by having endogenous tax rates of the income tax system. We define the tax revenues from each distortionary tax separately (and valid for all t)

5.4.1 Tax revenues

Observe that all tax revenues appear without having to keep track of the procurement of shares for each age group.

$$
\begin{aligned}
\overline{LT}_t &= t^W_t \sum_{g\in\{F,M\}} \sum_{b=17}^{61} \left[\left(1 - t^\ell_t - t^{ATP2}_t\right) \left(\left(1 - \vartheta^{PP}_{g,b,t} - \vartheta^{LP}_t\right) w_{g,b,t} - t^{ATP1}_t \right) \ell^g_{b,t} N^{gW}_{b,t} \right] \\
&\quad + t^\ell_t \sum_{g\in\{F,M\}} \sum_{b=17}^{61} \left(w_{g,b,t} - t^{ATP1}_t \right) \ell^g_{b,t} N^{gW}_{b,t}
\end{aligned}
\tag{5.21}
$$

$$
\overline{TATP}_t = \sum_{g\in\{F,M\}} \sum_{b=17}^{61} \left[t^{ATP1}_t \ell^g_{b,t} + t^{ATP2}_t \left(w_{g,b,t} - t^{ATP1}_t \right) \ell^g_{b,t} + \kappa_t t^{ATP1}_t \left(\bar{\ell} - \ell^g_{b,t} \right) \right] N^{gW}_{b,t} \tag{5.22}
$$

$$
\overline{BT}_t = t^b_t \left(b_t - \kappa_t t^{ATP1} \right) \left(\bar{\ell} - \ell_t \right) N^W_t \tag{5.23}
$$

$$
\overline{TTRT}_t = t^{TRT}_t \left(\sum_{b=17}^{101} N^F_{b,t} TRT^F_{b,t} + \sum_{a=17}^{101} N^M_{a,t} TRT^M_{a,t} \right) \tag{5.24}
$$

$$
\overline{PPT}_t = t^{PP}_t \sum_{b=62}^{72} N_{b,t} \left(f^{PP}_t + f^{LD}_t \right) \tag{5.25}
$$

$$
\overline{LPT}_t = t^{RP}_t \sum_{g\in\{F,M\}} \sum_{b=67}^{101} N^g_{b,t} \left(f^{g\overline{RP}}_{b,t} + f^{\overline{ATP_1}}_t + f^{\overline{ATP_2}}_t + f^{\overline{CSP}}_t \right) \tag{5.26}
$$

$$\overline{DPT}_t = t_t^{DP} \sum_{g\in\{F,M\}} \sum_{b=27}^{101} N_{b,t}^{g} \widehat{\hat{f}}_{b,t}^{gDP} \tag{5.27}$$

$$\overline{SPT}_t = t_t^{SP} \sum_{g\in\{F,M\}} \sum_{b=27}^{101} N_{b,t}^{g} \widehat{\hat{f}}_{b,t}^{gSP} \tag{5.28}$$

$$\overline{FGT}_t = t_t^{G} f_t^{G} \sum_{g\in\{F,M\}} \sum_{b=62}^{101} N_{b,t}^{g} \tag{5.29}$$

$$\overline{ET}_t = t_t^{a} \sum_{g\in\{F,M\}} \sum_{b=17}^{61} w_{g,b,t} \ell_{b,t} N_{b,t}^{gW} \tag{5.30}$$

$$\overline{VAT}_t = t^{VAT} \left(\left(p_t^{CP} + t_t^{CP} \right) C_t^{P} + \left(p_t^{G} + t_t^{CG} \right) C_t^{G} \right) \tag{5.31}$$

$$\overline{RC}_t = t_t^{CP} C_t^{P} + t_t^{CG} C_t^{G} \tag{5.32}$$

$$\overline{RT}_t^{h} = t_t^{r} r_t \left(\frac{1}{1+\omega} \right) A_{t-1}^{h} \tag{5.33}$$

$$\overline{RT}_t^{z} = t_t^{rz} r_t \left(A_{t-1}^{z} - n_{t-1}^{vz} V_{t-1} \right) \tag{5.34}$$

$$\overline{RT}_t = RT_t^{h} + RT_t^{z} \tag{5.35}$$

$$\overline{DT}_t^{h} = t^{d} D_t \frac{\omega}{1+\omega} \frac{A_{t-1}^{h}}{V_{t-1}} \tag{5.36}$$

$$\overline{DT}_t^{z} = t^{dz} n_t^{vz} D_t \tag{5.37}$$

$$\overline{DT}_t = \overline{DT}_t^{h} + \overline{DT}_t^{z} \tag{5.38}$$

$$\overline{CT}_t = t_t^{c} \left[p_t^{P} Y_t^{P} - \left(1 + t_t^{a}\right) W_t \ell_t N_t^{W} - p_t^{PM} M_t^{P} - \hat{\delta} \hat{K}_{t-1}^{P} - r_t g p_{t-1}^{PI} K_{t-1}^{P} \right] \tag{5.39}$$

$$\overline{GT}_t^{h} = \begin{cases} t_t^{g} \left(V_t - V_{t-1} \right) \frac{\omega}{1+\omega} \frac{A_{t-1}^{h}}{V_{t-1}} + t_0^{g} \left(V_0 - V_{t_0} \right) \frac{\omega}{1+\omega} \frac{A_{t_0}^{h}}{V_{t_0}} & for\ t = 1 \\ t^{g} \left(V_t - V_{t-1} \right) \frac{\omega}{1+\omega} \frac{A_{t-1}^{h}}{V_{t-1}} & for\ t > 1 \end{cases} \tag{5.40}$$

$$\overline{GT}_t^{z} = \begin{cases} t_t^{gz} \left(V_t - V_{t-1} \right) n_{t-1}^{vz} + t_0^{gz} \left(V_0 - V_{t_0} \right) n_{t_0}^{vz} & for\ t = 1 \\ t_t^{gz} \left(V_t - V_{t-1} \right) n_{t-1}^{vz} & for\ t > 1 \end{cases} \tag{5.41}$$

$$\overline{GT}_t = \overline{GT}_t^{h} + \overline{GT}_t^{z} \tag{5.42}$$

$$\overline{MT}_t = \sum_{i=P,G} t_t^{iM} \left(p_t^{P} M_t^{iPD} + M_t^{iPF} + p_t^{G} M_t^{iG} \right) \tag{5.43}$$

$$\overline{IT}_t = \sum_{i=P,G} t_t^{iI} \left(p_t^{P} I_t^{iD} + I_t^{iF} \right) \tag{5.44}$$

$$\overline{XT}_t = \sum_{i=P,G} t_t^{iX} p_t^{i} X_t^{i} \tag{5.45}$$

$$\overline{TT}_t = t_t^{t} \frac{1}{1+t_t^{t}} \left[C_t^{PF} + \sum_{i=P,G} \left(M_t^{iPF} + I_t^{iF} \right) \right] \tag{5.46}$$

where LT_t is the tax-revenue from labor income tax on employed, BT_t is the tax-revenue from labor income tax on unemployed. These revenues are differentiated as employed persons are subject to the labor market contribution tax whereas unemployed are not. TRT_t is the tax-revenue from taxation of age specific transfers, PT_t is the tax-revenue from taxation of public pensions, ET_t is the tax-revenue from the payroll tax, VAT_t is the tax revenue from value added consumption taxes, RC_t is the tax revenue from consumption quantity taxes, RT_t is the tax-revenue from the tax on interest payments, DT_t is the tax-revenue from the taxation of dividends, CT_t is the tax-revenue from corporate taxation, GT_t is the tax-revenue from the capital gains tax, MT_t is the revenue from the tax on materials, IT_t is the revenue from the tax on investment, XT_t is the negative revenue from the export subsidies, and finally, TT_t is the domestic tariff-revenue.

The capital gains tax revenue, GT_t, is simply calculated as the tax rate, t_t^g, times the increase in the value of the firm, $V_t - V_{t-1}$, during period t for all periods except the first. For the first period we add a capital gains tax for period 0, if the value of the firm at the end of period 0, V_0, has changed compared with the calibrated value at the same point in time, V_{t_0}. This is because the model is not solved for the tax revenue (and most other variables) in period 0.[2]

The total tax revenue is defined as

$$\begin{aligned} \overline{tax}_t &= \overline{LT}_t + \overline{TATP}_t + \overline{BT}_t + \overline{TTRT}_t + \overline{PPT}_t + \overline{LPT}_t + \overline{DPT}_t \\ &\quad + \overline{SPT}_t + \overline{FGT}_t + \overline{ET}_t + \overline{VAT}_t + \overline{RC}_t + \overline{RT}_t + \overline{DT}_t \qquad (5.47) \\ &\quad + \overline{CT}_t + \overline{GT}_t + \overline{MT}_t + \overline{IT}_t + \overline{XT}_t + \overline{TT}_t \qquad (5.48) \end{aligned}$$

To balance the budget we consider the tax rate, t_t^w as an endogenous variable. Furthermore we apply the following rule to tax rates on non-interest income

$$t_t^i - t_{t-1}^i = t_t^w - t_{t-1}^w, \text{ for } \quad i = TRT,\ RT,\ DP,\ G,\ b$$

so that the absolute change in the tax rates of public age-distributed transfers, labor market pensions, labor market disablement pension, social security pension and unemployment benefits are equal to the absolute change in the tax rate of wage income.

[2] See chapter 7.7 on the solution of the model for an elaboration of this point.

5.5 Equilibrium conditions

To close the model, we need the market clearing conditions for the domestic goods markets and the market for labor. These are given by

$$Y_t^P = \sum_{i=P,G} \left(M_t^{iPD} + I_t^{iD} \right) + C_t^{PD} + X_t^P \tag{5.49}$$

$$Y_t^G = \sum_{i=P,G} M_t^{iG} + C_t^G + \frac{\hat{g}\hat{Y}}{p_t^G} + X_t^G \tag{5.50}$$

$$L_t^s = L_t^P + L_t^G \tag{5.51}$$

The computer program of the model is coded in the GAMS language, see Brooke, Kendrick &Meeraus (1988). GAMS allows the user to call a number of different solution algoritms for the numerical simulation. The present model is solved using the CONOPT2 solver, see Drud (1985, 1997).

Chapter 6
EXOGENOUS FORECAST DATA

The dynamic simulation of DREAM which is taken to represent the Danish economy in the present phase of demographic transition and the gradual growth of funded labor market pension schemes is based on exogenous forecast of four categories of variables. These are: 1) Demographic forecasts 2) Forecast of contributions to labor market pension schemes and ATP pension, as well as forecast of pensions to former civil servants and the LD pensions. 3) Forecasts of public expenditures 4) Forecasts of tax rates and policy response functions. In addition the calibration procedure implies that some central elasticities are considered exogenously given and constant through time

The following section presents these exogenous variables and parameters.

6.1 Population and labor force

As mentioned a modified version of the official forecast of Statistics Denmark from 1998 is used to forecast the demographic changes. The modification is that all immigrants are assumed to be 17 years old at the time of immigration. This implies that each immigrant spends a larger part of the lifetime in Denmark. To keep the stock of first generation immigrants in line with the official forecast, the number of immigrants in "17 years equivalents" has to be lower that the annual number of immigrants in the official forecasts.

The official forecast implies that the expected lifetime of all age group and of both gender are expected to increase annually until 2040 which is the time-horizon of the forecast. To be able to use the forecast in DREAM it is necessary to prolong the horizon beyond 2040. This is done by assuming that the expected lifetime for each age and gender group remains constant at the 2040 level.

Defining the demographic age burden as the number of persons between 62 years and

99 years divided by the population between 17 years and 61 years, table 6.1 below shows the evolution in the burden over time as well as the evolution in each of the two population groups separately. Finally the table shows the development in the labor force measured in number of persons. The labor force in persons is forecasted by assuming that the labor market participation rate for each 5 year age-group and for each gender remains at the 1995 level.

Year	Demographic age burden	Population 62-	Population 17-61	Labor force
1995	0.289	100.0	100.0	100.0
2000	0.289	101.5	101.5	100.9
2005	0.301	105.1	100.9	99.3
2010	0.346	118.5	99.0	97.8
2015	0.371	127.0	98.9	97.4
2020	0.394	134.3	98.5	96.7
2025	0.417	140.5	97.4	94.9
2030	0.456	153.7	94.6	93.0
2035	0.472	152.2	93.2	92.2
2040	0.470	151.4	93.1	93.1
2050	0.421	140.0	96.1	94.4
2075	0.430	140.9	94.7	94.9

Table 6.1: Measures of the exogenous demographic evolution

Observe that the increase in the demographic age burden is made up of an increase in the number of elderly by more that 50 per cent in the period until 2030 and simultaneously a reduction in the part of the population between 17 and 61 years by around 7 percent. The reduction in the labor force is almost 8 percent. These two parts have different impacts on the economy. The increase in the number of elderly increases the public expenditures and therefore the financing burden of the working generations. The reduction in the labor force is an anticipated negative supply shock to the economy.

6.2 Pension schemes

6.2.1 Labor market pension schemes

The contributions to the labor market pension schemes are assumed to be an age independent percentage of the pre-tax wage sum. This implies that the average contributions vary across the life cycle due to the fact that both the wage and the labor market participation rate differ across the life cycle. Contributions to the labor market pension scheme are initiated at the age of 22 years and continues until the age of 62 years, where the generation enters the early retirement scheme. This implies that any future generation has a contribution period of 40 years, which probably is a relatively long period.

Age	Men	Women
	Dkr. per head	
22-26	57200	37200
27-31	41000	34900
32-36	30000	27700
37-41	29900	29500
42-46	28800	28600
47-51	28300	27400
52-56	24500	24100
57-61	19600	19300
63-66	12100	14500

Table 6.2: Initial pension undertakings

Age	Pensions
	Dkr. per head
67-72	12800
73-76	10900
77-82	10700
83-86	10000
87-92	7700
93-96	5800

Table 6.3: The average initial pensions

Persons, who are older than 22 years in 1995 are assigned with an initial pension undertaking, which depends both on their past contributions and the current contribution which by the forecat procedure of the pension fund is assumed to remain fixed until the generation in question reaches 62 years. The initial pension undertakings are shown in table 6.2. Recall that pension undertaking (as well as other income variables) are measured as the average of the whole population of the given age and gender group, which therefore includes persons who are not in the labor force.

The initial pension undertakings are constructed from data collected by the Danish Economic Council for this analysis. From this material pension undertakings for mature and new pension funds are constructed separately and aggregated to yield the aggregate undertakings shown in table 6.2. Similarly the labor market pensions of current pensioners are initiated using the same data sources. The pensions are given in table 6.3

The initial accumulated stock of financial wealth in the labor market pension fund is based on official data from the authroities (Finanstilsynet) and is 363.5 billion Dkr. in 1995.

The contribution rate is defined as an average rate defined as the total contributions divided by the wage sum for the relevant age groups. Total contributions to labor market pension schemes (net of payroll tax or labor market contribution tax) was in 1995 Dkr. 26.401 mill. (Finanstilsynet, 1996). The wage sum of persons who are at least 22 years old is in DREAM given as Dkr. 486.566 mill. This yields an average contribution rate of 5,77 per cent (including the payroll or labor market contribution tax rate of 6 per cent in 1995).

The average contribution rate is forecasted given an assumption that the relatively new pension funds for blue collar workers in the private sector will gradually increase the contribution rate to 9 per cent in year 2010. In addition we assume that the coverage of labor market pension schemes is gradually increased as the number of jobs that entitles to civil servants pension is reduced and converted into jobs with standard labor market pension schemes. This latter effect is assumed to increase the contribution rate by 0.035 per cent every five years until 2040. The forecast of the average contribution rate for persons who are at least 22 year is shown in table 6.4 below:

Year	Average contribution rate
1995	5.77
2000	6.75
2005	7.62
2010	8.49
2015	8.52
2020	8.57
2025	8.60
2030	8.63
2035	8.67
2040	8.71
Henceforth	8.71

Table 6.4: Labor market pension contribution rate

The real rate of interest measured in the price of the foreign good is equal to 4 percent in the entire time horizon. By assumption the rate of inflation in foreign prices is 2 percent in the entire time horizon. As the Danish capital income tax rates are nominal, we convert the nominal rate into real tax rates using the following procedure: The arbitrage condition between shares and bonds measured in nominal terms is given by

$$i_t \left(1 - t_t^i\right) \tilde{V}_{t-1} = \left(1 - t_t^{\tilde{d}}\right) \tilde{D}_t + \left(1 + t_t^{\tilde{g}}\right) \left(\tilde{V}_t - \tilde{V}_{t-1}\right)$$

where i_t is the nominal rate of interest which is equal to 6.08 per cent, t_t^i is the tax rate on interest income for pension funds, which is set equal to 26 percent. $\tilde{V}_{t-1}$ is the nominal value of shares, $t_t^{\tilde{d}}$ is the tax rate on dividends to the pension funds, which is set equal to 5% and finally $t_t^{\tilde{g}}$ is the tax rate on (accrued) capital gain in the pension funds, which is set equal to 5%.[1]

In DREAM the arbitrage condition between shares and bonds are measured in units of the foreign good.

$$r \left(1 - t^r\right) V_{t-1} = \left(1 - t_t^d\right) D_t + \left(1 - t_t^g\right) \left(V_t - V_{t-1}\right)$$

For the two arbitrage conditions to be equivalent we need the following definitions of the tax rates in the model

$$t^d = t_t^{\tilde{d}} \left(1 + \pi\right) - \pi = 0.031$$

$$t^g = t_t^{\tilde{g}} \left(1 + \pi\right) - \pi = 0.031$$

$$t^r = \frac{i \left(1 - \left(1 + \pi\right) \left(1 - t_t^i\right)\right) - \pi t^g}{\left(i - \pi\right)} = 0.3502$$

This yields a real rate of interest after tax for the pension funds equal to

$$r \left(1 - t^r\right) = 0.026$$

Observe that the real rate of interest after tax is same irrespectible of whether the financial wealth takes the form of bonds or shares. This follows from the arbitrage condition. Therefore the 2.6 percent is (also) the average yield after tax for the pension fund.

[1] The tax rates used apply to the Danish economy from 1999. Observe that with these tax rates the assumed pre-tax nominal yield on shares is equal to 3.1 percent annually.

6.2.2 Private pension schemes

Private pension schemes are modelled as savings. The interest on these savings receive the same favorable tax treatment as interest income in pension funds. By assumption 40 percent of the savings is withdrawn as the person in question reaches the age of 67 years. The rest of the savings are withdrawn as the person reaches 72 years. This split is calibrated to yield the actual withdrawal in 1995. The initial stock of savings in 1995 is shown in table 6.5 below

Age	Pension savings
	Billion Dkr.
17-21	0.781
22-26	2.494
27-31	4.903
32-36	9.309
37-41	13.867
42-46	19.549
47-51	27.589
52-56	37.208
57-61	43.882
62-66	54.749
67-61	48.533

Table 6.5: Initial value of the stock of private pension savings

Observe that the stock of private pension savings is not distributed according to gender. This is due to the assumption that the couple matrix is diagonal so that men are married to women of their own age. This implies that it is the same household that receive the withdrawal irrespectively of the private pension savings belong to a woman or a man.

Age	Men	Women
17-21	0.00308	0.00150
22-26	0.00696	0.00640
27-31	0.01307	0.01323
32-36	0.02485	0.02200
37-41	0.02941	0.02366
42-46	0.03422	0.02730
47-51	0.04339	0.03998
52-56	0.07883	0.07418
57-61	0.07811	0.07595
62-66	0	0

Table 6.6: Contribution rates to private pension schemes

Based on the reported age and gender distribution of contributions from the sample collected by the Economic Council and the aggregate contribution from Finanstilsynet (1996) we construct age and gender distributions of contributions for 1995 by scaling the sample to the total aggregate level. To forecast these contributions we assume that the rate of contributions to the total wage sum of the age and gender group in question remains constant through the projection period. The contribution rates of the different age and gender groups are shown table 6.6

6.2.3 Pensions to civil servants

The Danish ministry of Finance has forecasted the costs of pensions to civil servants measured in fixed 1995 prices. This forecast is shown in table 6.6 below. In DREAM the annual level of pensions to civil servant is found by indexing this forecast to the wage level in the economy.

Year	Pensions to civil servants
	Billion Dkr.
1995	11.500
1996-2000	15.410
2001-2005	17.417
2006-2010	19.319
2011-2015	19.782
2016-2020	18.542
2021-2025	16.251
2026-2030	13.336
2031-2035	10.282
2036-2040	7.961
Henceforth	7.961

Table 6.7: Annual expenses to civil servants' pensions in fixed prices

6.2.4 LD pensions

LD pensions are restricted to persons who received wage income in 1977-79. There are no contributions to these pensions. Since this implies that the owners of the stock of wealth in the fund are limited to the original contributors there is only payments from the fund in the first 7 five year periods of the forecast, i.e. until 2030. The payment is assumed to be a once and for all withdrawal and to take place at the time when the person in question turns 62 years.

The macroeconomic effects of the pension scheme is very limited. Therefore the calculation of the annual payments from *LD* is found in the simplest way possible. We assume that the payments per person is constant and given as the total payments from LD in 1995 divided by the number of non-disabled persons who had the age of 62 in 1995. Total payment from LD in the following 6 periods are then found as the constant payment per person times the number of non-disabled persons who turn 62 in the five year period in question.

6.2.5 ATP pensions

There exists two types of contributions to the ATP pension scheme. The first (and original part) depends on the amount of hours worked per person. The second part of the contribution is given as 1 per cent of the wage sum. The wage sum dependent contributions are modelled directly in DREAM. The employment dependent contribution is modelled as constant through time. The level of the contribution rate is found by dividing the revenue from contributions in 1995 by the total employment in DREAM in 1995.

Pensions from the ATP are exogenous in DREAM. Pensions from each of the two schemes are forecasted separately by the ATP fund. Pensions based on the employment related contributions are distributed to all pensioners. Pension based on the wage sum related contributions are distributed to pensions in the age group from 67 years to 76 years. The forecast of the wage sum related pensions is not (necessarily) consistent with the model-generated contributions. In fact the DREAM forecast implies a wage increase due to the reduction in the labor supply which generates increased contributions per person. The net-effect is that the wage sum is reduced. Pension are however forecasted given an assumption of a constant (real) wage. This implies that pensions from the wage sum related contributions may be divert from the level that would have appeared in a consistent fully funded system. The same kind of problem appears in the case of the employment related contributions.

Year	Employment dependent	Wage income dependent
1995	2.245	0.000
1996-2000	3.172	0.011
2001-2005	4.742	0.180
2006-2010	7.007	0.646
2011-2015	10.798	1.511
2016-2020	14.501	2.725
2021-2025	17.180	4.178
2026-2030	19.169	5.761
2031-2035	21.060	7.428
2036-2040	22.041	9.153
2041-2045	22.286	10.970

Table 6.8: Pensions from the two types of ATP pension

In the period after 2045 pensions from the two types of schemes are assumed to remain at the 2045 level per person.

6.3 Age dependent public transfers and public service expenditures

The part of the public expenditures that may be characterized as public services to inviduals are distributed across gender and agegroups. This distribution follows the generational accounting system of the Danish Ministry of Economic Affairs (1997). The types of public expenditures that are distributed in this way are expenditures to hospitals, education, health care, residential housing for the elderly and domestic nursery and caretaking. Table 6.9 shows the average expenditures to these activities per individual in age and gender groups. One observes that for the group of pensioners expenditures are rapidly increasing with the age of the individual. Therefore the fact that the population forecast implies that expected lifetime is increasing tend to imply a sginificant increase in this type of expenditures.

	Men	Women	Men	Women
	Total - Million Dkr.		Per head - 1000 Dkr.	
17-21	6389	6285	36	37
22-26	4408	5177	23	28
27-31	3644	4443	17	21
32-36	2858	3486	15	19
37-41	2800	3100	15	17
42-46	2851	3066	15	17
47-51	3300	3325	16	17
52-56	2703	2755	18	18
57-61	2555	2911	20	22
62-66	2922	3214	26	26
67-71	3391	3822	34	32
72-76	3205	4222	39	38
77-81	2844	4376	51	50
82-86	2275	4834	71	77
87-91	1062	3329	90	109
92-96	350	1402	120	150
97-101	94	302	229	195

Table 6.9: Public service

A similar distirbution of service expenditures are constructed for 1993 by the Committee for Social Welfare, Socialkommision (1993b). For the age-groups analyzed (individuals older than 59 years) the level of expenditures is higher than those presented in table 6.9.

	Men	Women	Men	Women
	Total - Million Dkr.		Per head - 1000 Dkr.	
17-21	1135	1511	6	9
22-26	1389	3069	7	17
27-31	1195	4859	5	24
32-36	996	4435	5	24
37-41	1109	3663	6	20
42-46	1284	3038	7	16
47-51	1801	2775	9	14
52-56	1866	2366	12	16
57-61	1988	2754	16	21
62-66	169	271	2	2
67-71	258	418	3	3
72-76	304	535	4	5
77-101	1770		6	

Table: 6.10: Public transfers

Age dependent public transfers include expenses to maternity leaves schemes, early retirement pension[2], child care transfers to families with children less than 18 years, and education benefits. Social retirment pension and early retirement pension are not included. These transfers are mostly directed towards agegroups who belong to the work force and therefore have tendency decrease in the projction period due to the ageing of the population.

6.4 Tax rates

Tax rates are assumed to be exogenous until 2005, which is the period where the adjustment in the tax rates are announced in the recent tax reform act of 1998. In this period differences in between government expenditures and the tax total revenue increases the public debt.[3] From 2005 the income tax rate is assume to be endogenous as explained in chapter 4.

	1995	1996-2000	2001-2005	Henceforth
Wage income *	42.7	41.2	39	Endogenous
Pay roll tax	6.0	7.8	8.0	8.0
Unemployment benefits	31.6	29.7	27.5	Endogenous
Social pensions	28.6	26.7	24.5	Endogenous
Disablement pensions	35.5	32.9	30.4	Endogenous
Retirement pensions **	39.1	36.1	33.6	Endogenous
Private pensions	40	40	40	40

*Note: * Inclusive labor market contribution rate (pay roll tax)*
*** This tax rate is also used when calculating the form spouse pensions, ATP pension and civil servants pension.*

Table 6.11: The initial tax rates

The effective average tax rate on wage income inclusive labor market contribution tax is calculated from the tax system and the distribution of personal income in 1995 (using 5000 Dkr. intervals). The effective average tax rate is calculated given an assumption that the relevant wage income has to be at least as high as the maximum annual unemployment benefits in 1995. (which is approximately 130,000 Dkr.) For the forecast period tax rate are calculated by introducing new tax rates and evaluating

[2]Early retirement pension differs from early retirement benefits. The former is awarded to individuals due to lack of working abilities. There are no age requirements in this scheme. The latter is a general early retirement scheme for individuals who have been member of the unemployment insurance scheme for at least 25 years and who are at least 60 years old.

[3]In fact the debt is not increased as the revenue from the taxes are adjusted to account for the fact that the tax system in DREAM cannot fully capture the effect of the recent tax reform.

the income brackets at the 1995 price level. Implicitly the relative distribution of personal income from 1995 is therefore assumed to be constant.

The deviation in the effective average tax rate for other income categories is partly due to the fact that these income categories are not subject to the labor market contribution tax, and partly due to different assumptions concerning the relevant part of the total distribution of the personal income for the type of income in question. The effective average tax rate of unemployment benefits is calculated as the average tax rate for a person who receives the annual maximum unemployment benefit rate and has no other types of personal income.

Table 6.12 shows the personal capital income tax rates. The tax rate are calculated as explained in Knudsen *et al.* (1998)

Year	Interest income tax rate	Dividend tax rate	Capital gains tax rate
1995	55.6	27.8	24.8
1996-2000	50.6	25.3	24.3
2001-2005	37.5	23.6	23.0
Henceforth	37.5	23.6	23.0

Table 6.12: Personal capital income tax rates

Finally the recent adjustment of the tax reform also affects the indexation of social pensions and unemployment benefits in the period where the adjustments are phased in. Therefore the automatic indexation rule, described in chapter 4 is abandoned in this period. The annual growth rate in unemployment benefits and social pensions (measure in 1995 prices) are shown in table 6.13

Year	Unemeployment benefits	Social pensions
1996-2000	0.0020	0.0036
2001-2005	-0.0017	-0.0009
Henceforth	Endogenous	Endogenous

Note: Measured in fixed prices

Table 6.13: Annual growth rate in unemployment benefits and social pensions

6.5 Dynamic calibration

Calibration is the process where numerical values are assigned to the parameters of the model. The usual static procedure for calibrating CGE-models is of little use in the present context as the Danish economy was not (even as a rough approximation) in a stationary state (or steady state) in the base year of 1995.[4] There are several reasons for this, for instance that the size and the composition of the population were changing, and also that important stock variables (financial and physical assets) showed non-steady alterations. Instead it is necessary to perform dynamic calibration of the model dealing specifically with the fact that the base (calibration) year is a temporary equilibrium on the path of temporary equilibria eventually converging to the final stationary state (or steady state).

In general, when solving the dynamic model it is assumed that the perfect foreseeing agents optimize just before the end of period zero making their plans for all their future periods. The model is solved at once for all periods from period one to the terminal period where a steady state eventually has been reached. Some forward looking variables (the value of the firms, the wealth of households and the shadow prices of capital) are solved for (the end of) period zero, too.

The principle of the dynamic calibrating procedure is to use the dynamic model to generate the entire dynamic path of the endogenous variables subject to the fact that the model solution for the first period exactly reproduces the values of the endogenous variables in the base year data set. Technically this procedure implies that base year observations of endogenous variable are added to the model as constraints. These additional constraints imply that a corresponding number of parameters is determined endogenously.

To be more specific, we set up a calibration variant of the model, which consists of the standard dynamic model equations enhanced with additional restrictions for period one. They take the form of either additional restrictions that exogenously fix some of the (normally) endogenous variables at their value in the base year set in period one, or of extra equations concerning variables in period one. These additional restrictions determine (normally) endogenous variables in period one which are already determined by existing, standard equations. This allows for determining a number of parameters and additional variables (corresponding to the number of additional

[4] The base year of 1995 is also referred to as the calibration year and period one.

restrictions) endogenously in the dynamic calibration. From period 2 and onwards the dynamic calibration model is identical to the standard dynamic model. In conclusion, period one is regarded as a temporary equilibrium which is aligned to the actual base data set by additional restrictions valid for period one only.

The fact that the dynamic calibration is performed by solving the model for all periods at once implies that any change in the future value of an exogenous variable (e.g. the population) foreseen at the end of period zero affects the outcome of the calibration. This is a direct consequence of the assumption of perfect foresight. Future shocks that are not anticipated at the end of period zero will of course not affect the calibration.

Due to the forward looking behavior of the agents of the model it is highly relevant to carefully interpret the agents expectations of economic policy in the future. In the present analysis we assume that the agents expect that current rules concerning the indexation of social transfers will be maintained in the future. The real value of public service to a person of a given age and gender is also assumed to remain at the initial level. Concerning tax rates it is assumed that the announcement of future tax rates in the recent adjustment of the tax system is carried out as announced. In this period it is expected that differences between tax revenues and public expenditures are financed by changing the debt position of the public sector. After the announcement period the agents expects that increases in public expenditures are financed through increases in the personal income tax rate.

Data for some variables presently has to be imputed by using the period one solution of the model to generate the base year values. This is either because actual data for these variables do not exist (e.g. the stock of human capital) or the actual data are compiled so that they are a poor indicator of the model-relevant concepts (e.g. the value of firms).

The majority of parameters concern virtually atemporal equations and may be determined using single equation calibration similar to the standard technique used in static models. However, the parameters of genuine dynamic equations require a more intricate treatment. We now focus on important parameters where the dynamic calibration differs from the static calibration:[5]

1. For the private production sector, the scale parameter, Φ^P, of the installation

[5] The procedure of item 1 and 2 below are discussed in details in Knudsen *et.al.* (1998). Therefore they are only briefly sketched here.

costs function, (3.5), is calibrated by fixing total private investments at the actual 1995 level.

2. For households, the bequest parameter, ξ, and the intertemporal elasticity of substitution, S, of the intertemporal utility function, (4.1), is calibrated jointly with the shares of the total "free" wealth at the end of period zero, A_{t_0}, of all generations except the last, by fixing the share of the last generations "free" assets at time zero, a_{12,t_0}, and the generations consumption, C_{b,t_1}, at period 1.

3. The initial levels of pension undertakings in the labor market pension funds are scaled such that the wealth of the pension fund converge to a stationary state in the long run. The initial distribution of the pension undertakings (the pre-scaling levels) are calculated using disaggregated data from Danish labor market pension funds (see the section on pension undertakings).

6.5.1 Result of the base case calibration

There are numerous parameters in the model, and we shall only report the most interesting. Table 6.14 shows some important parameters fixed *a priori*.

To save computer time, the model is solved for 5 year periods. Instead of multiplying all flows by 5 we adopt the convention of keeping all flows unchanged, dividing all stocks by 5 to keep the relation between flows and stocks correct. Correspondingly all rates (interest rates etc.) are multiplied by 5, i.e. the rate of time preference at 0.013 for a 5 year period corresponds to an annual rate of 1/4 percent.

The assumed foreign trade elasticities are numerically larger than the empirical estimates for Denmark. We asses that there are strong theoretical reasons for expecting the foreign trade elasticities of a small open economy to be large in the very long run and suspect that the empirical estimates on time series covering around 30 years at most underestimate the elasticities in the 60 years perspective relevant here. The foreign trade elasticities are therefore set equal to 5 in absolute terms.

r	Real rate of interest	0.200
θ	Rate of time preference	0.013
α_{12,t_0}	Last generations share of free assets at the end of period zero	0.150
ω	Household asset composition parameter	1.500
γ	Elasticity of labor supply with respect to real reward	0.100
σ_{iY}	ES between value added and materials (both sectors)	0.250
σ_{iYH}	ES between capital and labor (both sectors)	0.600
σ_{PYMP}	ES between dom. and imp. materials in private sector	5.000
σ_{GYMP}	ES between dom. and imp. materials in gov. sector	5.000
σ_{PI}	ES between dom. and imp. investment goods in private sector	5.000
σ_{PI}	ES between dom. and imp. investment goods in gov. sector	5.000
σ_{CP}	ES between dom. and imp. consumer goods	5.000
ε	Numerical export demand elasticity	5.000
ι	Convexity of adjustment costs of investments	1.000
g	Corporate debt share	0.600

Table 6.14: Important parameters fixed a priori

Table 6.15 displays the results of the calibration of the most important parameters.

S	Intertemporal elasticity of substitution	0.907
ξ	Preference for leaving bequest	2.326
ϕ	Scale parameter of installation costs of investments	2.126

Table 6.15: Important parameters calibrated

Chapter 7
THE PROJECTION OF THE DANISH ECONOMY

The present projection should not be considered a forecast of the Danish economy. The three main reasons are: First, impulses from the foreign economies are assumed to be identical to the impulses in the initial period. Thus the international interest rate remains at approximately 6 per cent through out the time horizon, the exchange rate vis a vis the trading partners is constant through out, the international inflation rate is assumed to remain at 2 per cent through out and finally the foreign demand curves for the domestic products are assumed to remain constant both with respect to the position (i.e. no increased international division of labor is assumed) and the elasticity. Second, the present analysis abstracts from underlying productivity growth. Finally, the projection assumes a policy reaction function that implies that the age dependent level of both public service and public income transfers to households remain at the present level per person measured in fixed prices. Furthermore the public deficit assumed to be zero from 2005, due to endogenous increases (reductions) in the income tax rate.

Therefore one interpretation is that the projection gives a measure the domestic response to the major changes in the demographic composition of the domestic population and the gradual increase in the labor market pension funds given a fairly static economic policy (measured by the services that each individual receives) which does not imply increase indebtedness of the public sector.

Before turning towards the actual calculated evolution of the economy a decomposition of the exogenous developments into three basic shocks is presented along with the economic response to each of the shocks.

7.1 The types of shocks and the economic impacts

The demographic evolution consists of a reduction in the number of persons in the labor force by approximately 7 to 8 per cent during the next 30 years (see chapter 6). At the same time the number of persons who are older than 62 year increase by approximately 50 per cent. The assumption that both the real value of public service expenditures and real public transfers are assumed to remain fixed per person of a given age, implies that both total real public service expenditures and total real public income transfers are increased with the growing number of elderly people.

During period until 2030 the contributions to labor market pension schemes increase by approximately 50 per cent from the level in 1995. Pension receipts from the pension funds are expected to increase by almost 300 per cent in the same period.

The reduction in the labor force may be considered a foreseen supply side shock to the economy. Both the increase in the public expenditures that is associated with the increase in the number of elderly and the growing significance of the pension fund may be interpreted as a foreseen demand shock to the economy.

7.1.1 Qualitative effects of the supply shock

If the reduction in the labor force is considered permanent (and for expositional simplicity a once and for all shock) then the impact effect is a permanent inward shift in the aggregate labor supply function. On impact this implies that producer real wages are increased and the demand for labor is reduced. The firms expect (with certainty) the new position of the aggregate labor supply curve to be permanent. This implies that the firms foresee a permanent reduction in the marginal product of capital due to the lower demand for labor. This induces the firms to reduce the current level of investment, as the firms are faced with convex cost of installation of capital and therefore are unable to profitably reduce the capital stock on impact. The consequence for domestic production is that it drops on impact and is gradually further reduced over time as the stock of capital gradually decreases to the new stationary level.

If no part of the capital stock was imported and if no costs of installation of capital were present the reduction in the capital stock would imply that the capital labor ratio remained unchanged and therefore that the relative reduction in output would be of similar magnitude as the reduction in the labor force. However, as a significant part of the capital is in fact imported and the terms of trade are improved due to the drop in domestic production a reduction in the user cost of capital is the outcome. This is enforced by the fact that the convex cost of installation of capital is assumed to be a function of the gross investments. The latter effect is however minor. The result is that the long run capital labor ratio is increased so that both capital and production is reduced less that the labor force.

On the demand side the lower production on impact and the expected further reduction yields a lower expected income for the consumers. Therefore total consumption initially drops. The total drop in demand is smaller than the drop in production and a reduction in the balance of payments on impact is the consequence. Over time this balance of payment reduction is gradually eliminated. The long term effect is a reduction in aggregate consumption. The consequence for the long run stock of wealth in the economy is negative as both the domestic stock of capital and the stock of foreign assets is reduced.

7.1.2 Qualitative effect of the demand shock

Increased expenditures to public services

Consider the increase in the number of elderly and suppose that it is a once and for all permanent shock. The age distribution of public services including hospitals and other health care expenses has a profile that implies a higher level of service per person for elderly people. Therefore public service expenditures increase. Since we assume that the public sector runs a balanced budget this implies that the tax rates on non-interest income are increased.

Increased tax rates have both a supply and a demand effect. The supply effects appears because of the assumption that the absolute increase in the effective tax rates of different types of non-interest income is identical. This implies that the absolute reward from being employed is reduced, which according to the individual labor supply function reduces the labor supply of each individual. Therefore private production is reduced on impact and the reduction is increased over time along with the gradual reduction in the private capital stock. Public production is increased due to the increased demand for public services and this leads to an increase in public employment and in the public stock of capital. Total employment is reduced whereas the effect on the total capital stock is ambiguous.

The effect on demand for private consumption is negative which is due to a dominating negative income effect stemming from the permanently increased tax rate. This also tends to reduce private savings. The reduced savings are further enforced by the fact that the absolute reduction in disposable income is higher for generations who are in the labor force. The long run stock of wealth is reduced due to the permanently lower accumulation of savings.

Increased public transfers to pensioners

The increase in the number of elderly also increase public transfers to households. The absolute dominating effect on transfers is the increase in public pension. Again this is financed through higher taxes on non-interest income.

Similar to the effect of increased public expenditures the higher tax rate has a negative supply effect that leads to lower production and capital stock in the private sector. The demand effect is however different as no negative income effect on the private consumption is present, since the increased revenue from taxes is shifted back to the household through the increased public transfers. As disposable wage income is reduced more than disposable social pensions the life-cycle motive for saving is reduced and therefore the long run effect is a reduction in the stock of wealth.

Increased savings in labor market pension schemes

In models with perfect foresight and perfect capital markets like DREAM, introduction of fully funded pension schemes does not change the equilibrium per se. The reason for this lack of effect is that the consumer is always in a position to borrow against the discounted value of future income as long as the consolidated budget constraint is fulfilled. The perfect capital market implies that there is a unique after tax interest rate and nothing is gained or lost when borrowing against the pension savings.

The Danish tax system implies that interest income on pension savings receive a tax relief and furthermore pension savings implies that a part of the tax burden is shifted forward in time. Due to the structure of the overlapping generation model Ricardian equivalence does not prevail and therefore the forward shift may affect the equilibrium given the policy response rule of government. For these two reasons alone one should expect the introduction of fully funded labor market pension schemes to alter the equilibrium in the economy.

However, the dominating effect of the labor market pension schemes is the fact that the precautionary principles of the pension fund implies a longer effective time horizon of the pension fund than the assumed time horizon of the households. This longer effective time horizon of the pension fund implies that the household considers a part of the contributions to the pension scheme as forced savings. The mechanism is the following: When the household enters the economy it has a time horizon that is equivalent to the expected lifetime of a 17 years old woman. When choosing the optimal consumption path the members of the household uses this time horizon, and does not consider an eventual period after the time horizon has expired. The precautionary principles of the pension fund on the other hand has to take into account the fact that the life time of a person may well exceed the expected life time at the time of the entrance to the pension fund. A part of the capitalized value of the contributions therefore is reserved to pension receipts after the expiration of the time horizon of the household. As these pension receipts does not affect the utility of the household the pension scheme is not considered balanced from the point of view of the household. The result is that the stock of human capital (measured until the expiration of the time horizon) at the age of 22 year is lower in the presence of a labor market pension scheme than with the absence of any schemes. This implies that *ceteris paribus* the level of consumption is lower at any point in time where the person has positive future contributions to the labor market pension scheme. Therefore the savings in each period is higher for each generation. Over time the increased savings generates a higher stock of wealth in the economy.

By the assumption that pension savings does not affect the labor supply the pension schemes has no direct supply effect. However, there is an indirect effect on the cost of capital of the firms from the tax relief on capital income from the pension savings. Since the institutional investors determine the value of shares the tax rates of these investors enters the first order condition of the firm. Compared to the situation were the capital income tax level is equal to the personal tax rates, this has only an insignificant effect on the cost of capital as both the capital gains tax rate and the interest income tax rate are lower for institutional investors.

The sum of all shocks

First, observe that from the time where the ageing problem begins to manifest itself around 2010, then the supply effects of all the shocks are similar: They all pull in the direction of a lower labor supply. Concerning private consumption the effects are more mixed as the increase in the public service expenditures, the increased labor market pension contributions, and reduced labor supply all tend to reduce private consumption on impact whereas the increased income transfers tend to increase private consumption. It is plausible to assume that the combined effect of the first three shock dominates. In the long run the effect from the increased labor market pension savings is reversed due to the accumulation of more wealth which generates higher consumption over time. Therefore the negative effects on private consumption is gradually reduced over time.

Concerning the fiscal pressure from the increased demographic ageing, we observe that in the period after 2040 the composition of the population is expected to remain fairly constant with an increase in the demographic ageing burden by approximately 40 per cent from the 1995 level. Also the population from 17-61 year remains fairly constant at a level approximately 5 per cent below the 1995 level. Therefore expenditure pressure remains slightly below the level reached at the top of the ageing burden in 2035. However, the gradual and rather significant increase in the pension receipts from the pension fund implies an automatic increase in the tax revenue over time that continues until the steady level of the pension fund is reached around year 2100. The fiscal burden on the working population therefor reaches a maximum in the year 2035 (where the demographic burden peaks) and is gradually decreased in the following period.

7.2 Quantitative economic effects of the demographic evolution

7.2.1 The macroeconomic evolution

The previous section outlined the supply part of the exogenous demographic shock to the economy as a negative shock. Indeed this is the medium term development. However, in the period until 2005 the labor force is increasing and the income tax rate is reduced. The latter is a consequence of the tax reform act of 1998 (see table 6.9. for the evolution in the tax rates). This tends to increase production and aggregate disposable income in the short run. This is strengthened by the fact that initial net investments are positive, therefore both the capital stock and the employment are increasing until 2005. This initial increase in production implies that production grows faster than demand for the domestic good and therefore a deterioration in the terms of trade is generated. The low relative price on domestic goods combined with the expectation of a fairly significant domestic price increase in the period after 2010 generates intertemporal price speculation in consumption that implies a tendency to increase consumption in this phase of the projection at the expense of future consumption (in a period where consumption of domestic products is more expensive). Even though production peaks in 2005, aggregate consumption is growing until 2020. During this time domestic prices are continuously lower than in the initial period, which is made possible by the rather dramatic decrease in investments which in 2020 are reduced by more than 10 percent from the initial level whereas the capital stock is still higher than the original level.

These slow gradual changes in the stock of capital stock are the main contributor to the domestic price effect that postpone the demand shock to the economy. Negative supply shocks from reduced labor force hit the economy already in 2005 but the employment remains higher than the initial level until 2015.

Observe that the quantitative size of these dynamic effects are connected to the assumptions (stated in the introduction to this chapter) on which the projection is based. First, the drop in investment both on impact and in the long run is strengthen by the fact that no labor productivity growth is present in the analysis. A positive labor productivity growth would imply positive net investments in the long run and therefore possibly a less significant drop in investment. Furthermore the relative price changes between domestic and foreign goods are affected by the assumption that no demographic shock hits the foreign economy.[1]

[1] Incorporation of the demographic development in the rest of the world may even reverse the effect on the terms of trade, as demographic shocks are expected to be larger in several of the other OECD countries that constitute the major trading partners of Denmark.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Private consumption	416	427 (2.6)	422 (1.3)	422 (1.4)	436 (4.7)	454 (9.1)
Real GDP	830	862 (3.8)	815 (-1.8)	827 (-0.4)	825 (-0.6)	828 (-0.2)
Employment, index	100.0	101.5 (1.5)	92.8 (-7.2)	95.7 (-4.3)	95.4 (-4.6)	95.5 (-4.5)
Capital stock	2755	2820 (2.4)	2753 (-0.1)	2757 (0.1)	2760 (0.1)	2769 (0.5)
Value of firms	2513	2430 (-3.3)	2315 (-7.9)	2347 (-6.6)	2366 (-5.9)	2375 (-5.5)
Foreign assets	-266	-101	218	319	455	682

Note: The numbers in parantheses are the percentage change compared to the initital value

Table 7.1: Dynamic evolution in main macroeconomic variables

From around 2020 to around 2045 the description of the economy in the preceding section fits the evolution. At this point the supply effect has stabilized with a reduction in the net-output from the private sector at around 3 per cent. Employment is around 6 percent lower, which implies a permanent reduction in the unemployment rate at about 1 per cent. The capital stock of the private sector remains almost at the initial level whereas the public capital stock is increased by 4 per cent due to the assumption of a fixed capital output relation in the public sector. The implication is that the capital labor ratio in the private sector is permanently increased due to the change in the demographic composition. This effect appears (mainly) due to the positive terms of trade effect that follows from the fact that domestic production is reduced relative to domestic demand for the domestic good. As the capital stock partly consists of imported goods the relative price of investment to domestic output is decreased by 7 per cent from the initial level. The generates a reduction in the user cost of capital that explains the observed increase in the capital labor ratio. The evolution in the capital labor ratio in the private sector is shown in Figure 7.1. Miles (1999) also reports an increase in the capital labor ratio for the United Kingdom as a consequence of the ageing population. However, in his single good model the increased capital labor ratio is due to a reduction in the international rate of interest generated by the changing population in the OECD countries. It is interesting to observe that the present (higher) long run level of the capital labor ratio is comparable to the reported level of the capital labor ratio for the UK in the eighties and nineties.

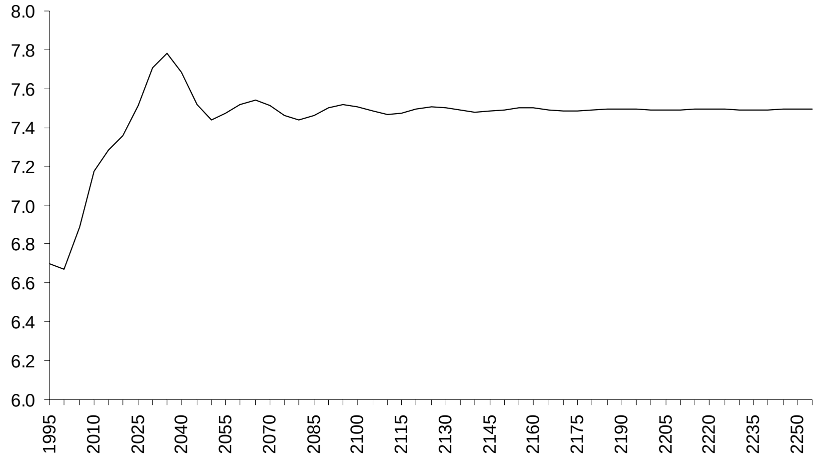

Figure 7.1: The capital labor ratio in private sector

The demand effect starts pulling the domestic demand upward from 2035. There are two reasons for this: First, the demographic age-burden peaks at this point. From 2035 to 2050 the number of elderly is reduced by 10 per cent and the number stabilizes around the 2050 level. Therefore the pressure from increased expenditures is reduced and the tax-rate on non-interest income may be reduced. Second, the increased labor market pensions also imply that the financing burden of the working generations may be reduced as the tax revenue from pension receipts keep increasing until around 2100. Simultaneously the increased income of very old pensioners and the expected future income to future pensioners imply that domestic consumption start rising from the local minimum in 2055.

In the very long run domestic aggregate production is almost constant even if employment has been reduced by more than 4 per cent. At the same time private consumption is increased by 9 per cent. This positive long run effect on consumption is generated by the increased savings rate that follows from the increased contributions to the labor market pension fund. This generates a higher stock of wealth in the economy which changes the foreign asset position from a deficit to a relatively large surplus, see table 7.1.

7.2.2 Evolutions in savings and wealth

It is a widespread presumption that the demographic development will induce a reduction in the interest rate, see e.g. Miles (1999). This may be taken to imply that the expected demand for capital is reduced relative to the expected supply of capital.

The demand for capital is negatively affected by the reduction in the labor force. On the other hand the supply of capital may be negatively affected by the fact that an increased number of persons become pensioners and therefore partly is expected to live from the accumulated stock of wealth. The stated presumption implies that the former effect is dominating.

In the case of the Danish economy the accumulation of wealth is affected by the fact that the funded pension system is in a growing phase in the period where the demographic transition takes place. In the present simulation the funded pension system affects the total accumulation of the private sector for two reasons. First, the gradually increased amount of deferred taxation along with the policy reaction rule that the government runs a balanced budget given the current tax revenue imply that the government accumulates a positive asset of outstanding future taxes in the private sector. Second the fact that part of the savings in the pension fund is considered forced savings by the private agents implies an increase in the accumulation of wealth along with the increased accumulated stock of contributions.

On the other hand the income of the economy is positively affected by the terms of trade effects mentioned in the previous subsection. The evolution in the private wealth to private income (net of tax) is shown in figure 7.2 along with the evolution of the part of the private wealth that is neither accumulated in labor market pension schemes nor private pension schemes. The figure shows that the total private wealth (including the deferred taxation) to income is increasing from a factor 7 to a factor 8 in the period where the labor market pension system matures, i.e. until around 2100. In the same period the "free" stock of financial wealth of the private sector relative to income (net of tax) is reduced from a factor 6 to a factor 3 reflecting the fact that the agents in the private sector substitutes "free" savings for savings in pension arrangements. The figure indicates that around year 2100 more than half of the private wealth (including deferred taxes) takes the form of funded pension savings.

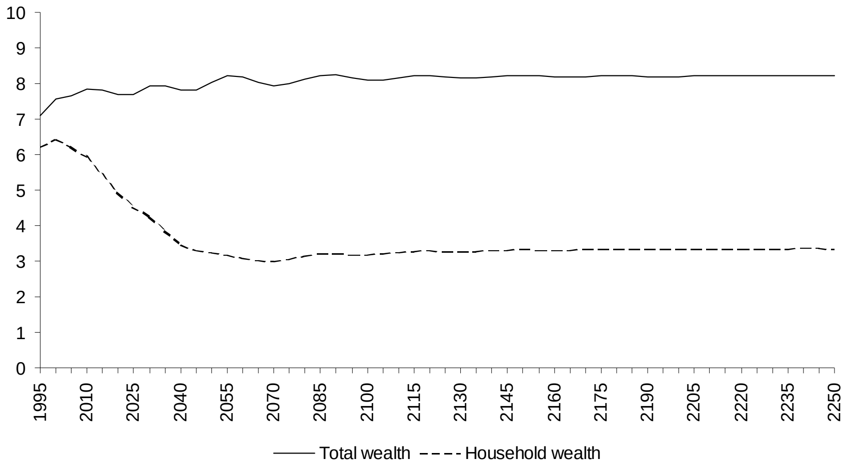

Figure 7.2: Ratio of wealth to income (net of tax)

The same development measured as the ratio of savings to income (net of tax) is shown in Figure 7.3. Savings are defined as changes in the stock of wealth.

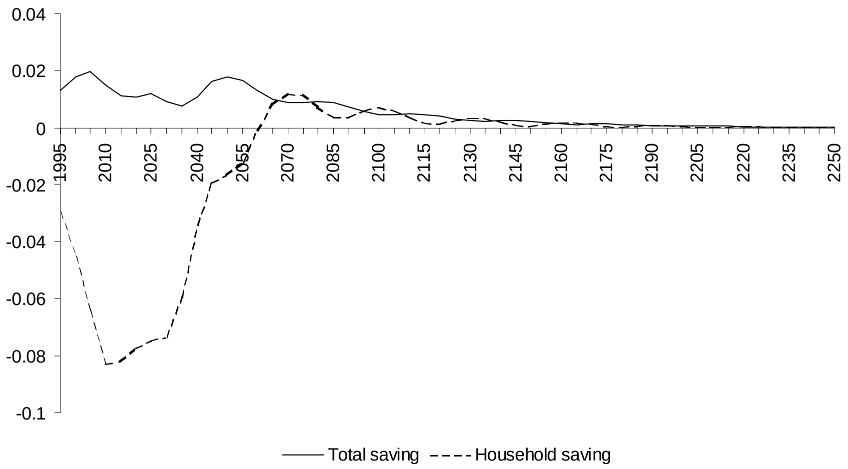

Figure 7.3: Ratio of savings to income (net of tax)

7.2.3 Evolution in the fiscal pressure

In this section we focus on the public expenditures and income. Table 7.2. and table 7.3 below (which are repetitions of table 1.1 and 1.2) give an overview

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Social security pension	73	74	114	99	101	102
Civil servants pension, ATP, LD	15	22	42	42	42	42
Age-dependent public service	155	162	193	188	186	188
Age-dependent public transfers	45	43	47	46	46	46
Total age-dependent public expenditures	**288**	**302**	**396**	**375**	**375**	**378**
Other expenditures	135	129	129	128	128	128
Total public expenditures	423	431	525	503	502	506
Gross domestic product (GDP)	969	1001	995	996	1002	1016
Total public expenditures as percent of GDP	43.7	43.0	52.7	50.5	50.1	49.8
Change in total public expenditures as percent of GDP from 1995		-0.6	9.1	6.8	6.5	6.1

Note: Average annual rates, deflated by foreign price level

Table 7.2: Public expenditures and Gross Domestic Product

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Tax on labor market pensions	5	5	20	30	36	35
Tax on civil servants pensions, ATP, LD	6	8	17	16	15	14
Tax on private pension schemes	7	5	13	12	13	13
Tax on social security pensions	21	18	37	28	27	26
Other revenues	381	395	438	417	412	418
Total revenue	**419**	**431**	**525**	**503**	**502**	**506**
Effective average wage-income tax rate	42.7	39.0	47.0	43.2	40.9	39.6

Note: Average annual rates, deflated by foreign price level

Table 7.3: Evolution in public revenues

From table 7.2. we observe that total age-dependent expenditures increase by approximately 33 percent or 107 bill Dkr. (in fixed foreign prices) from 1995 to the peak in 2031-2035. This figure overestimates the actual increase as the automatic reduction in social pensions due to increased labor market pensions is not included. Furthermore, disregarding the increase in the pension from the ATP pension fund the increase in age-dependent public expenditures reduces to 25 percent in the same period. This does however not affect the general conclusion which is that if future generations are to expect the same level of public service and public income transfers then there will exist a substantial need for additional revenue to the public sector if increased indebtedness is to be avoided. In the period after 2035 fiscal pression is reduced due to the reduced number of pensioners. From table 7.2 one observes that both social pensions and age dependent public transfers are reduced. Total public expenditures as percent of GDP is reduced from 53 percent in 2031-35 to around 50 in the new stationary state. Therefore the long run costs of the demographic development is significantly lower than the cost in the peak period from 2031-35.

From table 7.3. we observe that 1/3 of the revenue that is necessary to finance the increased expenditures (ignoring the ATP pension increase) is generated from increased revenues from taxes on pensions in 2031-35. The rest of the revenue has to be raised from the working part of the population.

As mentioned the projection is made given the assumption that the public sector runs a balanced budget in each period and that this is done by adjusting the tax rate on non-interest income in each period starting from 2005. The policy reaction rule implies that the tax rate is increased until 2035 where it peaks. At the peak the average effective tax rate has increased by 4.3 percentage points from 1995 or 8.0 percentage point from 2005. After the peak in 2035 the tax rate is gradually reduced. In 2050 the tax rate is almost back at the 1995 level. This is partly due to the reduced cost of the elderly as a consequence of the reduction in the number of elderly and partly due to increased tax revenue from labor market pension receipts. The increase in the taxbase continues beyond 2050 and in the new stationary state the effective average income tax rate is almost reced to the minimum level from 2001-2005.

Thus the financing burden of the demographic age-burden is in fact temporary - although the temporary period last for 50 years - even if the demographic burden is permanently increased. This feature is explained by the increased tax base due to the fact that gradually the pension receipts exceeds the pension contributions. This is enforced by the fat that part of the pension receipts follows from the forced savings in the labor market pension schemes.

Since the public expenditures peak rather rapidly from 2020 to 2035 an alternative policy response rule has been introduced as a surprise shock in 2005. The alternative rule is that the demographic age-burden is financed through a constant tax rate on non-interest income from 2010 to 2045. In this way the financing burden is stretched out over 10-15 extra years before the actual burden appear. Therefore the policy implies substantial budget surpluses in the first part of the period. This policy implies that the tax rate jumps 5 percentage points from the 2005 level (or remain 1.5 percentage point above the 1995 level).

7.2.4 Life cycle income for a specific generation

This section presents the composition of the life cycle income from different sources. As specified in chapter 4 the non interest income of is the sum of income from several sources. Observe, that we define pensions receipts from funded pension systems as part of the income profile and therefore part of the human capital. As the funded pension receipt is partially a reduction in accumulated savings the life cycle income profile may be misspecified for some purposes. Figure 7.4 shows the composition of the income before tax over the life cycle for the average individual who are 17 years in 1995. Observe that the figure does not include the withdrawal from private pension schemes that takes place at the age of 67 and 72 years. Interest income and inheritance from the parent generation are also excluded.

Figure 7.4. shows that in the age-group from 17 to 61 years where the individual is in the labor force, the main contributor to the total income is income from employment. The second largest contributor is age-dependent transfers from the government followed by unemployment benefits and disablement and spouse pension from the labor market pension system. By the assumption of the model the agent is forced to retire at the age of 62 years. In the following five year period the agent receives early retirement benefits and disablement and spouse pensions from the labor market pension scheme. As the agents turns 67 years he or she becomes entitled to social pension and retirement pension from various funded pension systems. The interesting feature of this life cycle income distribution is that the total replacement rate of pension to the peak income as employed is close to 100 per cent. One should, however, keep in mind that the social pension is overestimated in the simulation. There are two reasons for this: First, the total expenditures to early retirement benefits and social pension are added together and distributed as an equal "pension" per person in the age-group from 62 years. However, the receipt per person is higher in the early retirement benefit system than in the pension system. Second, the projection is based on the assumption that social pension per individual is indexed to the wage level but otherwise unaltered. However, current rules of the social pension system imply that a part of the social pension is reduced with increased pension receipts from the funded

pension system.

This is however not sufficient to substantially change the overall picture that on average the sum of the social pension and the pension from funded pension systems is approximately of the same size as the total average income of a middle-aged individual, when the labor market pension system has (almost) reached its equilibrium level.

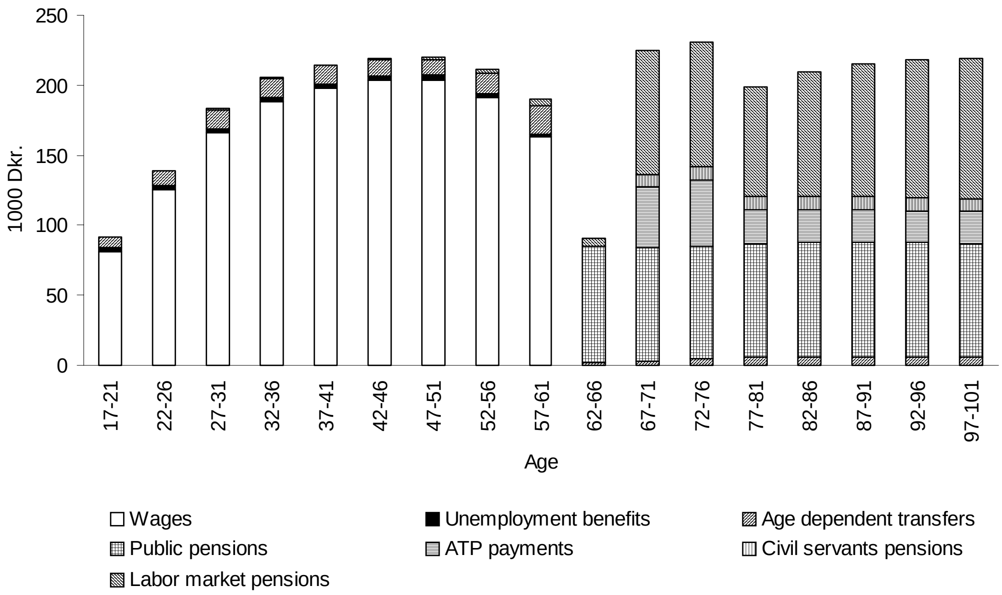

Figure 7.4: Life cycle income for an individual who is 17 years old in 1995

7.3 Decomposition 1: The effect of the labor market pension fund

As discussed in the introduction to the present chapter the economic projection is composed of three different shocks: 1) A demand shock from the increased savings and the deferred taxes due to the evolution of the labor market pension fund, 2) A demand shock from the increased public expenditures to the elderly and the associated financing through higher income tax rates, 3) A supply shock from the reduced labor force due to a lower number of persons in the relevant age-groups.

In the present and the following sections each of these shocks are eliminated in turn. In this way the marginal effect on the total economy of the specific shock may be revealed. The sum of these marginal effects do not add up to the total effect for several reasons, not the least due to a large interdependence between the shocks. However, the decomposition does yield a considerable insight into the relative importance of each of the elements in the total exogenous evolution.

In the present section a simulation of the economy without contributions to the funded labor market pension system is presented. The experiment is performed as a policy change which is announced ultimo 2005 and enacted from 2006. This implies that the stock market is allowed to clear conditioned on the new information at the end of 2005 and a jump in the value of firm therefore is observed. The policy change removes the tax relief on contributions to the labor market pension system and therefore contribution to the system is set to zero from 2006.[2] Therefore pension receipts are gradually reduced as the new generations of pensioners have a contribution period that gradually becomes shorter.

Current rules of indexation, see e.g. relation (4.19), imply that public transfers are indexed to the wage rate net of contributions to the labor market pension system. Hence a reduction in the contribution rate will generate an increase in public transfers. As this is not the intention of the experiment we simply fix the level of the transfers in the period from 2006-10 to the level that would have appeared in case of no shock. For the following periods we assume that public transfers are indexed according to the standard rule (with no contributions to labor market pension fund). In the base scenario the contributions to the labor market pension system increase after this period and therefore the indexation of public transfers is reduced in the base scenario relative to the present scenario where contributions to the labor market pension remain zero throughout.

The fact that contributions to the labor market pension scheme are removed imply that tax receipt from income taxation is increased on impact since the tax relief is removed. This generates a higher tax revenue that for a given level of public expenditures is used to reduce the average income tax rate so that the government budget remains balanced in each period.

As the labor market pension fund in the base scenario matures the tax revenue from pension receipts exceeds the lost revenue from the relief on pension contributions. This effect does not appear in the present scenario and therefore the long run effect is a reduction in the tax revenue compared to the base scenario. The policy reaction

[2] For simplicity it is assumed that the contribution rate to private pension schemes remain unchanged and that these still receive the tax releif.

function which keeps the government budget balanced implies that the average income tax rate is increased.

Furthermore, the fact that contributions to the labor market pension fund partly finances pension receipts beyond the time horizon of the household implies that the savings rate in the economy increases with the contribution rate to the labor market pension fund. Therefore the present scenario generates a lower total savings ratio and hence a lower stock of wealth in the long run. This effect dominates the macroeconomic evolution in the very long run.

Table 7.4 shows development in the main macro economic variable and compare the evolution to the base scenario.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Private consumption	416	427 (2.6)	432 (3.8)	419 (0.7)	393 (-5.6)	341 (-18.2)
Base case		(2.6)	(1.3)	(1.4)	(4.7)	(9.1)
Real GDP	830	862 (3.8)	817 (-1.5)	825 (-0.6)	811 (-2.2)	798 (-3.8)
Base case		(3.8)	(-1.8)	(-0.4)	(-0.6)	(-0.2)
Employment, index	100.0	101.5 (1.5)	93.1 (-6.9)	95.5 (-4.5)	93.9 (-6.1)	92.5 (-7.5)
Base case		(1.5)	(-7.2)	(-4.3)	(-4.6)	(-4.5)
Capital stock	2755	2820 (2.4)	2749 (-0.2)	2735 (-0.8)	2698 (-2.1)	2657 (-3.6)
Base case		(2.4)	(-0.1)	(0.1)	(0.1)	(0.5)
Value of firms	2513	2450 (-2.5)	2302 (-8.4)	2303 (-8.4)	2273 (-9.6)	2222 (-11.6)
Base case		(-3.3)	(-7.9)	(-6.6)	(-5.9)	(-5.5)
Foreign assets	-266	-101	-209	-442	-831	-1430
Base case		-101	218	319	455	682

Note: The numbers in parantheses are the percentage change compared to the initial value

Table 7.4: Macroeconomic variables with zero contributions to the funded pension

Table 7.4 reveals that the behavior of the economy in the very long is significantly different from the base scenario. Private consumption is reduced by more than 18 per cent from the 1995 value in the present case, whereas the base scenario exhibits an increase of 9 per cent. This change in total consumption is due to a reduction in the disposable human capital in the stationary state compared to the base scenario. The absence of pension contributions implies that no forced saving is present in the economy. Therefore the demand from the persons who are older than 77 is gradually reduced to the level of social pensions. The reduction in the disposable human capital to the households is explained by the fact that no pension receipts from the labor market pension fund is present. This of course has a direct effect on the human capital as measured in the present paper. In addition the lack of pension receipts generates a tax revenue loss in the long run as no deferred taxes have been accumulated. Therefore the policy reaction function implies that age-groups who belong to the work force are faced with increased average income tax rates. This tax increase has both a direct and an indirect effect on the disposable human capital. The latter appears since the absolute reward from employment is reduced, which induces a reduction in the labor supply. This generates a standard negative supply effect that in the long run yields lower capital stock and employment, so that the human capital is reduced.

Observe from table 7.4 that in the stationary state capital stock is reduced significantly more than the employment compared to the base scenario. Thus the capital labor ratio is reduced. This is explained by the absence of a price reduction on investment relative to the domestic output price. This follows from the fact that domestic consumption is reduced more than domestic production compared to the base case. Therefore the excess of domestic production to domestic demand for the domestic good is higher in the present scenario than in the base scenario. This generates a term of trade deterioration compared to the base scenario and there also an increase in the domestic output price relative to the investment price index.

In the period until 2035 the effect is reversed compared to the base scenario. This follows from the fact that this is the period where the funded labor market system is young so that contributions dominates the pension receipts and the forced savings are accumulated. Thus that fact that contributions to the labor market pension fund is forced to zero implies that income tax base is increased so that the average income tax rate may be reduced to obtain the same revenue. This implies a increase in the labor supply and the capital stock relative to the base scenario in this period. The fact that forced savings is reduced also increases the private consumption relative to the base scenario in this period.

Table 7.5 and 7.6 show the evolution in public expenditures and revenue respectively given that contributions to the labor maket pension fund is zero. Concerning the public expenditures one observes that these are almost unaffected compared to the base scenario. On the other hand the long run level of GDP is reduced relative to the base scenario due to a negative terms of trade effect that follows from the lower level of domestic consumption in the present scenario. Therefore the long run rate of expenditures to GDP increases significantly more than in base scenario: 9.6 percentage point compared to 6.1 percentage points.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Social security pension	73	74	114	99	100	98
Civil servants pension, ATP, LD	15	22	42	42	42	42
Age-dependent public service	155	162	193	188	184	182
Age-dependent public transfers	45	43	47	46	45	44
Total age-dependent public expenditures	**288**	**302**	**396**	**374**	**371**	**367**
Other expenditures	135	129	129	128	127	126
Total public expenditures	423	431	525	502	498	493
Gross domestic product (GDP)	969	1001	1002	992	966	926
Total public expenditures as percent of GDP	43.7	43.0	52.4	50.6	51.6	53.2
Change in total public expenditures as percent of GDP from 1995		-0.6	8.7	6.9	7.9	9.6

Note: Average annual rates, deflated by foreign price level

Table 7.5: Public expenditures and Gross Domestic Product

Concerning tax revenues the fact that contributions to the labor market pension is zero implies that revenue for pension receipts is reduced by 40 billion Dkr. in the long run. On the other hand the lack of contribution increases the tax base of wage income ceteris paribus. However, the absense of an increased tax base and the lower rate of savings imply that the economic effects of the demographic transition become permanent, and furthermore that long run costs relative to GDP are even higher than the costs at the period 2031-35 where the demographic age burden peaks. The increase in the long run effective average tax rate on wage income is no less than 10.6 percentage points compared to the 1995 level. This has to be compared to the long run decrease of 3.1 percentage points in the base scenario.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Tax on labor market pensions	5	5	11	16	2	0
Tax on civil servants pensions, ATP, LD	6	8	17	16	18	20
Tax on private pension schemes	7	5	13	12	13	12
Tax on social security pensions	21	18	35	29	33	38
Other revenues	381	395	449	429	432	423
Total revenue	**419**	**431**	**525**	**502**	**498**	**493**
Effective average wage-income tax rate	42.7	39.0	45.4	43.9	48.1	53.3

Note: Average annual rates, deflated by foreign price level

Table 7.6: Evolution in public revenues

The conclusion of the present scenario is that the presence of the funded pension system with the element of forced savings and the accumulation of deferred taxes generates a level of taxable pension receipts that increases the tax base sufficiently to finance the structural shift in the public expenditures that follows from the permanently increase age burden. Furthermore the increased savings (both in the public and the private sector) generates an increase in the stock of total financial wealth that is sufficient to finance a very large permanent increase in the domestic consumption.

7.4 Decomposition 2: Reduced social pensions

The second shock to the economy from the demographic evolution is the increased public expenditures that are financed through higher income tax rates. In this section an alternative policy reaction function is announced ultimo 2005 and introduced from 2006. In this case the income tax rate remains unchanged and the balanced government budget is achieved through annual changes in the social pensions.

In the period from 2010 to 2035 this financing rule generates relatively significant reductions in the social pensions and therefore in the replacement ratio of the pay-as-you-go pension system. The reduction is particularly high since no reductions in the age dependent public service are introduced. However in the period after 2040 the social pension are gradually increased due to the increased tax revenue from pension receipts from the funded pension system. In the long run the level of social pensions is comparable to the level in the base scenario. This of course reflects the fact that in the long run the fiscal pressure from the increased number of pensioners is neutralized by the increased tax revenue from the matured funded pension system.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Private consumption	416	427 (2.6)	427 (2.6)	439 (5.3)	455 (9.3)	457 (9.8)
Base case		(2.6)	(1.3)	(1.4)	(4.7)	(9.1)
Real GDP	830	862 (3.8)	826 (-0.4)	836 (0.7)	830 (0.0)	829 (-0.1)
Base case		(3.8)	(-1.8)	(0.4)	(-0.6)	(-0.2)
Employment, index	100.0	101.5 (1.5)	94.2 (-5.8)	96.5 (-3.5)	95.7 (-4.3)	95.6 (-4.4)
Base case		(1.5)	(-7.2)	(-4.3)	(-4.6)	(-4.5)
Capital stock	2755	2820 (2.4)	2787 (1.2)	2789 (1.2)	2777 (0.8)	2773 (0.6)
Base case		(2.4)	(-0.1)	(0.1)	(0.1)	(0.5)
Value of firms	2513	2445 (-2.7)	2349 (-6.5)	2373 (-5.6)	2383 (-5.2)	2380 (-5.3)
Base case		(-3.3)	(-7.9)	(-6.6)	(-5.9)	(-5.5)
Foreign assets	-266	-101	437	608	700	729
Base case		-101	218	319	455	682

Note: The numbers in parantheses are the percentage change compared to the initital value

Table 7.7: Macroeconomic variables with reduced social pensions

The fact that the reduction in the social pensions is temporary and only affects pensioners in the period from 2010 to 2050 with at peak at 2035 implies that persons who are relatively young in 2010 do not increase savings to compensate for lower social pensions when they become pensioners as social pensions are not lower at that time. Therefore the reduction in private consumption due to the lower social pensions is reduced to persons who are pensioners in the temporary period where the pension is reduced and to older persons in the labor market who will become pensioners in a period with lower social pensions.

On the other hand the reduction in the average income tax rate induces increased labor supply and higher disposable wages. Therefore the human capital of all age-groups who are in the labor force in the period is increased. Since most of the generations face no increased life cycle motive for savings this increases private consumption of the age-groups in the labor market.

In total the composition of consumption is shifted in favor of the younger generations in the period and the net effect on total consumption is positive primarily due to the fact that the income tax is a distortionary whereas the implicit tax on the present generations of pensioners is comparable to a non distortionary tax. In total there is an efficiency gain to the economy. For the period after 2050 the evolution of the economy is similar to the base scenario.

Table 7.8 and 7.9. shows the development in the public expenditures and tax revenues associated with the present scenario where social pensions are the source of finance of the costs associated with the demographic transition. Observe that the necessary reductions in the social security pension are very large: In 2031-35 where the number of pensioners is 50 percent higher than in 1995 the total expenditures to the social pensions has to be approximatively 25 percent lower than in 1995 to keep the government budget balanced and the tax rates fixed.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Social security pension	73	74	51	75	98	99
Civil servants pension, ATP, LD	15	22	42	42	42	42
Age-dependent public service	155	162	192	189	187	188
Age-dependent public transfers	45	43	46	46	46	46
Total age-dependent public expenditures	**288**	**302**	**331**	**352**	**374**	**375**
Other expenditures	135	129	127	128	128	128
Total public expenditures	423	431	458	480	502	503
Gross domestic product (GDP)	969	1001	1004	1012	1018	1018
Total public expenditures as percent of GDP	43.7	43.0	45.7	47.4	49.3	49.4
Change in total public expenditures as percent of GDP from 1995		-0.6	2.0	3.7	5.6	5.7

Note: Average annual rates, deflated by foreign price level

Table 7.8: Public expenditures and Gross Domestic Product

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Tax on labor market pensions	5	5	16	27	34	34
Tax on civil servants pensions, ATP, LD	6	8	14	14	14	14
Tax on private pension schemes	7	5	13	12	13	13
Tax on social security pensions	21	18	13	18	24	24
Other revenues	381	395	403	409	416	417
Total revenue	**419**	**431**	**458**	**480**	**502**	**503**
Effective average wage-income tax rate	42.7	39.0	39.0	39.0	39.0	39.0

Note: Average annual rates, deflated by foreign price level

Table 7.9: Evolution in public revenues

The conclusion of this scenario is that given the fact that the introduction of the funded pension system generates an increase in the income tax base that is sufficient to finance the increased public expenditures in the long run then the effect on the macro economy of the shift in the source of finance towards reduced replacement ratio in the pay-as-you-go system is very limited.[3] However, the reductions in the social security pensions in the period from 2010 to 2040 are very large. This is partly explained by the reduction in the labor force and partly by the increase in the expenses to public services to the elderly that follows from the increased number of pensioners.

7.5 Decomposition 3: Increased labor supply

The final exogenous shock to the economy from the demographic evolution is the reduction in the labor supply. In the present section the labor supply is kept constant at the 2005 level throughout the time horizon. As in the previous cases the change is announce at the end of 2005 and enacted from 2006. The change in the total labor supply is obtained by an exogenous increase in the labor market participation ratio of all age groups of both gender. The participation ratios are increased in the same relative proportion. This scenario is not considered a policy scenario (as no policy actions are specified) but is only intended to show the marginal effect of the reduced labor supply.

The experiment increases the labor supply gradually until 2035 where it is increased

[3] This is very different from the result in Knudsen et al. (1998b) where the same type of policy is analysed in a version of the model that does not incorporate the funded pension system.

by approximately 7 per cent compared to the base scenario. From the point on the increase becomes small and converges to approximately 5 per cent in the long run. (see table 6.1).

The higher aggregate labor supply on impact generates a lower producer wages and therefore a higher level of employment. In the usual way this induces higher investments and capital stock that feeds back into an even higher employment and a higher producer wage. This increase in production and income generates a larger tax base for the income tax. Therefore the policy reaction function induces a lower average income tax rate that enforces the expansive effect on production relative to the base case. In total both employment and the capital stock is increased. Observe that also the amount of force savings is increased as the contributions to the labor market pension system increase in proportion to the wage sum.

As both current and future expected income is increased for the generations of agents in the labor market, these groups increase consumption and savings. The same is true for the pensioners - although at a smaller scale - due to the reduced income tax rate compared to the base scenario.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Private consumption	416	427	442	446	468	496
		(2.6)	(6.2)	(7.2)	(12.4)	(19.0)
Base case		(2.6)	(1.3)	(1.4)	(4.7)	(9.1)
Real GDP	830	862	872	873	877	879
		(3.8)	(5.1)	(5.3)	(5.7)	(5.9)
Base case		(3.8)	(-1.8)	(-0.4)	(-0.6)	(-0.2)
Employment, index	100.0	101.5	100.7	100.6	100.8	100.9
		(1.5)	(0.7)	(0.6)	(0.8)	(0.9)
Base case		(1.5)	(-7.2)	(-4.3)	(-4.6)	(-4.5)
Capital stock	2755	2820	2916	2930	2941	2946
		(2.4)	(5.8)	(6.3)	(6.7)	(6.9)
Base case		(2.4)	(-0.1)	(0.1)	(0.1)	(0.5)
Value of firms	2513	2528	2480	2498	2510	2528
		(0.6)	(-1.3)	(-0.6)	(-0.1)	(0.6)
Base case		(-3.3)	(-7.9)	(-6.6)	(-5.9)	(-5.5)
Foreign assets	-266	-101	51	228	470	821
Base case		-101	218	319	455	682

Note: The numbers in parantheses are the percentage change compared to the initial value

Table 7.10: Macroeconomic variables with constant labor supply

Table 7.10 shows the expansionary effects of the increased labor force. Observe that the long run level of consumption is increased by around twice as much as the permanent increase in the labor supply compared to the base scenario. This is in part generated by the permanent reduction in the income tax rate and in part by the increased forced savings due to the higher annual wage sum each year.

On the supply side table 7.7 shows that the long run increase in the capital stock is even higher than the increase in employment. Hence the capital labor ratio is slightly increased compared to the base scenario. Again this is explained by the terms of trade effect. The fact that long run domestic consumption increases more than domestic production generates a lower net output to the world market of the domestic product which turns into an increase in the relative price of the domestic product and therefore a reduction in the investment price index relative to the domestic output price so that user cost of capital is reduced in the long run.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Social security pension	73	74	107	98	100	101
Civil servants pension, ATP, LD	15	22	41	42	42	42
Age-dependent public service	155	162	184	186	184	186
Age-dependent public transfers	45	43	44	45	45	46
Total age-dependent public expenditures	**288**	**302**	**377**	**371**	**371**	**375**
Other expenditures	135	129	133	131	131	132
Total public expenditures	423	431	510	503	502	507
Gross domestic product (GDP)	969	1001	1035	1039	1054	1073
Total public expenditures as percent of GDP	43.7	43.0	49.3	48.4	47.6	47.2
Change in total public expenditures as percent of GDP from 1995		-0.6	5.6	4.7	4.0	3.5

Note: Average annual rates, deflated by foreign price level

Table 7.11: Public expenditures and Gross Domestic Product

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Tax on labor market pensions	5	5	18	29	34	32
Tax on civil servants pensions, ATP, LD	6	8	16	15	14	13
Tax on private pension schemes	7	5	13	12	13	14
Tax on social security pensions	21	18	31	26	23	22
Other revenues	381	395	432	422	418	426
Total revenue	**419**	**431**	**510**	**503**	**502**	**507**
Effective average wage-income tax rate	42.7	39.0	43.6	40.9	37.9	35.9

Note: Average annual rates, deflated by foreign price level

Table 7.12: Evolution in public revenues

In this case the tax base increases sufficiently in the long run not only to finance the permanently increased public expenditures but also to finance a 6.8 percentage point reduction in the average income tax rate compared to 1995. This is more than the double reduction compared to the base scenario. Observe also that at the peak of demographic burden in 2031-35 almost half of the necessary increase in the income tax rate is due to accumulated effects of the reduction in the labor force up until this point.

The conclusion to this scenario is that the reduction in the labor force is a major contributor to the over all demographic burden that appears in the first half of the twentyfirst century. Secondly, the combined assumption of a constant labor supply and the evolution of the funded pension system almost eliminates the ageing problem, since the possible permanent reduction in the necessary level of the income tax rate is of almost the same size as the maximum increase in the income tax rate at the peak of the demographic problem.

7.6 Decomposition 4: 1 percent decrease in the rate of interest

In this section it is assumed that the rate of interest is reduced by an unanticipated shock from 2006. The change is revealed to the economy at the end of 2005, so that foreward looking variables are adjusted in this period.

Ceteris paribus the lower rate of interest implies that furture pension receipts are reduced and therefore the increase in the future tax base due to deferred taxes is lower.

Hence this effect in isolation tends to reduce the positive long effect of the funded pension system. On the other hand the reductions in the rate of interest generate positive shocks to the economy by reducing the cost of capital and by reducing the discount rate of future income. Therefore firms increase investments on impact and accumulate a higher stock of capital in the long run. The question analyzed in the present section is whether the positive shock to the economy is sufficient to generate an increase in the tax base that outweights the reduction that follows from the lower after tax yield on pension savings. The answer is a qualified yes. In the long run the average tax rate on wage income is only slightly higher in case of a low interest rate. However, in the in the first 50 years a low interest rate implies a significantly increased fiscal pressure.

The main macoreconomic variable are displayed in Table 7.13. The reduction in the interest rate generates an immidiate increase in the level of investment that increases the capital stock rather rapidly. Compared to the base scenario this implies an increase in domestic production already from the period 2006-2010. This supply effect continues through out. One the demand side several effect counteract one another. First, the reduction in the rate of interest increases the value of the non-human capital accumulated in the households. Similarly the discounted value of future wage income is increased both due to the higher wages and employment that appears due to the increased stock of capital and the reduced discount rate. On the other hand future income from the labor market pension fund is reduced at least for older generations on the labor market who will not gain sufficiently from the increased wages and for persons who have already become pensioners. On impact there is a small positive effect on consumption compared to the base scenario. However, in the periods from 2031-35 to 2046-50 consumption is reduced compared to the base scenario. This negative effect is generated by lower pension receipts from the labor market pension fund for persons who had a major part of the contribution before the increase in the capital stock. This generates a lower tax base and therefore a higher effective average income tax rate which tends to reduce consumption of the working part of population. These two interrelted effects are sufficient to affect the macro level of consumption negatively for a substantal period, but gradually the effect is reversed by the increased pensions that follows from increased contributions due to the higher stock of capital in the economy..

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Private consumption	416	427	410	407	434	473
		(2.6)	-(1.5)	-(2.3)	(4.2)	(13.7)
Base case		(2.6)	(1.3)	(1.4)	(4.7)	(9.1)
Real GDP	830	862	838	860	869	877
		(3.8)	(1.1)	(3.6)	(4.7)	(5.7)
Base case		(3.8)	(-1.8)	(-0.4)	(-0.6)	(-0.2)
Employment, index	100.0	101.5	92.7	95.6	95.8	96.3
		(1.5)	-(7.3)	-(4.4)	-(4.2)	-(3.7)
Base case		(1.5)	(-7.2)	(-4.3)	(-4.6)	(-4.5)
Capital stock	2755	2820	3019	3069	3114	3144
		(2.4)	(9.6)	(11.4)	(13.0)	(14.1)
Base case		(2.4)	(-0.1)	(0.1)	(0.1)	(0.5)
Value of firms	2513	2796	2608	2644	2682	2705
		(11.2)	(-1.3)	(-0.6)	(-0.1)	(7.6)
Base case		(-3.3)	(-7.9)	(-6.6)	(-5.9)	(-5.5)
Foreign assets	-266	-101	-309	-151	211	717
Base case		-101	218	319	455	682

Note: The numbers in parantheses are the percentage change compared to the initial value

Table 7.13: Macroeconomic variables with lower interest rate

Table 7.14 shows the effect on public expenditures and Gross Domestic Product in this case where the rate of interest is reduced by 1 percent. Observe that expenditures are increased more rapidly than production so that total public expenditures as percent of GDP increases more than in base scenario until 2071-75 where the effect is reversed. This is due to the indexation of social security pensions and the fact that wages in the public sector follow the increased private sector wages. Furthermore, production of age dependent public service is more labor intensive than production in the private sector. Therefore the public expenditures to this activety increases more compared to the base scenario than does GDP. However in the long run this effect is reversed.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Social security pension	73	74	122	107	110	112
Civil servants pension, ATP, LD	15	22	43	43	43	43
Age-dependent public service	155	162	201	197	196	200
Age-dependent public transfers	45	43	50	50	50	51
Total age-dependent public expenditures	**288**	**302**	**416**	**397**	**399**	**406**
Other expenditures	135	129	129	128	128	129
Total public expenditures	423	431	545	525	527	534
Gross domestic product (GDP)	969	1001	1021	1027	1050	1081
Total public expenditures as percent of GDP	43.7	43.0	53.4	51.1	50.2	49.4
Change in total public expenditures as percent of GDP from 1995		-0.6	9.7	7.5	6.6	5.8

Note: Average annual rates, deflated by foreign price level

Table 7.14: Public expenditures and Gross Domestic Product

The evolution in the public expenditures implies that (compared to the base scenario) there is a need for increased revenue in the first 50 years after the reduction in the rate of interest. In the same period the tax base from pension receipts is reduced compared to the base scenario. This implies that the fiscal pressure in this period is significantly strengthened. From 1995 to 2031-35 the effective average tax rate on wage income is increased by no less than 8.4 percentage point in the present scenario compared to the 4.3 percentage point in the base scenario. In the long run this difference is reduced to 0.9 percentage points so that also the present scenario ends up having a lower long run effective average tax rate on wage income than the 1995.

Billion Dkr.	1995	2001-05	2031-35	2046-50	2071-75	New stationary state
Tax on labor market pensions	5	5	13	23	40	40
Tax on civil servants pensions, ATP, LD	6	8	20	18	16	15
Tax on private pension schemes	7	5	12	10	12	12
Tax on social security pensions	21	18	45	36	32	29
Other revenues	381	395	456	438	428	438
Total revenue	**419**	**431**	**545**	**525**	**527**	**534**
Effective average wage-income tax rate	42.7	39.0	51.1	47.9	43.2	40.5

Note: Average annual rates, deflated by foreign price level

Table 7.15: Evolution in public revenues

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